TIME-FREQUENCY DOMAIN ANALYSIS OF EXCHANGE RATE MARKET INTEGRATION IN SOUTHERN AFRICA DEVELOPMENT COMMUNITY: A HILBERT-HUANG TRANSFORM APPROACH

by

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Abstract

The desire of most African economic communities to introduce a common currency has persisted for years. As postulated by the Optimum Currency Area hypothesis, coordination of policy indicators among member countries is desirable for stable monetary union. In this regard, the integration of exchange rate markets has been studied and cited as one of the key indicators that could signal economic integration. Therefore, analysis of similarities, interdependence, and information transfer across exchange rate markets in Southern African Development Community (SADC) is a necessity to measure the extent of integration in the region. However, the intrinsic complexity of exchange rate data generation and its stylised characteristics of non-stationarity and non-linearity influence the modelling of such data in terms of the accuracy of the analysis and the embedded policy direction. In response, this thesis proposes empirical mode decomposition-based market integration analysis to address the limitations of the existing literature which fails to recognise the heterogeneity of market participants and data generation of the exchange rate in SADC.

The data employed for the thesis are the daily real exchange rates from 15 out of 16 member countries of the SADC from 3rd January, 1994 to 7th January 2019. The choice of study window and countries was based on the availability of adequate and consistent data for robust analysis and the period after South Africa, the largest economy, joined SADC. Based on the criteria, Zimbabwe was excluded from the analysis.

To achieve the purpose of this thesis, a four-step approach was used. The first step reviewed and explored the non-stationarity and non-linearity stylised facts about the data and observed that exchange series in SADC are non-stationary and non-linear. The second stage compared the performance of two Hilbert-Huang Transforms (EMD and EEMD) to decompose SADC exchange rate markets of which EEMD emerged superior. The components of the decomposed series were examined for dominance and ability to define the exchange rate trajectory in SADC. The residue of all the markets explained over 80% of the variation of the original series except Angola. The short- and long-term comovement was analysed through the analysis of the characteristics of IMFs and residues. The analysis of the IMFs and residues obtained from EEMD showed that exchange rate markets in SADC are driven by economic fundamentals and 12 out of 15 countries examined showed some level of similarity in the long-term trend.

In the third stage, EEMD-DCCA based multifrequency network was introduced to study the dynamic interdependence structure of the exchange rate markets in SADC. This was done by first decomposing all series into intrinsic mode functions using EEMD and reconstructing the series into three frequency modes: high, medium, and low frequency, and residue. The DCCA method was used to analyse the cross-correlation between the various frequencies, residues and original series. These were meant to address the non-linearity and non-stationarity in observed exchange rate data. A correlation network was formed from the cross-correlation coefficients to reveal rich information

than would have been obtained from the original series. The results showed similarities between the nature of cross-correlation between high-frequency series mimicking the original series. There was also a significant cross-correlation of long-term trends of most SADC countries' exchange rate markets.

The final stage proposed EEMD-Effective transfer entropy-based model to study exchange rate market information transmission in SADC at various frequencies. The combination of Ensemble Empirical Mode Decomposition (EEMD) and the Rényi effective transfer entropy techniques to investigate the multiscale information transfer helped quantify the directional flow of information at four frequency domains, high-, medium-, and low-frequencies, representing short-, medium-, and long-terms, respectively, in addition to the residue (fundamental feature). This revealed a significant positive information flow in the high frequency, but negative flow in the medium and low frequencies.

Based on the findings of this thesis we recommend that EEMD based method be used in the analysis of financial data that susceptible to non-linearity and non-stationary to elicit the time-frequency information. In terms of policy towards monetary formulation, we recommend a stepwise approach to monetary integration in SADC.

Keywords: Hilbert-Huang Transform; exchange rate Market integration; Empirical mode decomposition; wavelet; Fourier Transform.

Declaration

I, **Anokye Mohammed Adam,** hereby declare that the thesis for the Doctor of Philosophy (Applied Mathematics) at the University of Venda, hereby submitted by me, has not previously been submitted for a degree at this or any other university, and that it is my own work in design and execution and that all reference material contained therein has been duly acknowledged.

Anokye Mohammed Adam (Student). .... Date: ....27/09/2022....

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Acronyms

ADF	Augmented Dickey-Fuller
ANG	Angola
BOT	Botswana
CMA	Common Monetary Area
COM	Comoros
COMESA	Common Market for Eastern and Southern Africa
CWT	Continuous Wavelets Transform
DCCA	Detrended Cross-Correlation Analysis
DF	Dickey-Fuller
DRC	Democratic Republic of Congo
DWT	Discrete Wavelets Transform
EAC	East African Community
ECOWAS	Economic Community of West African States
EEMD	Ensemble Empirical Mode Decomposition
EMD	Empirical Mode Decomposition
EMH	Efficient Market Hypothesis
ERS	Elliot, Rothenberg and Stock
ESW	Eswatini
ETE	Effective Transfer Entropy
HFRQ	High Frequency Series
HHT	Hilbert–Huang Transforms
HMH	Heterogeneous Market Hypothesis
KPSS	Kwiatkowski, Phillips, Schmidt, and Shin
LES	Lesotho
LFRQ	Low Frequency Series
LM	Lagrange Multipliers
LR	Likelihood Ratio
MAD	Madagascar
MAL	Malawi
MAU	Mauritius
MFRQ	Medium Frequency Series
MOZ	Mozambique
NAM	Namibia
NN	Neural Network
NP	Ng-Perron

OCA	Optimum Currency Area
PP	Phillips-Perron
RET	Rational Expectation Theory
SA	South Africa
SADC	Southern Africa Development Community
SEY	Seychelles
SPWD	Smoothed Pseudo-Wigner Distribution ()
STFT	Short Time Fourier Transform (STFT)
TANZ	Tanzania
UNECA	United Nations Economic Commission For Africa
USD	United State Dollar
WT	Wavelet Transform
WVD	Wigner–Ville Distribution
ZAM	Zambia

Dedication

To my wife, Priscilla and Children, Kobby, Ewurama and Nana

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My heartfelt gratitude goes to the creator of the world and the maker of humanity for His blessings and intelligence showered upon me. It could be an element of ungratefulness and acclamation of undue pride should I accord myself all the sweet success of this work. Several personalities directly or indirectly contributed to the success of the work. The utmost gratitude goes to my Supervisors, Dr. Kwabena Kyei, Dr. Simiso Moyo, Prof. Ryan Gill and Dr. Emmanuel N. Gyamfi, for painstakingly guiding me throughout this thesis. May the Good Lord replenish your lost energies and time. I am also grateful to the anonymous reviewers of the published chapters for their comments which have helped improve the quality of arguments and discussions. However, any remaining errors and shortcomings in this thesis are my responsibility. The rest are my friends, Jerry Boano Danquah and Emmanuel Antwi, and special assistant -Emmanuel Asafo-Adjei. Finally, I would like to express my indebtedness to my wife and children for enduring my obsession with work during the thesis preparation period at all hours. Thank you all for your contribution.

Summary

The thesis is based on the following original manuscripts of which all have been published in accredited journals. These papers make original contribution to the modelling of financial data characterised by non-stationarity and non-linearity induced by noise from market participants. A summary of the specific contributions can be found at the main contributions section of the study in chapter seven.

Published

1. **Adam, A.M.**, Kyei, K, Moyo, S., Gill, R., & Gyamfi, E.N (2021). Similarities in Southern African Development Community (SADC) Exchange Rate Markets Structure: Evidence from the Ensemble Empirical Mode Decomposition, *Journal of African Business,* https://doi.org/10.1080/15228916.2021.1874795

2. **Adam, A.M.**, Kyei, K, Moyo, S., Gill, R., & Gyamfi, E.N (2022). Multifrequency network for SADC exchange rate markets using EEMD-based DCCA. *Journal of Economic and Finance 46* (1),145-166. https://doi.org/10.1007/s12197-021-09560-w

3. **Adam, A.M.**, Kyei, K, Moyo, S., Gill, R., & Gyamfi, E.N (2021). A New EEMD-Effective Transfer Entropy-based Methodology for Exchange Rate Market Information Transmission in SADC, *Complexity*, 2021, 3096620

The generation of financial market data has been acknowledged to result from complicated multi-processes that relate to economic factors and the characteristics of the markets. That complicates modelling of some components of the market data. This complexity of financial market questioned the validity of natural law of scale-invariance or fractals of a self-similar process. This thesis contributes to the literature in this area by proposing three approaches to examine similarity in structure, independence, and information transfer with application with exchange rate markets in SADC. The thesis is organised into seven chapters. The chapter one introduces the general background of the study. The review of empirical mode decompositions and its competing models such as Fourier transform, and variations of wavelet transforms are presented in chapter two. The review showed that empirical mode decomposition outperforms Fourier and Wavelet transforms because of its adaptiveness, ability to handle non-linear and non-stationary series.

In chapter three, the non-stationarity and non-linearity properties of exchange rate data were examined using a variety of methods. The ADF, PP and KPSS were used to test for non-stationarity. The results of these tests showed that the SADC exchange rate data are non-stationary. Tests from BDS test, NN test, Keenan and Tsay tests, TAR-LR test and Engle LM test also showed that SADC exchange rate data are non-linear.

Chapter four proposes a new way of analysing short- and long-run comovement based on the analysis of the characteristics of IMFs and residue. The performance of EMD and EEMD in the decomposing SADC exchange rate markets was assessed and found EEMD to be superior. The EEMD was then used to decompose all the exchange rate market into IMFs and residues to determine which component explains the exchange rate trajectory in SADC. The comparison of the residue showed similarity in structure in 12 out of 15 countries examined.

In Chapter Five, a multifrequency network-based EEMD-DCCA was introduced to study the dynamic interdependence structure of the exchange rates market in SADC. This was done by first decomposing all series into intrinsic mode functions using EEMD and reconstructing the series into three frequency modes: high, medium and low frequency, and residue. The DCCA method was used to analyse the cross-correlation between the various frequencies, residues and original series. These were meant to address the non-linearity and non-stationarity in observed exchange rate data. A correlation network was formed from the cross-correlation coefficients in all cases which revealed richer information than would have been obtained from the original series. It was observed that dissimilarities in exchange rate markets are driven by high-frequency series induced by speculative activities.

Chapter six introduces the EEMD-effective transfer entropy-based model to study exchange rate market information transmission in SADC at different frequencies. The Ensemble Empirical Mode Decomposition (EEMD) and the Rényi effective transfer entropy techniques employed to investigate the multi-scale information that might be disregarded, and further quantify the directional flow of information. The study reveals a significant positive information flow in the high frequency, but negative flow in the medium and low frequencies.

Finally, Chapter seven present the general conclusion in the form of contributions, recommendations, limitations and suggestions for further studies

CHAPTER 1

General Introduction

Chapter Summary

This chapter presents the general framework of the thesis. The intrinsic complexity of financial data in general and how that could affect data modelling are discussed. The relevance of such modelling in Southern African Development Community (SADC) policy towards monetary unions was elaborated. The introduction, problem statement, rationale of the study, objectives, expected contributions from the study, and the scope of the study are also discussed.

1.1 Introduction

The dynamics of African financial markets has witnessed increased investigation, in part due to its growing importance in the world economy and, more importantly, the availability of high-frequency market data. The availability of high-frequency financial market data allows for the identification of microstructures of these markets (Dacorogna et al., 2001; Nava, Di Matteo and Aste, 2016). The generation of financial market data results from complicated multiprocesses that are related to economic factors and the characteristics of the markets which makes its data series difficult to decompose it into its implicit categories: noise, cycles at different time points and trend (Di Matteo et al., 2003; Dacorogna et al., 2001).

The complexity of financial market questions the validity of natural law of scale-invariance or fractals of a self-similar process, which posits that occurrence of similar patterns at different time scales are invariant and therefore exhibit invariant probabilistic properties at different time scales (Calvet and Fisher, 2002). Nava, Di Matteo, and Aste (2016) observed that financial time series data exhibit statistical properties which are time point variant resulting from a heavy-tailed probability distribution and autocorrelation structure of the data. As noted by Muller et al. (1993), these multiscale properties reflect the behavioural market theory of heterogeneous market hypothesis (HMH). The HMH sees market participant as heterogeneous with different information, objective and varying investment horizon ranging from seconds to years, and therefore react to the market differently. As a result, financial market time series is likely to exhibit non-linearity, non-stationarity, and long memory because financial markets data series are highly mixed and noisy. Nonstationarity and nonlinearity have thus become a stylised fact of financial time series.

The non-linearity in the data generating process of financial time series data makes the use of a standard linear models inappropriate for modelling financial time series. The performance of existing tests for non-linearity in detecting diverse types of the artificially generated non-linear structure have been reviewed and critiqued by Chen et al. (2001). The complex nature of the financial time series data has increased the use of time-frequency representations recently. The frequently used representations include short time Fourier transform (STFT) (Gröchenig, 2001), Wavelet transform (WT) (Szu, Sheng & Chen, 1992), S-transform (ST) (Stockwell et al., 1996), Wigner–Ville distribution (WVD) (Chen, 2007; Lokenath, 2002), matching pursuit (Chen et al., 2007; Stéphane and Zhang, 1993), adaptive optimum kernel time-frequency representation (Liu et al., 2008) and Smoothed Pseudo Wigner distribution (SPWD) (Qiao, 2010). They are used to model the behaviour of the series via identifying which frequencies are present, the strength of the frequencies, and the variation over time (Nava, Di Matteo and Aste, 2018a; 2018b). However, these methods have been found to be deficient in handling the complex dynamics of financial data. For example, Fourier analysis and the S transform are effective in studying periodic and stationary time series whose properties are time-invariant. On the other hand, under noisy conditions, the wavelet transform is inaccurate in detecting the complete properties. The limitations of these methods are apparent in the literature (Azeemsha and Nasimudeen, 2012; Manjula and Sarma, 2012; Xiao et al., 2017) In addition, these methods require a priori-basis selection, which confounds the economic interpretation or meaning of the results of the analysis.

Apparently, the above methods have been used extensively in analysing financial market data integration in Africa despite their weaknesses discussed. This leads to inaccurate identification of hidden structures embedded in the data. Thus, this brings to the fore the validity of a conclusion drawn from such an analysis and its policy implications.

1.2 Problem Statement

Huang et al. (1996, 1998, 1999) proposed Hilbert–Huang transforms (HHT) for analysing signals characterised by non-linear and non-stationary behaviour; and it has become the focus of recent literature. HHT consists of two parts: empirical mode decomposition (EMD) and Hibert transform (HT). The uniqueness of HHT is the formation of the concept of intrinsic mode function (IMF) as the basis function of EMD (Addison et al., 2009; Macelloni et al., 2011; Yang et al., 2007; Wang et al., 2012). The EMD is a multiresolution decomposition method that decomposes non-stationary and non-linear signals into basis functions, IMFs, that are adapted from the signals themselves (Ayenu-Prah and Attoh-Okine, 2009). Then, the Hilbert transform (HT) is afterwards executed. By these approaches, the rational physical significance of the instantaneous amplitude and instantaneous frequency obtained by each component is ensured (Wang et al., 2012). Furthermore, because EMD uses an adaptive basis which is extracted from the data itself to transform, no *a priori* basis functions are defined for decomposition. These address the weakness of competing models in correctly identifying the hidden structures embedded in the time series data. Despite these strengths, the standard EMD suffers from mode-mixing, making the physical meaning of individual IMF unclear.

A class of empirical mode decompositions which are improvements to the standard empirical mode decomposition (EMD) is introduced to address the shortcoming of EMD. The ensemble empirical mode decomposition (EEMD) corrects the issue of mode mixing but introduces the problem of exact reconstruction of signals. Wei et al. (2013) observed through empirical analysis that performance in reconstructing frequencies does not follow the extension sequence. This implies that the performance of these classes depends on the behaviour of the financial time series. Although EMD and its improved versions are seen to provide a meaningful and superior understanding of time series, its application in financial time series is minimal and non-existent in exchange rate markets. The exchange rate markets provide understanding of inflation and interest rate dynamics in most African economies. In addition to providing important transmission channel for monetary policy, it is key indicator that facilitate monetary integration. The volatile nature of exchange rate markets in most African countries and the heightened interest of economic blocs in Africa toward currency union make exchange analysis timely.

The purpose of this study is to assess the performance of a class of empirical mode decomposition methods in modelling of selected exchange rate markets and comovements of the decompositions from a class of empirical mode decomposition.

1.3. Rationale of the Study

The growth in African financial market relative to the global financial market and availability of data has attracted the interest of global investors, policymakers and academics. The perceived less integrated African financial market compared to the global market offer a diversification opportunity for global investors and, therefore, a complete understanding of these markets is necessary for portfolio selection (Agyei-Ampoma, 2011; Adam and Gyamfi, 2015). Furthermore, the seemingly interest of regional blocs in the formation of currency unions in Africa requires a complete understanding of the interaction of the financial markets to aid policymakers. The exchange rate is influenced by the extent to which domestic prices adjust to exchange rate changes to provide understanding of inflation dynamics. Most especially, it provides an important transmission channel for monetary policy, in addition to the standard aggregate demand channel (Frimpong & Adam, 2010).

This study questions the ability of empirical mode decomposition to reveal the hidden patterns in different frequency modes. This has implications for the correct understanding of the behaviors of the African financial market for both policy and theoretical formulation.

1.4 The Aim of the Study

The study aims at identifying the driving force of the financial market series data generation process in Africa by applying the H-H transform to reveal hidden patterns in the African high-frequency financial markets series data. The nature of integration of the financial market series data at various time points is also investigated.

1.5 The Objective of the Study

The following broad objectives are considered.

1. Assess the performance of a class of empirical mode decompositions in decomposing Africa exchange rate series data
2. Examine the intra-SADC exchange rate series data integration using a class of empirical mode decomposition

1.6 Specific Objectives of the Study

The study addresses the following specific objectives.

1. Compare EMD and EEMD to investigate the underlying factors affecting the exchange rate markets and propose a framework for examining similarities in economic structures.
2. Develop a EEMD-cross-correlation-based network for analysis of multi-frequency comovement of exchange rate markets.
3. Introduce empirical mode decomposition based non-linear information flow method to analyse the interdependence of exchange rate markets.

1.7 Contributions from the Study

This thesis makes important contribution to literature and in practice, especially, in areas of financial markets interconnectedness modelling and implied policies. The study will provide further insight into the ongoing debate in literature about modelling asymmetric relationships, similarity, interdependence, and information transfer in financial markets. Firstly, the correct formulation of a policy thrives on the accuracy of information emanating from the analytical framework. Understanding the intrinsic characteristics of exchange rate data could inform policymakers' decision toward monetary integration. Secondly, the proposed model for examining the long term fundamental dependencies and information transfer will contribute to literature on time-frequency domain analysis in this area and could form the basis for further literature development in this area. Thirdly, the models developed could be useful in providing a clearer picture of the readiness of the SADC to form the monetary union. Lastly, the findings of this research have implications for practical implementation of the trading strategy of the exchange rate market participants in SADC.

1.8 Scope of the Thesis

The analytical framework proposed in this study are multistage instead of integrative. The accuracy of the model hinges much on the adequacy of the selected decomposition approach, which is EEMD. The independence and information transfer will be in the reconstructed series from group IMFs which could confound some details of the original series data. This was partly due to the variation of the original series explained by the individual IMFs. The data used covered daily real exchange rates from 15 out of 16 member countries of the SADC from 3 January 1994 to 7 January 2019. The SADC was selected for this study because of its level of integration and level of intra-regional community trade. According to UNCTAD (2019) economic development report, SADC recorded highest intra-regional community trade at $34.7 billion and deeper levels of integration of about 84.9 per cent. In addition, SADC has made significant strides toward currency union formation. These make SADC an ideal bloc with respect study of exchange markets connectedness. The choice of study window and countries is based on the availability of adequate and consistent data for robust analysis and the period after South Africa, the largest economy, joined SADC. Based on these criteria, Zimbabwe was excluded from the analysis.

1.9 Conclusion

In this chapter we have been introduced to the data driven in financial markets such as exchange rate due to the heterogeneity of its participants and possibly stylised fact of such data. The discussion on rationale of the study, problem statement, research objectives which include both broad and specific objectives, scope, and expected contributions of the study to the literature set the tone for the thesis.

CHAPTER 2

Literature Review

Chapter Summary

This chapter presents the review of the empirical mode decomposition and its competing models, Fourier and Wavelet transforms. The various forms of Fourier and wavelet are discussed. We finally compare the performance of EMD to Fourier and wavelet on the basis of its adaptiveness, ability to handle non-linear data, localisation of frequency, and leads to an empirical inquiry which is verifiable.

2.1. Introduction

The quest to understand the data generating structure to detect and investigate any cyclical behaviour of its generating process at different time scales has heightened the interest of the econometrics community in spectral theory (Lacobucci, 2003). This is evidenced by the increase in spectral analysis methods in financial time series studies. The behaviour of financial time series may be decomposed into three main parts: long-run, medium-run and short-run behaviours; reflecting the heterogeneous agents contributing to the generation of financial time series (Muller et al., 1993). These three parts are respectively associated with slowly evolving secular movements (the trend), a faster-oscillating part (the seasonality) and a rapidly varying, often irregular component (the noise). In the absence of testable a priori hypothesis on the data generating process, this separation is very complicated.

The use of a single spectral frequency to study the properties of such time series can be misleading. Therefore, recent empirical studies have relied on ad hoc detrending and smoothing techniques to extract the business cycle. Despite its fundamental correctness, these techniques cannot exactly decompose the series. Also, they inaccurately define the business cycle based on some required and adjustable characteristics. This has increased the number of spectral analysis tools used. Fourier and wavelet transforms are the most common for spectral analysis. Although substantial studies have used Fourier transform to analyse financial time series data, Wavelet transform has assumed importance over Fourier transform because of its attractiveness to uniquely provide a complete representation of a time series from both the time and the frequency domains. The application of wavelet analysis in the finance literature is enormous (Owusu Junior, Adam and Tweneboah, 2017; Owusu, Tweneboah and Adam, 2019). The introduction of HHT by Huang et al. (1998) as a complete adaptive time-frequency representation has come as an improvement in spectral analysis. The following sections review the commonly used transforms in the finance literature which are: Fourier, wavelet, and Hilbert-Huang transforms.

2.2 Fourier Series

A real-valued periodic function $f(t)$ can be represented by a Fourier series as:

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t), \quad 2.1$$

where $\omega_0 = \frac{2\pi}{T}$ is the fundamental frequency. The real quantities a_0, a_k and b_k are defined as

$$a_0 = \frac{2}{T}\int_0^T f(t)dt \quad 2.2$$

$$a_k = \frac{2}{T}\int_0^T f(t)\cos(k\omega_0 t)\,dt \quad 2.3$$

$$b_k = \frac{2}{T}\int_0^T f(t)\sin(k\omega_0 t)dt \quad 2.4$$

with $k = 1, 2, \dots \dots, \infty$

2.2.1 The Fourier Transform

For a function $f(t) \in L^2(R)$ of a real variable t , the Fourier transform is defined by the integral

$$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt, \quad 2.5$$

that is, for a frequency ω, the function $\hat{f}(\omega)$ represents the components of $f(t)$ at ω. If the determination of all the frequency components of $f(t)$ is possible, the original function should be reconstructed by a superposition of all these components as:

$$\hat{f}(t) = \frac{1}{2\pi} + \int_{-\infty}^{+\infty} \hat{f}(\omega)e^{+i\omega t}d\omega, \quad 2.6$$

If the variable t represents time then, $\hat{f}(\omega)$ is spectrum of t. The major weakness of Fourier transform is the inability of the Fourier spectrum to provide any time domain information. The combination of both the time and frequency localisations, and a possible solution to this is the short-time Fourier transform.

2.2.2 The Short-Time Fourier Transform

Assume that a signal $f(t)$ is stationary when seen through a window $\phi(t)$, centred at a time location b, the Fourier transform of the windowed segment $f(t)\phi(t-b)$ is the short-time Fourier transform (STFT) defined as

$$SF(\omega, b) = \int_{-\infty}^{+\infty} f(t)\phi(t-b)e^{-i2\pi\omega t}dt \quad 2.7$$

and reconstructed signal from the transform is given by

$$f(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} SF(\omega, b)\phi(t-b)e^{i2\pi\omega t}dbd\omega, \quad 2.8$$

where the window function $\phi(t)$ is allowed to be complex and must have a non-zero spectrum at $\omega = 0$, hence behaving like a low-pass filter. The window function $\phi(t)$ can be the rectangular, the Hanning, the Hamming or Gaussian window (Durak & Arikan, 2003). The length of the window function determines the time and the frequency resolution. As indicated by the Heisenberg-Gabor uncertainty principle, the shorter the window the fine the representation in time but coarse in the frequency domain, and the vice-versa for both time and frequency domains (Goswani and Chan, 2011). Consequently, wavelet analysis was introduced to improve time-frequency localization (Gossmann and Morlet, 1994).

2.3 The Wavelet Transforms

Wavelet transforms are based on group theory and square integrable representations, which allow one to unfold a signal, or a field, into both time and frequency. Wavelet transforms use analytical functions, called wavelets, which are localized in space and gives both good frequency and temporal resolutions (Farge, 1992; Ayenu-Prah and Attoh-Okine, 2010).

Mathematically, a wavelet function $\psi(t) \in L^2(\mathbb{R})$ has an average value equal to zero, $\int_{-\infty}^{+\infty} \psi(t)dt = 0$, which the admissibility condition and the square of $\psi(t)$ integrates to unity, $\int_{-\infty}^{+\infty} \psi^2(t)dt = 1$ (Vidakovic, 2009). The function $\psi(t)$ is called "mother wavelet", and generates the family of continuously translated with a factor, dilated with a scale λ, and rotated wavelets:

$\psi_{k,\lambda}(t) = \frac{1}{\sqrt{\lambda}}\psi(\frac{t-k}{\lambda})$, where $k, \lambda \in \mathbb{R}$. 2.9

This implies that the wavelet transforms of a time series evolving in time is a function of two variables, time and frequency.

The use of the translated version of the mother wavelet helps to achieve time localisation. The translated version as well as the scale version of mother wavelets, are used to measure the correlation of the time series to be analysed. The coarse features of the input time series are highlighted if the signal correlates at large scales. On the other hand, a strong correlation recorded at small scales reveals fine features of the input time series.

The wavelet differs from the STFT by the shapes of the analytical functions, as the wavelet transform uses width adapted functions, the STFT uses functions with the same width. Vidakovic (2009) noted that low-frequency wavelets are broader compared to high-frequency wavelets Two wavelet transforms are commonly discussed: continuous wavelet transform, and discrete wavelet transform

2.3.1 Continuous Wavelet Transforms (CWT)

For a function $f(t) \in L^2(\mathbb{R})$ CWT is defined as a function of two variables, k (time) and λ (scale), as:

$$W_{\psi,f}(k,\lambda) = \frac{1}{\lambda}\int_{-\infty}^{+\infty} f(s)\bar{\psi}_{k,\lambda}(s)ds \qquad 2.10$$

where the wavelet function ψ has a complex conjugate, denoted by $\bar{\psi}$. The wavelet function is dilated, as λ is increased, and when k is varied, the wavelet is translated in time. The implication is that function $W_{\psi,f}$ can be computed on the entire time-frequency plane when (k,λ) changes (Vidakovic, 2009).

The function $f(t)$ can be reconstructed from its wavelet transform when the admissibility condition $C_\psi = \int_{-\infty}^{+\infty} \frac{|\psi_\omega|^2}{\omega} d\omega < \infty$ 2.11

of Fourier transform $\psi(\omega)$ is satisfied, and the reconstruction is given by (Vidakovic, 2009):

$$f(t) = \frac{1}{C_\psi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{\lambda^2}W_{\psi,f}(k,\lambda)\bar{\psi}_{k,\lambda}(t)dkd\lambda. \qquad 2.12$$

The parameters λ and k vary continuously (with the constraint $\lambda = 0$) making the continuous wavelet transform a redundant transformation. The correlation information can be minimized by selecting discrete values for k and λ.

2.3.2 Discrete Wavelet Transform

The discrete wavelet transform (DWT) (Mallat, 1989) was obtained by discretizing the parameters a and b. In its most common form, the DWT employs a dyadic sampling with parameters λ and k based on powers of two: $\lambda = 2^j$, and $k = m2^j\ ,with\ j,m \in \mathbb{Z}$. By substituting in $\psi_{k,\lambda}(t) = \frac{1}{\sqrt{\lambda}}\psi(\frac{t-k}{\lambda})$, where $k,\lambda \in \mathbb{R}$., we obtained the dyadic wavelets:

$$\psi_{j,m}(t) = 2^{-j/2}\psi(2^{-j}t - m). \qquad 2.13$$

The DWT can be written as

$$d_{j,m} = \int_{-\infty}^{+\infty} f(t)2^{-j/2}\psi\left(2^{-j}t - m\right)dt = \langle f(t), \psi_{j,m}(t)\rangle \qquad 2.14$$

where $d_{j,m}$ are known as wavelet coefficients at level m. These coefficients are used to construct the future series.

The main drawback of the wavelet transform is that its performance depends on the explicit and *a priori* selection of the mother wavelet. This selection may influence the frequency analysis. The Hilbert-Huang transform proposed by Huang et al. (1998) was introduced as a complete adaptive method to address this weakness.

2.4 Hilbert-Huang Transform

The Hilbert-Huang transform was designed as an alternative analysis tool to analyse non-linear and non-stationary time series. The HHT consists of two components: a decomposition algorithm called empirical mode decomposition (EMD) and a spectral analysis tool called Hilbert transform (HT) (Huang et al., 1998). The HHT is a decomposition based on the local characteristics of the data and it is able to capture non-linear characteristics with respect to amplitude and frequency. These characteristics make HHT attractive and appealing in many research areas.

2.4.1 Empirical mode decomposition

EMD is a dyadic filter bank in the frequency domain (Flandrin, Rilling and Goucalves, 2004). The goal of the empirical mode decomposition is to decompose the original data (non-stationary and non-linear data) into the IMFs and the residue. The EMD is a fully data-driven decomposition method and IMFs are derived directly from the signal itself. As indicated by Huang et al. (1998). An IMF must satisfy two criteria:

1. The number of extrema and the number of zero crossings must either be equal or differ at most by one.
2. At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

The first condition forces an IMF to be a narrow-band signal with no riding waves. The second condition ensures that the instantaneous frequency will not have fluctuations arising from an asymmetric waveform (Huang *et al*,1998).

The IMFs are obtained through a process called the sifting process which uses local extrema to separate oscillations starting with the highest frequency. Given a time series $x(t), t = 1,2,3, \ldots., N$, the process decomposes it into a finite number of functions, denoted by $IMF_k(t), k = 1, 2, 3, \ldots\ldots, n$ and a residue $r_n(t)$. The residue is the non-oscillating drift of the data. If the decomposed data consist of uniform scales in the frequency space, the EMD acts as a dyadic filter and the total number of IMFs is approximately equal to $n = log_2(N)$ (Flandrin, Rilling and Goucalves, 2004). At the end of the decomposition process, the original time series can be reconstructed as:

$$x(t) = \sum_{i=1}^{n} IMF_k(t) + r_n(t). \quad 2.15$$

According to Huang et al. (1998), the EMD comprises the following steps.

1. Initialise the residue to the original time series $r_0(t) = x(t)$ and set the IMF index $k = 1$.
2. To extract the kth IMF:
    (a) initialise $h_0(t) = r_{k-1}(t)$ and the iteration counter $i = 1$;
    (b) find the local maxima and the local minima of $h_{i-1}(t)$;

(c) create the upper envelope $E_u(t)$ by interpolating between the local maxima (lower envelope $E_l(t)$ by interpolating the local minima, respectively);

(d) calculate the mean of both envelopes as $m_{i-1}(t) = \frac{E_u(t)+E_l(t)}{2}$;

(e) subtract the envelope mean from the input time series, obtaining $h_i(t) = h_{i-1}(t) - m_{i-1}(t)$;

(f) verify if $h_i(t)$ satisfies the IMFs conditions:

- If $h_i(t)$ does not satisfy the $IMF's$ conditions, increase $i = i + 1$ and repeat the shifting process from step 2b;
- If $h_i(t)$ satisfies the $IMF's$ conditions, set $IMF_k(t) = h_i(t)$ and let $r_k(t) = r_{k-1}(t) - IMF_k(t)$.

3. When the residue $r_k(t)$ is either a constant, a monotonic slope or contains only one extrema, stop the process, otherwise continue the decomposition from step 2, setting $k = k + 1$.

The standard form of EMD has a problem called mode mixing. This is defined as either a single IMF consisting of widely disparate scales, or a signal of similar scale captured in different IMFs. To overcome the problem of mode mixing, Ensemble Empirical Mode Decomposition (EEMD) method emerged as improvement of EMD. The EEMD adds a fixed percentage of white noise to the signal before decomposing it and thus solves the mode-mixing problem. For a time series $x(t)$, the EEMD includes the following steps:

a. Generate a new signal $y(t)$ by adding to $x(t)$ a randomly generated white noise with amplitude equal to certain percentage of the standard deviation of $x(t)$.
b. Apply the EMD algorithm on $y(t)$ to obtain the IMFs,
c. Repeat steps a to b for m times with white noise with different standard deviations to obtain an ensemble of IMFs $\{IMF_k^1(t), k = 1, 2, \dots, n\}$, $\{IMF_k^2(t), k = 1, 2, \dots, n\}, \dots$, $\{IMF_k^m(t), k = 1, 2, \dots, n\}$.
d. Calculate the average of IMFs $\{\overline{IMF_k(t)}, k = 1, 2, \dots, n\}$, where $\{\overline{IMF_k(t)} = 1/m \sum_i^m IMF_k^i(t)\}$.

The intuition of the process is that observed data are a combination of true time series and noise and that the ensemble means of data with different noises are closer to the true time series. Therefore, the addition of white noise as an additional step to EMD steps may help to extract the true IMF by offsetting the noise through ensemble averaging (Chen and Pan, 2016).

The choice of empirical mode decomposition is justified by the accuracy in detecting the event under noisy conditions.

2.4.2 Hilbert Transform

The Hilbert transform can then be performed on the collection of IMFs that result from the EMD process. This is the second part of the HHT algorithm. The Hilbert transform defined in the time domain is a convolution between the Hilbert transformer $^1/_{\pi t}$ and a function $f(t)$ (Allez and Bouchaud, 2011). The Hilbert transform $\hat{f}(t)$ of a function $f(t)$ is defined for all t by

$$\hat{f}(t) = \frac{P}{\pi}\int_{-\infty}^{+\infty}\frac{f(\tau)}{t-\tau}d\tau \qquad 2.16$$

when the integral exists.

It is normally not possible to calculate the Hilbert transform as an ordinary improper integral because of the pole at $\tau = t$. However, the P before the integral denotes the Cauchy principal value that expands the class of functions for which the integral defined exists.

The conclusion from the review of Fourier, wavelet and Hilbert-Huang transforms are presented in Table 2.1 below. The summary matrix shows that Hilbert-Huang transform outperforms Fourier and Wavelet transforms because of its adaptiveness, ability to handle non-linear data, localisation of frequency and empirically focused. Therefore, using Hilbert-Huang transform as a transformation technique in financial time series will improve the accuracy of the results and implied policy.

Table 2.1 Comparison of Fourier, Wavelet and Hilbert-Huang Transforms

Transform	**Fourier**	**Wavelet**	**Hilbert-Huang**
Basis	a priori	a priori	adaptive
Frequency	convolution: global, uncertainty	convolution: regional, uncertainty	differentiation: local, certainty
Presentation	energy-frequency	energy-time-frequency	energy-time-frequency
Non-linear	No	no	yes
Non-stationary	No	yes	yes
Feature Extraction	No	discrete: no, continuous: yes	yes
Theoretical Base	theory complete	theory complete	empirical

Note: Presentation energy-frequency means displays both energy and frequency whereas energy-time-frequency denotes energy, time and frequency

2.5 Conclusion

In this chapter, the literature on empirical mode decompositions including its variants were reviewed and compared to its competitors such as the Fourier transform and the wavelet transform. We assessed the performance of EMD relative to others and observed that Huang transform outperforms Fourier and Wavelet transforms. This is because Huang transform is effective for adaptiveness, handling non-linear data, localisation of frequency, and useful for most financial time series to warrant its empirical basis.

CHAPTER 3

Statistical properties of SADC Exchange rate markets

Chapter Summary

The statistical properties of financial data play key role in its modelling (Adam and Owusu Junior, 2017). As discussed in the previous chapters the financial market time series is likely to exhibit non-linearity, non-stationarity and long memory because financial markets data series is highly mixed and noisy. Non-stationarity and non-linearity have thus become a stylised fact of financial time series. These stylised facts in the data generating process of financial time series influence the appropriateness of the use of certain class of models in modelling financial time series. In this chapter, stationarity and linearity properties of exchange rate series data are explored.

3.1 Stationarity Tests

Stationarity is a term used in empirical time series econometrics. Most financial time series data are collected as discrete data over time, and every individual observation in a time series is viewed as "just happen to be" or random or stochastic. Therefore, a time series is a collection of values of random variables orderred in time. Although all the individual observations are good representatives of their populations at particular values of, to be able to generalise this representativeness to other or future times, the order of "the representativeness" in time is required to be well behaved. That is, the mean, the variance and the covariance of the time series are required to be constant over time. The covariance is allowed to vary if and only if the covariance is across two or more time periods. If the above requirements are not satisfied, the time series is called a non-stationary time series. Studies using non-stationary time series data become ad hoc studies and the results cannot be generalised to other time periods (Gujarati, 2003, p.798).

Several techniques can be employed to investigate the stationarity properties of a time series. The simplest method would be the graphical examination of the series against time and/or the autocorrelation function of the time series.

The graphical examinations of time series are normally not sufficient, and further investigation into the stationarity by a number of tests required. Several unit root test methods have been proposed to test for stationarity such as the Dickey-Fuller (DF) test, the Augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test, the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test, the Elliot, Rothenberg and Stock (ERS) point optimal test, and the Ng-Perron

(NP) test. Some of the tests are specific to a particular time series scenario and their application is limited. For example, the ERS point optimal test cannot be used if a time series has less than 50 observations. For simplicity and practicality, we discuss the ADF test, the PP test and the KPSS test which have been widely used. Both the ADF and the PP test assume a nonstationary series under the null hypothesis (unit root under the null hypothesis). The ADF test is a simpler and straightforward test, and the PP test produces more conservative test results, which tend to identify a time series as a unit root process where the ADF test fails to identify. Unlike the ADF and the PP tests, the KPSS test assumes no unit root process in the time series (stationary under the null hypothesis). We first test the individual country data series with the popular ADF test and then the PP test. Any series found to have no unit root process by the ADF test and/or the PP test will be further tested by the KPSS test. The reason for employing this procedure is that we want to be sure that those series found to be stationary by the ADF test, and the PP tests are really stationary. Thus, the ADF and the PP tests serve as screening procedures. However, the screening may still have some chance to go wrong. In order to verify the test results, we redefine the null hypothesis and then test it by the KPSS test.

We now provide a detailed discussion of the test procedures and their results. The popular procedure to test stationarity is to investigate the ρ coefficient in the following autoregressive model:

$$Y_t = \rho Y_{t-1} + \delta t + \mu_t \,, \ \forall \ -1 \le \rho \le 1, \qquad 3.1$$

where δt is an exogenous variable, such as constant and/or constant and trend, and μ_t is pure random error. If $\rho = 1$, that is, the relation between an observation at time t and observation Y at time $t-1$ is unitary, then Y_t is a unit root non-stationary stochastic process. For simplicity and in line with other coefficient tests in regressions analysis, subtract Y_{t-1} from both sides of Equation (3.1) to get:

$$Y_t - Y_{t-1} = \rho Y_{t-1} - Y_{t-1} + \delta t + \mu_t \,, \qquad 3.2$$

which can be written as:

$$\nabla Y_t = \alpha Y_{t-1} + \delta t + \mu_t \text{ ,} \qquad 3.3$$

where ∇ is the first difference operator and $\alpha = (\rho - 1)$. The null hypothesis which says that the series has a unit root is equivalent to $\alpha = 0$ which is the same as $\rho = 1$, and it is the unit root test null hypothesis of the DF test. In this regard, rejection of the null hypothesis means that the series has no unit root.

3.1.1 Augmented Dickey Fuller (ADF) test

The first unit root test to be considered is the Augmented Dickey Fuller test which is the modified version of the Dickey Fuller test (DF test) proposed by Dickey and Fuller (1979). The DF test is based on the regression equation

$$\nabla Y_t = \delta t + \rho Y_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \text{ ,} \qquad 3.4$$

The null hypothesis in the DF test is presence of unit root $H_0 : \rho = 0$ against the alternative hypothesis of no unit root $H_1 : \rho < 0$. Hence, rejection of the null hypothesis means that the series has no unit root. The Dickey Fuller test based on AR(1) model (3.4) assumes that error term ε_t follows a white noise process. The ADF test t-statistic for an AR(p+1) model coupled with a trend is developed as

$$\nabla Y_t = \rho Y_{t-1} + \delta t + \sum_{i=1}^{p} \rho_i \Delta Y_{t-i} + \varepsilon_t \text{ ,} \qquad 3.5$$

where δt is time trend.

Like the DF test, the ADF test also tests for the presence of unit root $H_0 : \rho = 0$ against the alternative hypothesis of no unit root $H_1 : \rho < 0$. Also, rejection of the null hypothesis of the ADF test t-statistic means that the series has no unit root. The ADF test with or without trend does not follow standard t-distribution, the critical values are derived by simulation.

3.1.2 Phillips and Perron (PP) test

An alternative unit root test that controls serial correlation in the error term was proposed by Phillips and Perron (1988). Unlike the ADF, The Phillips-Perron test (PP test) is based on a nonaugmented Dickey Fuller test equation that allows for autocorrelated residuals as follows

$$\nabla Y_t = \delta t + \rho Y_{t-1} + \varepsilon_t \,, \tag{3.6}$$

where ε_t ~serially correlated.

The PP test modifies the t-ratio of ρ such that the asymptotic distribution of the test statistic is not affected by the serial correlation. The PP test t-statistic is calculated as

$$t = \sqrt{\frac{r_0}{h_0}} t_\theta - \frac{(h_0 - r_0)}{2h_0\sigma} \rho\sigma \,, \tag{3.7}$$

where r_0 is the estimate of the variance of $(Y_t - Y_{t-1})$, h_0 is the estimate of the variance of ($Y_t - Y_{t-n}$). The t_θ and $\rho\sigma$ are the t-statistics and standard error of ρ respectively. Like the DF test and the ADF tests, the PP also tests for the presence of unit root $H_0 : \rho = 0$ against the alternative hypothesis of no unit root $H_1 : \rho < 0$. Therefore, rejection of the null hypothesis of the PP test t-statistic means that the series has no unit root.

3.1.3 The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test

The KPSS test (1992) reversed the null hypothesis by assuming that the series is stationary (in contrast to the ADF test with the null hypothesis of nonstationarity). In this manner, rejection of the null hypothesis of the KPSS test t-statistic means that the series has a unit root. It is a Lagrange Multiplier (LM) test based on the Ordinary Least Squares (OLS) residuals from the regression of

$$Y_t = \delta t + \mu_t \,, \tag{3.8}$$

Since δt is the exogenous term of either constant or constant and trend specification, model (3.8) is of two forms

$$Y_t = r_t + \mu_t \,, \tag{3.9}$$

where r_t is a random walk, μ_t is a stationary error, and time $t = 1,..., T$

and

$$Y_t = r_t + \beta t + \mu_t \,, \tag{3.10}$$

Where βt deterministic trend

The test statistic is:`

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{T^2 f_0}, \qquad 3.11$$

where f_0 is an estimator of the residual spectrum and $S_t = \sum_{r=1}^{t} \hat{\mu}_r$

To specify the KPSS test, similar to the PP test, there is a need to specify whether an intercept or a trend and intercept are present in the test regression. There is also a need to select the method of estimating f_0.

3.2 Testing Linearity

The second documented stylised fact about financial time series is non-linearity. This can greatly influence the choice of a model to alter the behaviour of the financial time series. It is imperative to investigate this characteristic of financial time series. The next sections review the various tests employed in this study under the general hypotheses

$$\begin{cases} H_0: The\ series\ is\ linear \\ H_1: The\ series\ is\ not\ linear \end{cases} \qquad 3.12$$

3.2.1 BDS Interdependence test

The Broock, Dechert and Scheinkman (BDS) test by Broock et al. (1996) is one of the most popular tests for non-linearity. It is rooted within chaos theory and a nonparametric test. Originally, the BDS test was developed to test for independence and identical distribution (iid), but it has shown to have power against a large number of linear and non-linear alternatives (Brock and Dechert, 1991). The BDS statistic is based on the correlation integral, which is a measure of how many times a temporal pattern appears in the data. Consider a time series $X_t, t = 1, 2, \ldots, n$, and define its m-history as $X_t^m = (x_t, x_{t-1}, \ldots, x_{t-m+1})$, the correlation integral at the embedding dimension m is

$$C_{m,T}(\epsilon) = \sum_{t<s} I_\epsilon(X_t^m, X_s^m) \left\{\frac{2}{T_m(T_{m-1})}\right\}, \qquad 3.13$$

where $T_m = T - (m-1)$ and $I_\epsilon(X_t^m, X_s^m)$ is an indicator function which equals 1 if the sup norm $||X_t^m - X_s^m|| < \epsilon$ and equals 0 otherwise. Fundamentally, $C_{m,T}(\epsilon)$ counts the number of m-histories that lie within a hypercube of size of each other. Thus, the correlation integral estimates the probability that any two m-dimensional points are within a distance of ϵ of each other, i.e.,

$$P(|X_t - X_s| < \epsilon, |X_{t-1} - X_{s-1}| < \epsilon, \dots, |X_{t-m+1} - X_{s-m+1}| < \epsilon). \qquad 3.14$$

If the X_t are iid, this probability should be equal to the following in the limiting case

$$C_{1,T}(\epsilon)^m = P(|X_t - X_s| < \epsilon)^m. \qquad 3.15$$

Broock et al. (1996) define the BDS statistic as

$$V_{m\epsilon} = \sqrt{T}\,\frac{C_{m,T}(\epsilon) - C_{1,T}(\epsilon)^m}{S_{m,T}}, \qquad 3.16$$

where $S_{m,T}$ is the standard deviation and can be estimated reliably as documented by Broock et al. (1996). Under moderate regularity conditions, the BDS statistic converges in distribution to a $N(0,1)$ distribution. The BDS test has a null hypothesis of a linear series. Therefore, rejection of the null hypothesis of the BDS test means that the series is nonlinear.

3.2.3 White (1989) and Terasvirta et al. (1993) Neural Network tests

The Neural Network (NN) Test for Neglected Non-linearity (White, 1989) is based on neural network models. The most common is the single hidden layer feedforward network where a unit input sends a vector of signals $X_i, i = 1, \dots, k$, along links (connections) that attenuate or amplify the original signals by a factor γ_{ij} (weights). The intermediate or hidden processing unit j receives the signals $X_i\gamma_{ij}, i = 1, \dots, k$ and processes them. In general, the incoming signals are summed by the hidden units, and an activation function is used to generate an output $\Phi(\tilde{X}', \gamma_j)$, where Φ is typically the logistic function and $\tilde{X} = (1, X_1, \dots, X_k)$, passed to the output layer

$$f(X, \boldsymbol{\delta}) = \beta_0 + \sum_{j=1}^{q} \beta_j\, \Phi\left(\tilde{X}'\gamma_j\right), \;\; q \in N, \qquad 3.17$$

where $\beta_0, \dots, \beta_q$ are hidden to output weights and $\boldsymbol{\delta} = (\beta_0, \dots, \beta_q, \gamma_1', \dots, \gamma_q')'$.

The NN test, in particular, uses a single hidden layer network with input-to-output links. The output o of the network is

$$o = \tilde{X}'\theta + q\sum_{i=1}^{q} \beta_j\, \Phi\left(\tilde{X}'\gamma_j\right) \qquad 3.18$$

and the null hypothesis of linearity corresponds to the optimal weights of the network being equal to zero, that is the null hypothesis of the NN test is $\beta_j = 0$ for $j = 1, 2, \dots, q$ for given q and γ_j. Hence, rejection of the null hypothesis of the NN test means that the series is nonlinear.

Operatively, the NN test can be implemented as a Lagrange multiplier test

$$\begin{cases} H_0: E(\Phi_t e_t^*) = 0 \\ H_1: E(\Phi_t e_t^*) \neq 0 \end{cases}, \qquad 3.19$$

where the element $\boldsymbol{\Phi_t} \equiv (\Phi(\tilde{X}_t'\Gamma_1, \dots, \Phi(\tilde{X}_t'\Gamma_q))$ and $\boldsymbol{\Gamma} \equiv (\Gamma_1, \dots, \Gamma_q)$ are chosen a priori, independently of $\boldsymbol{X_t'}$ and for given q. To practically carry out the test, the element e_t^* are replaced by the OLS residuals $e_t = yt - \tilde{X}'\hat{\theta}$, to obtain the test statistic

$$M_n = \left(n^{-1/2} \sum_{t=1}^{n} \Phi_t \hat{e}_t\right)' \widehat{W}_n^{-1} \left(n^{-1/2} \sum_{t=1}^{n} \Phi_t \hat{e}_t\right), \qquad 3.20$$

where $\widehat{\boldsymbol{W}}$ is a consistent estimator of $\boldsymbol{W}^* = covar(n^{-\frac{1}{2}} \sum_{t=1}^{n} \Phi_t \hat{e}_t)$. Under H_0 $M_n \xrightarrow{d} x^2(q)$. To circumvent multicollinearity of Φ_t with themselves and $\boldsymbol{X_t'}$ as well as computational issues when obtaining $\widehat{W}_n$, two practical solutions are adopted. First, the test is conducted for $q^* < q$ principal components of $\Phi_t, \Phi_t \hat{e}_t^*$. Second, to avoid having to calculate, the following equivalent test statistic is employed of $\widehat{W}_n$

$$nR^2 \xrightarrow{d} x^2(q), \qquad 3.21$$

where R^2 is the uncentred squared multiple correlation from a standard linear regression of $\hat{e}_t$ on $\Phi_t, \tilde{X}_t$.

Teräsvirta et al. (1993) demonstrated that the presence of the intercept in the power of the logistic function used as activation function affects the outcome of this test. Furthermore, he verified a loss of power because of the γ parameters being chosen at random. Building on this, Teräsvirta et al. (1993) replaced the expression q$\sum_{i=1}^{q} \beta_j \Phi(\tilde{X}'\gamma_j)$ in Eq. (3.18) with an approximation based on the Taylor expansion and derived an alternative LM test that has been shown to have better power properties.

3.2.4 Keenan's (1985) and Tsay's (1986) tests

Keenan (1985) proposed a non-linearity test for time series that uses $\hat{X}_t^2$ only and modifies the second step of the Regression Equation Specification Error Test (RESET) test to avoid multicollinearity between $\hat{X}_t^2$ and X_{t-1}. Keenan (1985) assumed that the series can be approximated (Volterra expansion) as follows

$$X_t = \mu + \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_u \, a_{t-u} + \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_{uv} \, a_{t-u} a_{t-v} \; . \qquad 3.22$$

Clearly, if $\sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_{uv} \, a_{t-u} a_{t-v}$ is zero, the approximation is linear, so Keenan's idea shares the principle of an *F* test. The process is similar to Ramsey's test in terms of steps. Firstly, select (with a selection criterion, for example Akaike Information Criterion (AIC)) the value p of the number of lags involved in the regression, then fit X_t on $(1, X_{t-1}, \dots, X_{t-p})$ to obtain the fitted values $(\hat{X}_t)$, the residuals set $(\hat{a}_t)$ and the residual sum of squares regression (SSR). Then regress $\hat{X}_t^2$ on $(1, X_{t-1}, \dots, X_{t-p})$ to obtain the residuals set $(\hat{\zeta}_t)$. Finally calculate

$$\hat{\eta}_t = \frac{\sum_{t=p+1}^{n} \hat{a}_t \hat{\zeta}_t}{\sum_{t=p+1}^{n} \hat{\zeta}_t^{\,2}}. \qquad 3.23$$

The test statistic equals

$$\hat{F} = \frac{(n-2p-2)\hat{\eta}^2}{(SSR-\hat{\eta}^2)}. \quad 3.24$$

Under the null hypothesis of linearity, i.e.,

$$H_0: \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_{uv}\, a_{t-u} a_{t-v} = 0 \quad 3.25$$

and the assumption that (a_t) are independent and identically distributed Gaussian, asymptotically $\hat{F} \sim F_{1,n-2p-2}$. In this regard, rejection of the null hypothesis of the Keenan (1985) test means that the series is nonlinear.

Tsay (1986) improved on the power of the Keenan (1985) test by allowing for disaggregated non-linear variables (all cross products $X_{t-i}X_{t-j}, i,j = 1, \ldots, p$) thus generalising Keenan test by looking for quadratic serial dependence in the data. While the first step of Keenan test is unchanged, in the second step of Tsay test, instead of $(\hat{X}_t)^2$, the products $X_{t-i}X_{t-j}, i,j = 1, \ldots, p$ are regressed on $(1, X_{t-1}, \ldots, X_{t-p})$. Hence, the corresponding test statistic $\hat{F}$ is asymptotically distributed as $F_{m,n-m-p-1}$,, where $m = p(p-1)/2$, but with the same decision decision rule as the Keenan (1985) test.

3.2.5 TAR-Likelihood Ratio test

For discerning a specific subset of the self-exciting Threshold Autoregressive (TAR) models, Chan and Tong (1986) offer a likelihood ratio (LR) test, i.e., $\mathrm{TAR}(2, \mathrm{p}, \mathrm{p})$ from linear AR models when p, and d are known (or assumed). Using the identical notation as in the preceding section, $H_0: X_t \sim AR(p)$, is tested against H_1:

$$X_t = \begin{cases} \phi_{1,0} + \sum_{i=1}^{p} \phi_{1,i} X_{t-i} + a_{1,t} & if\ X_{t-d} < r \\ \phi_{2,0} + \sum_{i=1}^{p} \phi_{2,i} X_{t-i} + a_{2,t} & if\ X_{t-d} \geq r \end{cases}, \quad 3.26$$

where r is the threshold. Assuming that a_t is iid independent of $X_s, S < t$, the Chan and Tong LR test is given by

$$LR_1 = \left\{\frac{\sigma^2(NL,r)}{\sigma^2}\right\}^{\frac{n-p+1}{2}}, \quad 3.27$$

where $\sigma^2(NL,r)$ and σ^2 are the respective estimators of the error variance from $TAR(2,p,p)$ and $AR(p)$ models. Under the null hypothesis of linearity, the AR coefficients in the TAR regimes will not significantly be different, i.e., $H_0: \phi_i^1 = \phi_i^2$ $(\mathrm{i} = 0,1, \ldots, p)$, and $-2\log\,(LR_1)$ is asymptotically distributed as x_{p+1}^2. In practice, r is generally unknown and needs to be estimated. The LR test becomes

$$LR_2 = \left\{\frac{\sigma^2(NL)}{\sigma^2}\right\}^{\frac{n-p+1}{2}}. \quad 3.28$$

As a consequence, the likelihood function is irregular, and the asymptotic distribution of the statistics is no longer x^2. However, Chan and Tong (1986) provide a numerical estimate of the likelihood function and a likelihood ratio test based on it. Theoretical results allow tabulation of the asymptotic null distribution of LR_2 (see Moeanaddin and Tong, 1988; Chan and Tong, 1990).

3.2.6 Engle (1982) LM test

Engle (1982) developed the Lagrange multipliers (LM) test of test for ARCH effects, mostly because to its computational simplicity as the LM test only requires estimation of the linear model. It is analogous to the F statistic in the regression of the squared residuals from the fit of a linear model on the lagged (up to m) values of the same squared residuals to test for the null hypothesis of the coefficients in eqn 3.29 which indicates that the coefficients are not substantially different from zero.

$$\hat{a}_t^2 = a_0 + a_1\hat{a}_{t-1}^2 + \cdots + \epsilon_t, \qquad t = m+1, \dots, n \qquad 3.29$$

Once the quantities $SSR_0 = \sum_{t=m+1}^{n}(a_t^2 - \bar{a})^2$ and $SSR_1 = \sum_{t=m+1}^{n}\hat{\epsilon}^2$ are computed, the F statistic is given by eqn 3.30

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(n-2m-1)}, \qquad 3.30$$

which is asymptotically distributed as X_m. Consider that, as it is an LM test, it is possible to resort to nR^2 that asymptotically has the same distribution F.

3.3 Data

The data employed for the study is daily real exchange rates series of 15 out of 16 member countries of the SADC from 3rd January, 1994 to 7th January 2019, obtained from Thomson Reuters DataStream. Daily local currency per USD for Angola, Comoros, Botswana, Democratic Republic of Congo, Eswatini (formerly Swaziland), Lesotho, Madagascar, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Tanzania and Zambia were used. The choice of the study window and countries was informed by the availability of data and the period South Africa joined SADC. Based on these inclusion criteria, Zimbabwe was excluded from the analysis. The real exchange rate (Rex) used was nominal domestic currency per US dollar (USD) multiplied by the domestic consumer price index divided by the US consumer price index (US CPI), i.e.

$$Rex = \frac{Domestic\ currency}{US\ Dollar} \cdot \frac{Domestic\ CPI}{US\ CPI}. \qquad 3.31$$

The choice of USD for this analysis is justified by the dominance of USD in international trade by these countries and the extent of dollarisation of most SADC countries. In spite of recent de-dollarisation in Angola, Mozambique and Zambia, the dollar remains dominant in international trade globally including SADC and provides a means of standardising units of pairs of currencies (Corrales et al., 2016).

3.4 Results of the Unit Root tests for Stationarity

Table 3.1 presents the results of the stationarity tests. The outcomes of the stationarity tests – Philip Peron (PP), Augmented Dicky-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) are presented. It can be seen in Table 3.1 that almost all the conclusions from the ADF, the PP and the KPSS that the series are not stationary at levels. This is contrary to the conclusions from the ADF, the PP and the KPSS at first difference, indicating that the series are stationary.

Table 3.1: Results of Unit root tests for stationarity of exchange rates series of SADC

Country	At Level			At First Difference		
	with constant	with constant & trend	without constant & trend	with constant	with constant & trend	without constant & trend
Philip Peron (PP)						
Ang	-15.6059***	-15.7253***	-15.5583***	-173.3063***	-173.1426***	-173.3355***
Bot	-0.7001	-2.5896	1.3528	-77.717***	-77.7097***	-77.4364***
Com	-2.1903	-1.8711	0.1683	-72.8918***	-72.875***	-72.8961***
DRC	0.1166	-1.3081	3.1359	-78.2157***	-78.2424***	-78.1626***
Esw	-0.9914	-1.9254	0.6512	-70.0845***	-70.0832***	-70.0829***
Les	-0.9175	-1.9469	0.5730	-69.3417***	-69.3418***	-69.3192***
Mad	-0.0902	-2.3154	1.7923	-100.0044***	-100.0246***	-99.7446***
Mal	0.8217	-1.2986	2.6497	-85.9553***	-85.675***	-86.5336***
Mau	-2.1124	-2.985	0.6906	-104.5472***	-104.5406***	-104.4732***
Moz	0.3627	-0.8981	2.3315	-94.9533***	-95.0093***	-94.4992***
Nam	-0.9175	-1.9469	0.5730	-69.3417***	-69.3418***	-69.3192***
Sey	-0.9175	-1.9469	0.5730	-69.3417***	-69.3418***	-69.3192***
SA	-1.3897	-2.5673	0.2915	-94.6826***	-94.6733***	-94.6468***
Tanz	0.1504	-2.1539	3.1748	-85.5764***	-85.6723***	-83.7848***
Zam	0.0613	-1.4376	1.4814	-64.0479***	-64.0542***	-64.0109***
Augmented Dicky-Fuller (ADF)						
Ang	-9.68***	-9.9009***	-9.605***	-26.8917***	-26.8888***	-26.8945***
Bot	-0.7257	-2.3064	1.3216	-77.4437***	-77.4364***	-77.416***
Com	-2.129	-1.829	0.1776	-72.8498***	-72.8653***	-72.8543***
DRC	0.1728	-1.2022	3.2975	-54.1718***	-54.1719***	-53.9776***
Esw	-1.0067	-1.9656	0.6512	-70.0842***	-70.0832***	-70.0826***
Les	-0.9764	-1.9955	0.5199	-69.2846***	-69.2841***	-69.2822***
Mad	0.1278	-2.0174	2.0942	-49.676***	-49.6842***	-49.61***
Mal	0.7415	-1.3372	2.5012	-16.1313***	-16.2073***	-15.9016***
Mau	-2.0132	-2.7375	0.6394	-22.6497***	-22.6479***	-22.636***
Moz	0.4176	-0.8074	2.4452	-31.0641***	-31.0847***	-30.9473***
Nam	-0.9764	-1.9955	0.5199	-69.2846***	-69.2841***	-69.2822***
Sey	-0.9764	-1.9955	0.5199	-69.2846***	-69.2841***	-69.2822***
SA	-1.3736	-2.4655	0.3083	-59.2319***	-59.226***	-59.2277***
Tanz	0.1665	-1.8867	3.1593	-17.7017***	-17.7142***	-17.3821***
Zam	0.0599	-1.4424	1.4704	-64.0034***	-64.0164***	-63.9746***
Kwiatkowski-Phillips-Schmidt-Shin (KPSS)						
Ang	0.7491***	0.2828***	N/A	0.0257	0.0258	N/A
Bot	7.6516***	0.9922***	N/A	0.0578	0.0452	N/A
Com	2.3178***	1.7544***	N/A	0.2162	0.0352	N/A
DRC	7.4896***	0.4155***	N/A	0.1565	0.1285	N/A
Esw	5.0280***	1.3384***	N/A	0.1011	0.0444	N/A
Les	5.1245***	1.4219***	N/A	0.1091	0.0420	N/A
Mad	7.5338***	0.6975***	N/A	0.1034	0.0382	N/A
Mal	7.1360***	1.9163***	N/A	0.4164*	0.0867	N/A
Mau	4.6719***	0.4521***	N/A	0.0430	0.0432	N/A
Moz	5.7232***	1.1159***	N/A	0.2863	0.1319	N/A
Nam	5.1245***	1.4219***	N/A	0.1091	0.0420	N/A
Sey	5.1245***	1.4219***	N/A	0.1091	0.0420	N/A
SA	7.1517***	0.7332***	N/A	0.0484	0.0454	N/A
Tanz	8.0756***	1.2200***	N/A	0.1059	0.0443	N/A

Zam	6.0056***	1.5316***	N/A	0.1903	0.0429	N/A

Note: (*) Significant at the 10%; (**) Significant at the 5%; (***) Significant at the 1%.

3.5 Results of the Non-linearity Test

Table 3.2 shows the results of the univariate non-linearity tests – Teraesvirta's Neural Network test, White Neural Network test, Keenan's one-degree test for non-linearity, Tsay's Test for non-linearity and the Likelihood ratio test for threshold non-linearity. The non-linearity tests indicate that most of the country exchange rate series exhibit non-linear relationships at varying levels of significance.

Table 3.*2*: Results of nonlinearity tests for exchange rates series of SADC

Returns	Teraesvirta's Neural Network test	White Neural Network test	Keenan's one-degree test for non-linearity	Tsay's Test for non-linearity	Likelihood ratio test for threshold non-linearity
Ang	228.45***	74.896***	10.291**	41.610***	29.375***
Bot	2.151	2.278	46.189***	25.520***	63.569***
Com	5.182*	3.952	4.038**	5.374***	86.450***
DRC	3.861	3.791	18.714***	0.695	3.190
Esw	5.064*	3.706	1.204	0.005	6.419
Les	5.064*	3.742	1.204	0.005	6.419
Mad	30.408***	19.918***	0.249	9.123***	116.923***
Mal	5.576*	5.763*	1.029	49.75***	91.950***
Mau	4.846*	4.349	6.131**	3.995***	161.598***
Moz	11.211***	12.156***	0.661	80.380	147.825***
Nam	5.064*	4.411	1.204	0.005	6.419
Sey	40.534***	45.139***	2.572	18.350***	303.000***
SA	5.064*	4.309	1.204	0.005	6.419
Tanz	0.788	1.389	0.878	16.200***	36.161***
Zam	11.691***	10.291**	0.328	1.431	13.159**

[*], [**], and [***] indicate significance at 10%, 5% and 1% levels respectively for the logged series.

The BDS test in Table 3.3 is shown to assess the independent and identically distributed assumption of time series (Adam and Owusu Junior, 2017). This test further detects non-linear structures within the observations. The result in Table 3.3 reports the BDS statistic for embedding dimensions 2 and 3 for epsilon values from 1 to 4. The BDS test in Table 3.3 strongly rejects the null hypothesis of independent and identically distributed exchange rates fluctuations at 1% significance level.

Consequently, the nonstationarity and nonlinearity of the exchange rate in SADC require that they be modelled with techniques that capture the influence of noise and are able to deal with non-stationarity (Chevallier, 2011; Owusu Junior et al., 2021; Enayayi Taebi et al., 2021) to ensure reliable estimates for policy decision making. In response to this, the current study proposes empirical based models to study similarities, interdependency and information transfer with SADC exchange rate markets.

Table 3.3: Results of BDS test for nonlinearity of the exchange rate series of SADC

m	eps	statistic	eps	statistic	eps	statistic	eps	statistic
				Ang				
2	1	105.519***	2	97.067***	3	89.007***	4	82.764***
3	1	119.990***	2	97.414***	3	85.232***	4	76.943***
				Bot				
2	1	1252.868***	2	575.875***	3	342.600***	4	263.050***
3	1	2549.866***	2	772.030***	3	377.230***	4	259.091***
				Com				
2	1	435.935***	2	220.095***	3	175.203***	4	169.169***
3	1	797.863***	2	274.234***	3	185.893***	4	166.465***
				DRC				
2	1	195.753***	2	102.398***	3	80.340***	4	88.133***
3	1	256.345***	2	102.698***	3	77.676***	4	84.605***
				Esw				
2	1	559.246***	2	475.830***	3	304.879***	4	221.935***
3	1	1042.189***	2	617.942***	3	335.802***	4	221.560***
				Les				
2	1	559.246***	2	475.830***	3	304.879***	4	221.935***
3	1	1042.189***	2	617.942***	3	335.802***	4	221.560***
				Mad				
2	1	608.311***	2	374.349***	3	321.095***	4	250.981***
3	1	1111.481***	2	535.218***	3	353.210***	4	242.341***
				Mal				
2	1	340.778***	2	425.453***	3	289.690***	4	213.801***
3	1	592.472***	2	549.492***	3	321.653***	4	220.149***
				Mau				
2	1	444.523***	2	271.102***	3	221.156***	4	229.333***
3	1	850.495***	2	350.546***	3	242.244***	4	226.909***
				Moz				
2	1	223.238***	2	151.076***	3	141.742***	4	148.885***
3	1	357.106***	2	178.072***	3	150.391***	4	148.383***
				Nam				
2	1	559.246***	2	475.830***	3	304.879***	4	221.935***
3	1	1042.189***	2	617.942***	3	335.802***	4	221.560***
				Sey				
2	1	518.111***	2	1881.823***	3	511.165***	4	276.466***
3	1	764.911***	2	2500.096***	3	622.854***	4	292.112***
				SA				
2	1	559.246***	2	475.830***	3	304.879***	4	221.935***
3	1	1042.189***	2	617.942***	3	335.802***	4	221.560***
				Tanz				
2	1	1176.602***	2	398.221***	3	283.229***	4	247.950***
3	1	2472.961***	2	539.039***	3	307.114***	4	244.652***
				Zam				
2	1	254.629***	2	185.351***	3	161.765***	4	169.514***
3	1	413.925***	2	222.674***	3	174.451***	4	170.546***

Note: m and eps denote the embedding dimension and epsilon respectively. *[*], [**], and [***] indicate significance at 10%, 5% and 1% levels respectively.*

3.6 Conclusion

The chapter reviewed ADF, PP and KPSS as tools for examining stationarity property of time series, BDS test, NN test, Keenan and Tsay tests, TAR-LR test and Engle LM test as non-linear test tool. The stylised facts of non-stationarity and non-linearity of exchange rate data were examined. The results of these tests showed that SADC exchange rate data are non-stationary and non-linear.

CHAPTER 4

Similarities in Southern African Development Community (SADC) Exchange Rate Markets Structure: Evidence from the Ensemble Empirical Mode Decomposition

Chapter Summary

The need for exchange markets coordination in Africa is rooted in the quest of most economic blocs to form a monetary union characterised by a single currency and has therefore attracted the attention of researchers. The intrinsic complexity of the exchange rate market hinders researchers from producing consistently reliable results. The empirical mode decomposition (EMD) is a data-driven signal analysis method for non-linear and non-stationary data. The empirical mode decomposition method can be used to divide non-linear signal sequences into a group of well-behaved intrinsic mode functions (IMFs) and a residue, so that we can compare the similarities. In this chapter, EMD and ensemble empirical mode decomposition (EEMD), a modified version of EMD, are applied to the exchange rate series of the Southern African Development Community (SADC). By analysing the intrinsic mode functions (IMFs) of the EMD and the EEMD, we find that the EEMD method performs better on the orthogonality of IMFs than the EMD. We propose a new way of analysing short and long-run comovement through the analysis of the characteristics of IMFs and residue. The analysis of the IMFs and residue obtained from EEMD show that the exchange rate series of the SADC are driven by economic fundamentals, and 12 of the 15 countries examined show some level of similarity in the long-term trend. Our findings have implications for the direction of future SADC monetary union.

4.1 Introduction

The agenda for economic integration by African countries was enshrined in Article II of the disbanded Organization for African Unity Charter and Article 3 of the Africa Union Constitution. The ultimate goal was to introduce a common currency by the year 2021. Integration was seen as a development tool, in particular, for harnessing resources and capabilities toward industrial policy. To fast track the integration process, a two-way approach was adopted where each economic community had been encouraged to form a monetary union with a single currency which will eventually be brought together to form an economic union for the entire continent (Alagidede, Tweneboah and Adam, 2008).

Accordingly, most economic communities in Africa such as the Economic Community of West African States (ECOWAS), the Common Market for Eastern and Southern Africa (COMESA), the Southern African Development Community (SADC), and the East African Community (EAC) have embraced the common currency agenda and vigorously pursuing it (UNECA, 2011). The potential benefits to member states of such union have been extensively studied (Mwenda and Muuka, 2001; Misati, Ighodaro, Were and Omiti, 2015). As postulated by the Optimum Currency Area hypothesis by Mundell (1961), McKinnon (1963) and Kenen (1969), coordination of policy indicators among member countries is desirable for stable monetary union. In this regard, exchange rate markets integration has been studied and cited as a key indicator for stable monetary union and therefore accurate analysis of exchange rate markets integration in Africa is a necessity (Adam, Agyapong and Gyamfi, 2010; Musila and Al-Zyoud, 2012; Coulibaly and Gnimassoun, 2013; Zehirun, Breitenbach and Kemegue, 2015; Zehirun, Breitenbach and Kemegue, 2016).

The implications of exchange rate coordination on the possible monetary union in SADC, the largest regional economic grouping in Africa, have been studied (Khamfula and Huizinga, 2004; Agbeyegbe, 2009; Zehirun, Breitenbach and Kemegue, 2015; Zehirun, Breitenbach and Kemegue, 2016). The findings and recommendations of these studies have been mixed (see Asongu, Nwachukwu and Tchamyou, 2015). From the perspective of the heterogeneous market hypothesis (HMH), the exchange rate market is made up of heterogeneous participants (speculators, central banks, dealers, individuals, etc.) with different information, objective, interest and investment behaviour. Hence, the physical measurement of exchange rate data suffers from noise, too short span, non-linearity, non-stationarity, and long memory (Xu et al., 2016). These characteristics make observed exchange rate data difficult to model, limit its usage in research and practice (Muller et al., 1993; Ferreira, Moore and Mukherjee, 2019; Owusu Junior, Adam and Tweneboah, 2019).

The empirical mode decomposition (EMD) proposed by Huang et al. (1998) for analysing signals characterised by non-linear and non-stationary behaviour presents itself as an improved tool for analysing these types of time series. The EMD is a multiresolution decomposition method that decomposes non-stationary and non-linear signals into basis functions, IMFs, that are adapted from the signals themselves (Ayenu-Prah and Attoh-Okine, 2009). The innovation of EMD is the formation of the concept of intrinsic mode function (IMF) as the basis function of EMD (Addison et al., 2009; Macelloni et al., 2011; Yang et al., 2007; Wang et al., 2012; Hassan and Haque, 2016). The EMD uses adaptive basis which is extracted from the data itself, no *a priori* basis functions are defined for decomposition. The EMD overcomes the weakness of the competing model to correctly identify the hidden structures embedded in the data structure. In spite of these strengths, the EMD suffers from mode-mixing, a single intrinsic mode function (IMF) is either comprised signals of widely disparate scales or a signal of a similar scale residing in different IMF components, making physical meaning of individual IMF unclear (Hassan and Subasi, 2017; Hassan and Bhuiyan, 2016a; 2016b). To solve the problem of mode-mixing, noise-assisted extensions were developed. The ensemble empirical mode decomposition (EEMD) proposed by Wu and Huang (2009) corrects the issue of mode-mixing. Although EMD and EEMD are seen to provide a meaningful and superior understanding of time series, their application in exchange rates is minimal and non-existent in SADC exchange rate markets. Identification of the underlying characteristics of exchange rate price formation in each SADC country could reveal drivers of exchange rate market convergence or otherwise within the region and aid in policies toward monetary union. Therefore, to accurately understand SADC exchange rate markets fundamental behaviour and similarities, they must be decomposed into meaningful components of noise, trend and cycles at different timescale. This approach will improve the financial time series integration and comovement over detrend fluctuation analysis (DFA) (Stošić, Stošić, Stošić, and Stanley 2015; Ferreira, da Silva and de Santana 2019) and flavours of wavelet transform (Owusu Junior, Adam and Tweneboah, 2017; Meng and Huang, 2019) employed in the literature. Consequently, EMD has been used in time series modelling in recent literature (Hassan and Bhuiyan, 2017; Xian, He, Wang and Lai, 2020).

In this chapter, we propose an innovative way of assessing the similarities in the structure of SADC exchange rate data series by studying the underlying characteristics through descriptive statistics of the IMFs and residue from the discomposed series. We initially compared the performance of EMD and EEMD in obtaining well-behaved IMFs from the decomposition of exchange rates series in each of the SADC countries for further analysis. The study contributes to the literature by comparing the performance of EMD and EEMD in decomposing exchange rate data for the first time and studying the underlying characteristics of exchange rates in SADC using the descriptive statistics of the IMFs and residue. This improves the analysis compared to traditional methods like cointegration and reveals comovement structure at different timescales.

We observed through analysing the intrinsic mode functions (IMFs) of EMD and EEMD that EEMD method performs better on the orthogonality of IMFs than EMD. The analysis of the IMFs and residue obtained from EEMD showed that exchange rate markets in SADC are driven by economic fundamentals and 12 out of 15 countries examined showed some level of similarity in the long-term trend. Our findings have implications for the direction of future SADC monetary union.

The rest of the chapter is structured as follows. Section 2 introduces the methods employed in the study and section 3 describes the exchange rate data of SADC used in the study. Section 4 presents the analysis and the results, and section 5 provides the conclusions and recommendations.

4.2 Empirical mode decomposition methodology

EMD is a dyadic filter bank in the frequency domain (Flandrin, Rilling and Goucalves, 2004). The goal of the empirical mode decomposition is to decompose the original data (non-stationary and non-linear data) to the IMFs and the residue. The EMD is a fully data-driven decomposition method and IMFs are derived directly from the signal itself. As indicated by Huang et al. (1998), an IMF must satisfy two criteria:

1. The number of extrema and the number of zero crossings must either be equal or differ at most by one.
2. At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

The first condition forces an IMF to be a narrow-band signal with no riding waves. The second condition ensures that the instantaneous frequency will not have fluctuations arising from an asymmetric waveform (Huang et al.,1998).

The IMFs are obtained through a process called a sifting process which uses local extrema to separate oscillations starting with the highest frequency. Given a time series $x(t), t = 1,2,3, \dots.., M$, the process decomposes it into a finite number of functions, denoted as $IMF_k(t), k = 1, 2, 3, \dots\dots, n$ and a residue $r_n(t)$. The residue is the non-oscillating drift of the data. If the decomposed data consist of uniform scales in the frequency space, the EMD acts as a dyadic filter and the total number of IMFs is approximately equal to $n = log_2(M)$ (Flandrin, Rilling and Goucalves, 2004). At the end of the decomposition process, the original time series can be reconstructed as

$$x(t) = \sum_{i=1}^{n} IMF_k(t) + r_n(t). \qquad 4.1$$

According to Huang et al. (1998), the EMD comprises the following steps:

1. Initialise the residue to the original time series $r_0(t) = x(t)$ and set the IMF index $k = 1$.
2. To extract the kth IMF:
   (a) initialise $h_0(t) = r_{k-1}(t)$ and the iteration counter $i = 1$;
   (b) find the local maxima and the local minima of $h_{i-1}(t)$;

(c) create the upper envelope $E_u(t)$ by interpolating between the local maxima (lower envelope $E_l(t)$ by interpolating the local minima, respectively);

(d) calculate the mean of both envelopes as;

$$m_{i-1}(t) = \frac{E_u(t)+E_l(t)}{2} \qquad 4.2$$

(e) subtract the envelope mean from the input time series, obtaining;

$$h_i(t) = h_{i-1}(t) - m_{i-1}(t) \qquad 4.3$$

(f) verify if $h_i(t)$ satisfies the IMFs conditions:

- If $h_i(t)$ does not satisfy the $IMF's$ conditions, increase $i = i + 1$ and repeat the shifting process from step b.
- If $h_i(t)$ satisfies the $IMF's$ conditions, set $IMF_k(t) = h_i$ and define

$$r_k(t) = r_{k-1}(t) - IMF_k(t). \qquad 4.5$$

3. When the residue $r_k(t)$ is either a constant, a monotonic slope or contains only one extremum, stop the process, otherwise continue the decomposition from step 2, setting $k = k + 1$.

The standard form of EMD has a problem called mode mixing. This is defined as either a single IMF consisting of widely disparate scales, or a signal of similar scale captured in different IMFs. To overcome mode mixing, two noise assisted methods have emerged as improvements of EMD. Ensemble Empirical Mode Decomposition (EEMD) adds a fixed percentage of white noise to the signal before decomposing it and thus improves the mode-mixing problem. For time series, $x(t)$, the EEMD includes the following steps:

a. Generate a new signal of $y(t)$ by superposing to $x(t)$ a randomly generated white noise with an amplitude equal to a certain ratio of the standard deviation of $x(t)$.
b. Perform the EMD algorithm on $y(t)$ to obtain the IMFs.
c. Repeat steps 1 to 2 for m times with different white noise to obtain an ensemble of IMFs $\{IMF_k^1(t), k = 1,2,\dots,n\}$, $\{IMF_k^2(t), k = 1,2,\dots,n\}$,…, $\{IMF_k^m(t), k = 1,2,\dots,n\}$.
d. Calculate the average of IMFs $\{\overline{IMF_k(t)}, k = 1,2,\dots,n\}$, where $\overline{IMF_k(t)} = 1/m \sum_i^m IMF_k^i(t)$.

The importance of the process is that the observed data are a combination of true time series and noise and that the ensemble means of data with different noises are closer to true time series. Therefore, the addition of white noise as an additional step to the EMD steps may help to extract the true IMF by offsetting the noise through ensemble averaging (Chen and Pan, 2016). Though the empirical mode decomposition has been questioned about the exactness reconstruction (Hassan and Haque, 2015; Hassan and Subasi, 2017), it has some attractive properties which make it preferred in decomposing signals. First, it has the ability to decompose non-stationary and non-linear data into simple independent IMFs, making it attractive. Again, since the decomposition is based on the local characteristic time scale of the data and only extrema are used in the sifting process, it is local, self-adaptive, concretely implicational and highly efficient (Huang et al., 1998; Zhang et al., 2008).

4.3 Data Description

The data employed for the study is daily real exchange rates from 15 out of 16 member countries of the SADC from 3rd January, 1994 to 7th January 2019, obtained from Thomson Reuters DataStream. The countries included are Angola, Botswana, Comoros, Democratic Republic of Congo, Eswatini (formerly Swaziland), Lesotho, Madagascar, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Tanzania and Zambia. The choice of the study window and countries was based on the availability of adequate, consistent data for robust analysis and the period after which South Africa, the largest economy, joined SADC. Based on the criteria, Zimbabwe was excluded from the analysis. The real exchange rate used was nominal domestic currency per the US Dollar (USD) multiplied by the domestic consumer price index divided by the US consumer price index. The USD is chosen for this analysis because of the dominance of USD in international trade by these countries and the extent of dollarisation in most SADC countries. Figure 4.1 shows the plot log of daily national currency to US dollar from 3rd January, 1994 to 7th January 2019.

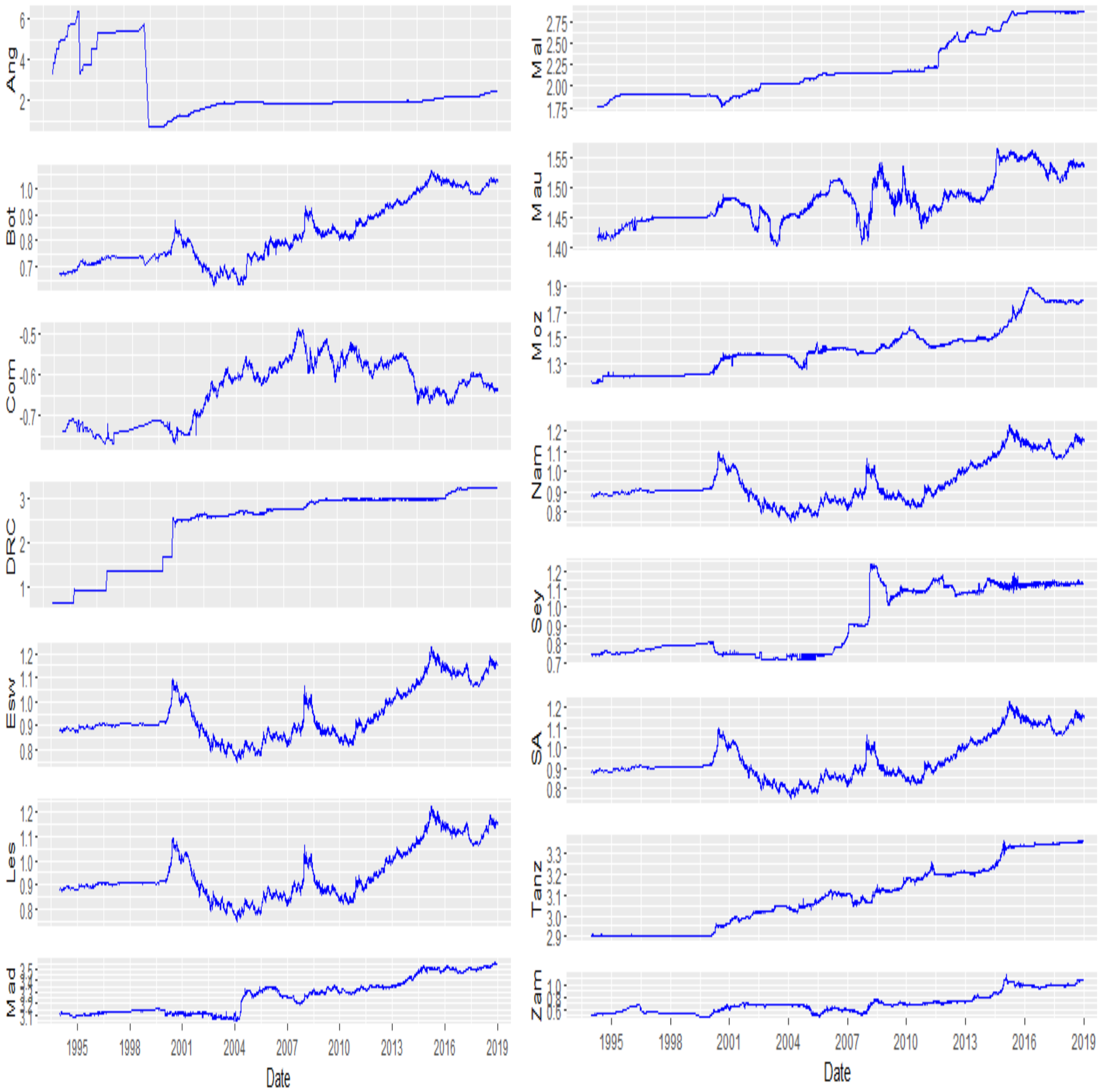


Figure 4.1: Plot of Log of National Currency to US dollar Exchange Rate of 15 SADC Countries

4.4 Results and Analysis

We begin the analysis by first applying the EMD and EEMD methods to the daily real domestic currency/USD exchange rate to obtain individual 11 IMFs for each country's exchange rate for both methods. The corresponding exchange rates obtained through EMD and EEMD look similar, but not identical. Figures 4.2 and 4.3 respectively display the IMFs obtained by EMD and EEMD for the case of South Africa. The figures show an indication of the difference in the behaviour of IMFs concerning EMD and EEMD. We followed Xu et al. (2016) to first assess the orthogonality of IMFs from both EMD and EEMD by computing the correlation between any two different IMFs and presented, for example, those of Botswana in Figures 4.4 and 4.5

respectively. We observed from Figure 4.4 that IMF8-IMF11 from EMD correlation coefficients are the largest compared to the rest in the figure. The plots for all other countries are presented in Figure 4.6 as supplementary plots for EMD. On the contrary, EEMD obtained correlations between any two IMFs which are closer to 0 as shown in Figure 4.5 for Botswana. The same was true for all other countries. The plots of all other countries are presented in Figure 4.7 as supplementary plots for EEMD. Next, we find the sum of the absolute correlation coefficients between any two IMFs from EMD and obtained 4.98 and 4.12 from EEMD for the case of South Africa. A similar pattern was observed in other countries. A decomposition with least absolute correlation coefficients between any two IMFs is deemed to perform better. We conclude on the performance of the EMD and EEMD by examining the equality of the means of the average absolute correlation coefficient between any two IMFs using the paired t-test. The results showed that there was no significant difference between the values from EMD $(Mean\ (M) = 0.094, Standard\ deviation(\sigma) = 0.00053)$ and values from EEMD $(M = 0.098, \sigma = 0.00024)$; $t(14) = 0.841, p = 0.2072)$(see Table 4.1) Based on the variance, we find EEMD to be more suitable for analysing the similarity structure of exchange rate markets in SADC because it performs more stably for the 15 exchange rate markets.

Table4. 1: Test of Equality of means of Absolute Correlations of EEMD and EMD

t-Test: Paired Two Sample for Means		
	EEMD	*EMD*
Mean	0.098069494	0.094123111
Variance	0.000000571	0.000002809
Observations	15	15
Pearson Correlation	0.11529037	
Hypothesized Mean Difference	0	
df	14	
t Stat	0.840667525	
P(T<=t) two-tail	0.207279271	
t Critical two-tail	2.144786688	

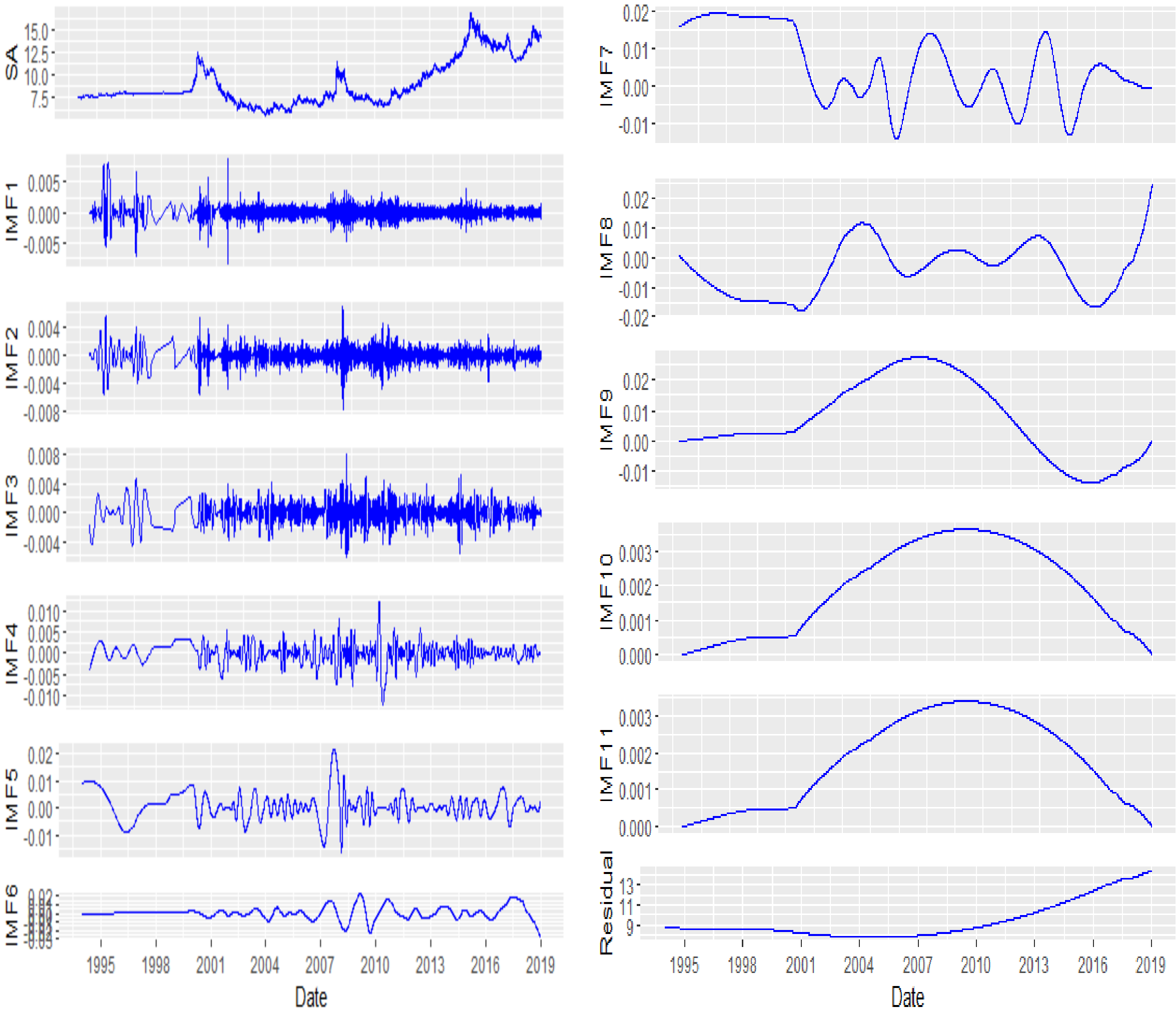


Figure 4.2 IMFs of real exchange rate of South Africa obtained by EMD

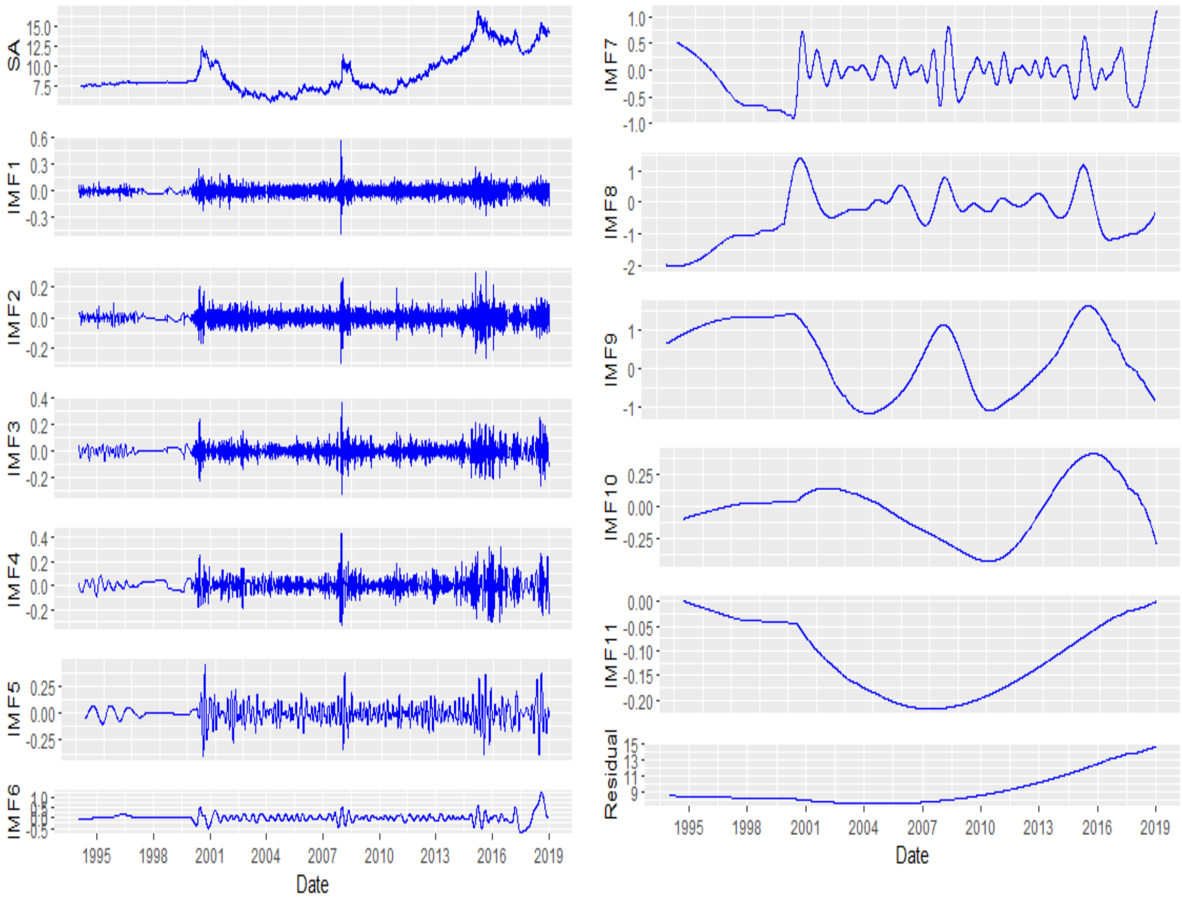


Figure 4.3 IMFs of real exchange rate of South Africa obtained by EEMD

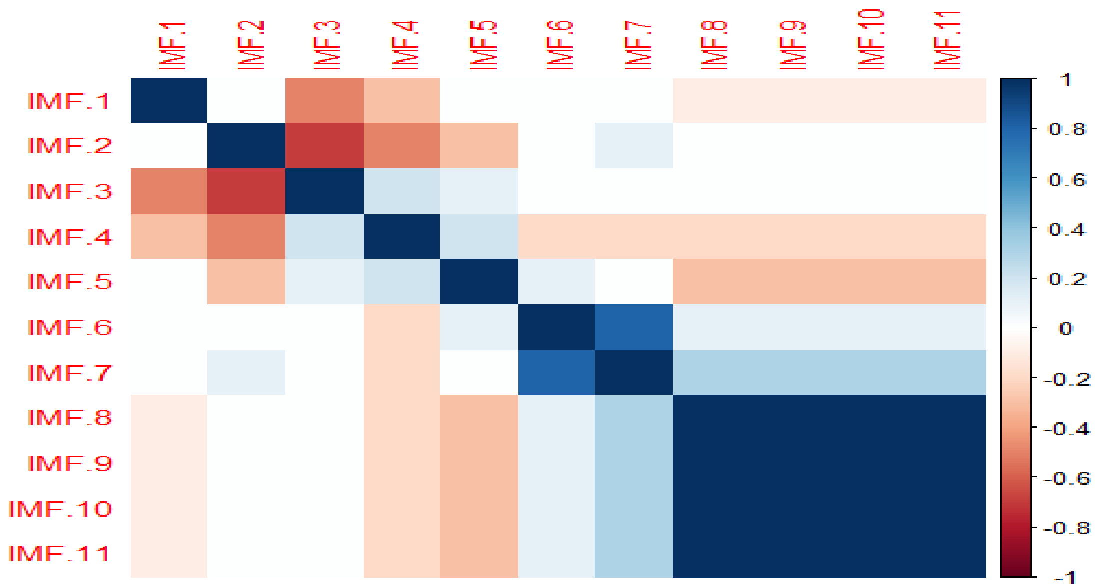


Figure 4.4 The correlation coefficients between any two IMFs of Botswana from EMD

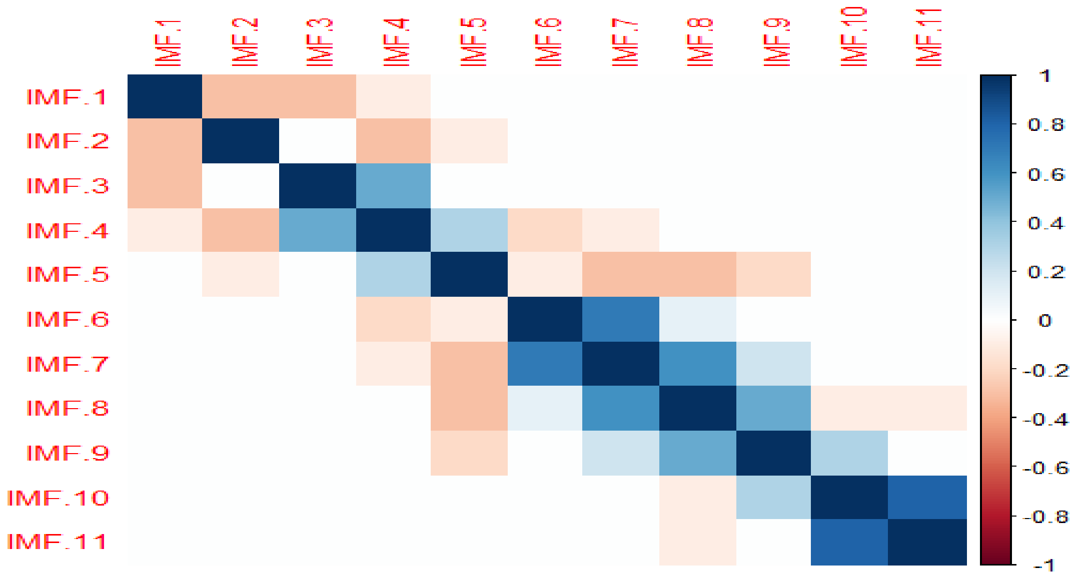


Figure 4.5 The correlation coefficients between any two IMFs of Botswana from EEMD.

We proceed to analyse the similarities of the exchange rate series of SADC by using statistics of the IMFs obtained from EEMD and the structure of the residue. These two measures have been used to analyse similarity in structure in previous studies (see Zhang, Lai and Wang, 2008; Bai, Zhang and Zheng, 2011; Zhang, Zheng, Bai and Liu, 2014). The advantage of this approach over the traditional comovement and cointegration approaches lies in its ability to efficiently decompose the series so that similarity of the structure at different modes is analysed devoid of noise or irregularity in the series.

The following statistics of IMFs are computed: mean period of each IMF, correlations between each IMF and the original data series, variance percentage of each IMF relative to that of the original data series, and also to the sum of all IMFs and residue. The mean period is defined as the ratio of the total number of points to the number of peaks for each IMF, and the correlation is the Pearson product moment correlation coefficient. As noted by Zhang et al. (2008), variance percentage of each IMF to the original data series can be used to explain the contribution of each IMF relative to the total volatility of the original data.

Table 4.1 presents the statistics of each IMF from EEMD of 15 countries included in this study. The evidence from Table 4.1 indicates that the dominant modes of the observed data are the residue in most cases. The Pearson correlation coefficient between the residue and the observed data for all the countries ranges between 0.74 and 0.97 except for Angola, which is

0.26. The dominant modes of Angola are the IMF6 and IMF7, with each having a correlation coefficient of more than 0.5. Again, we observed that the variances of the residue explain a greater proportion of the total volatility in the observed data series. As has been documented in the empirical mode decomposition literature (Huang et al., 1998; Bai et al., 2011; Xu et al., 2016) the residue depicts the deterministic long-term behaviour. This implies that exchange rate markets in SADC are driven mostly by fundamentals, which, in turn, are most likely rooted in macroeconomic economic fundamentals (Redda and Muzindusti, 2017). The IMFs statistic of four countries Eswatini, Lesotho, Namibia and South Africa are the same for the period studied (see Table 4.2). This is not surprising as there had been the existence of Common Monetary Area (CMA) made up of these four countries, rooted in *de facto* currency area since 1975, in which currencies of Eswatini, Lesotho and Namibia are issued at par with South African Rand (Jordaan, 2015; Masha, Wang, Shirono and Harris, 2007).

Table 4.2 Measures of IMFs and the residue for SADC exchange rate series derived through EEMD

Country	IMF 1	IMF 2	IMF 3	IMF 4	IMF 5	IMF 6	IMF 7	IMF 8	IMF 9	IMF 10	IMF 11	Residue
Angola												
μ	1.41	2.42	4.41	8.02	14.98	30.84	58.63	143.9	791.5	2374.1	2374.1	
ρ	0.00	0.14	0.41***	0.37**	0.17	0.51**	0.56***	0.25**	0.09	-0.06	-0.05	0.26**
σ_1^2	4.99%	14.72%	12.08%	7.96%	25.36%	20.83%	1.64%	1.83%	5.37%	40.25%	13.87%	144.78%
σ_2^2	4.99%	14.72%	12.08%	7.96%	25.36%	20.83%	1.64%	1.83%	5.37%	40.25%	13.87%	144.78%
Botswana												
μ	1.41	2.73	5.22	10.08	20.38	42.03	110.44	263.83	949.8	1187.25	4749	
ρ	0.01	0.02	0.02	0.04	0.04	0.12**	0.14**	-0.07	0.57***	0.80***	0.47***	0.94***
σ_1^2	0.03%	0.01%	0.02%	0.03%	0.06%	0.18%	0.61%	3.12%	4.79%	0.20%	0.03%	77.17%
σ_2^2	0.03%	0.01%	0.02%	0.03%	0.06%	0.18%	0.61%	3.12%	4.79%	0.20%	0.03%	77.14%
Comoros												
μ	1.39	2.78	5.23	10.01	20.64	45.66	105.53	215.86	431.2	678.42	949.8	
ρ	0.03	0.03	0.04	0.06	0.06	0.10*	0.23**	0.28**	0.65***	0.48***	0.80***	0.85***
σ_1^2	0.05%	0.03%	0.05%	0.10%	0.29%	0.69%	3.66%	4.09%	5.45%	5.41%	0.16%	48.32%
σ_2^2	0.05%	0.03%	0.05%	0.10%	0.29%	0.69%	3.65%	4.09%	5.45%	5.41%	0.16%	48.29%
DRC												
μ	1.39	2.49	4.68	8.91	19.15	42.78	94.98	237.45	593.63	1187.25	2374.5	
ρ	0.01	0.01	0.01	0.02	0.03	0.03	-0.27*	0.18*	0.28**	-0.26**	-0.69***	0.94***
σ_1^2	0.02%	0.01%	0.01%	0.01%	0.03%	0.05%	0.22%	1.01%	2.05%	5.82%	0.99%	133.16%
σ_2^2	0.02%	0.01%	0.01%	0.01%	0.03%	0.05%	0.22%	1.00%	2.04%	5.82%	0.99%	133.08%
Eswatini												
μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1586	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22**	0.18**	0.07	0.61***	0.63***	0.77***	0.87***
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%
σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%

Table 4.2 Continued

Lesotho												
μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1586	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22**	0.18**	0.07	0.61***	0.63***	0.77***	0.87***
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%
σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%
Madagascar												
μ	1.39	2.62	4.85	10	20.83	47.97	110.44	226.14	678.43	1187.25	4749	
ρ	0.02	0.02	0.02	0.03	0.06	0.11**	0.08	0.02	0.16	0.46***	0.42***	0.94****
σ_1^2	0.04%	0.01%	0.01%	0.02%	0.06%	0.18%	0.58%	1.31%	3.21%	2.87%	0.01%	86.37%
σ_2^2	0.04%	0.01%	0.01%	0.02%	0.06%	0.18%	0.58%	1.31%	3.21%	2.87%	0.01%	86.33%
Malawi												
μ	1.39	2.49	4.73	8.59	19	37.99	91.33	197.88	593.63	1583	4749	
ρ	0.00	0.00	0.01	0.02	0.04	0.02	0.05	-0.09	0.39***	0.18**	0.58***	0.97***
σ_1^2	0.01%	0.00%	0.00%	0.00%	0.01%	0.07%	0.19%	0.31%	0.78%	2.69%	0.05%	90.95%
σ_2^2	0.01%	0.00%	0.00%	0.00%	0.01%	0.07%	0.19%	0.31%	0.77%	2.69%	0.05%	90.89%
Mauritius												
μ	1.38	2.66	5.09	10.19	20.92	54.58	163.75	365.31	678.4	1583	4749	
ρ	0.04	0.04	0.04	0.06	0.11**	0.18**	0.32**	0.37***	0.20**	0.10**	0.10**	0.74***
σ_1^2	0.17%	0.06%	0.04%	0.11%	0.42%	0.84%	5.69%	12.71%	10.89%	1.98%	0.02%	83.27%
σ_2^2	0.17%	0.06%	0.04%	0.11%	0.42%	0.84%	5.69%	12.70%	10.88%	1.98%	0.02%	83.22%
Mozambique												
μ	1.36	2.58	4.82	9.52	20.74	40.94	98.94	296.81	678.4	2374.1	2374.1	
ρ	0.01	0.01	0.01	0.02	0.03	0.00	-0.06	0.14**	0.74***	0.00	-0.33***	0.88***
σ_1^2	0.03%	0.01%	0.01%	0.02%	0.05%	0.14%	0.78%	3.33%	7.20%	2.38%	0.33%	83.03%
σ_2^2	0.03%	0.01%	0.01%	0.02%	0.05%	0.14%	0.78%	3.33%	7.19%	2.38%	0.33%	82.98%

Table 4.2 Continued

Namibia												
μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1583	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22**	0.18**	0.07	0.61***	0.63***	0.77***	0.87***
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%
σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%
Seychelles												
μ	1.37	2.57	4.81	9.37	19.07	43.97	98.94	215.86	593.63	1583	2374.5	
ρ	0.03	0.02	0.03	0.04	0.04	0.05	0.12	0.17	-0.01	0.31***	0.84***	0.87***
σ_1^2	0.07%	0.03%	0.04%	0.06%	0.11%	0.26%	2.45%	2.27%	4.92%	5.38%	1.57%	78.57%
σ_2^2	0.07%	0.03%	0.04%	0.06%	0.11%	0.26%	2.44%	2.27%	4.92%	5.38%	1.57%	78.51%
South Africa												
μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1583	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22***	0.18**	0.07	0.61***	0.63***	0.77***	0.87***
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%
σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%
Tanzania												
μ	1.4	2.5	4.88	9.52	19.62	39.91	105.53	226.14	593.63	1583	4749	
ρ	0.01	0.01	0.02	0.03	0.02	0.03	0.00	-0.27**	0.26**	-0.11**	-0.16**	0.98***
σ_1^2	0.02%	0.01%	0.01%	0.01%	0.02%	0.04%	0.13%	0.68%	1.65%	0.77%	0.04%	104.69%
σ_2^2	0.02%	0.01%	0.01%	0.01%	0.02%	0.04%	0.13%	0.68%	1.65%	0.77%	0.04%	104.63%
Zambia												
μ	1.39	2.6	5.12	9.89	21.3	42.4	103.24	226.14	474.9	678.43	949.8	
ρ	0.02	0.02	0.02	0.03	0.07	0.14**	0.05	-0.35***	0.55***	0.13**	0.10**	0.91***
σ_1^2	0.03%	0.02%	0.03%	0.07%	0.17%	0.47%	0.99%	6.67%	4.49%	2.88%	0.12%	103.79%
σ_2^2	0.03%	0.02%	0.03%	0.07%	0.17%	0.47%	0.99%	6.67%	4.48%	2.88%	0.12%	103.74%

μ = mean period (days), ρ = Pearson correlation coefficient, σ_1^2 = variance as % of observed, σ_2^2 = variance as % of the sum of all IMFs and Residue. Note *,**,and *** indicate significance at 10%, 5% and 1% levels.

Having observed that the dominant mode in almost all cases is the residue, we follow Bai et al. (2011) to compare the residues. The closer the residues are, the more similar the exchange rate markets are. The residual graph offers an easy way to judge the similarities of the exchange rate market in SADC. Figure 4.6 shows that there is a similarity in the structure of 12 out of 15 countries included in the study with CMA countries being stronger. The only exceptions are Angola, Comoros and Seychelles and the obvious outliers are Angola and Comoros. Comoros recently joined SADC and that could explain its deviation from most of the SADC countries. The case of Angola could be originated from Angola's nonparticipating in the SADC free trade area until August 2019 and Common Market for Eastern and Southern Africa (COMESA) which could result in the economic integration of member countries and price convergence, thereby increasing comovement of the exchange rate (Alagidede et al., 2008). The similarity in structure of exchange rate markets of at least 12 countries in SADC presents some hope for the possible monetary union by expanding the existing CMA to accommodate those new countries orientating toward CMA countries. The monetary policy response to prices have always been to core prices (Eckstein,1981; Wang et al., 2019) and therefore findings that there are similarities in the core exchange rate gives policymakers comfort as far as the exchange rate is concerned.

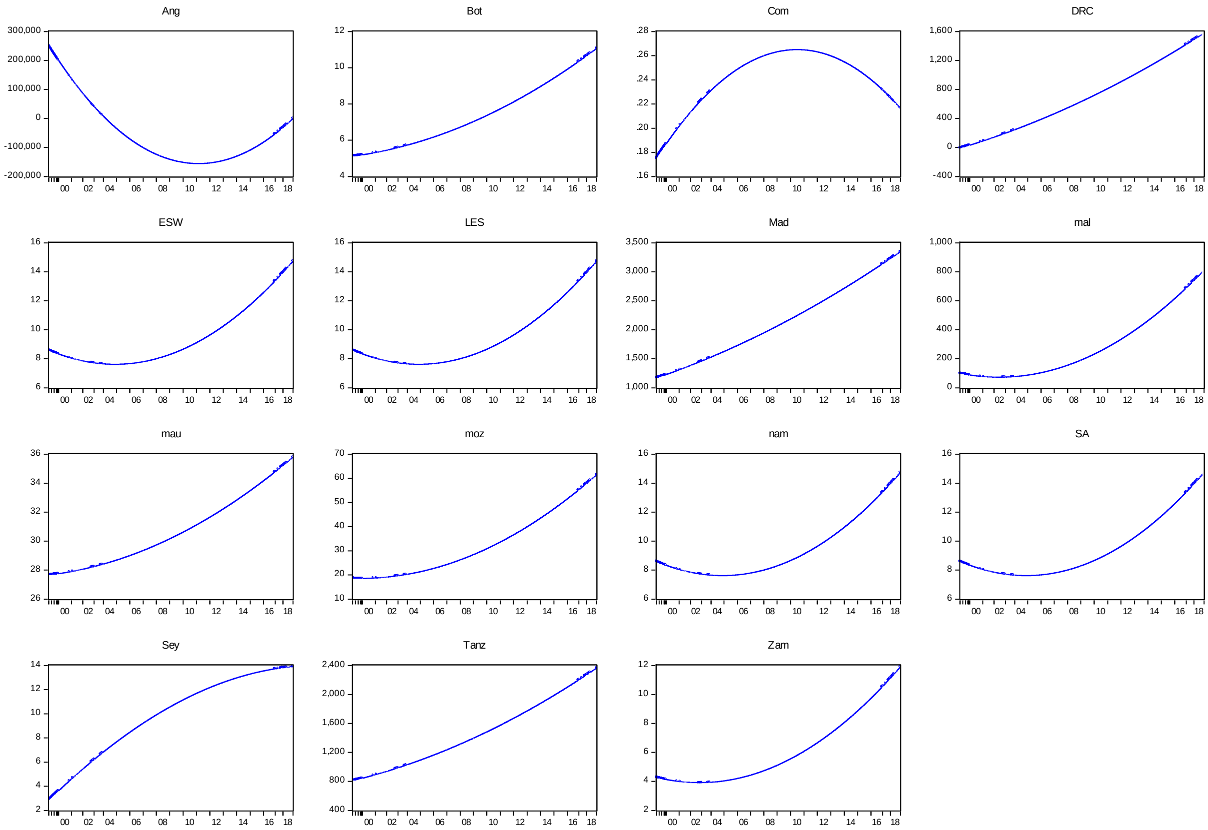
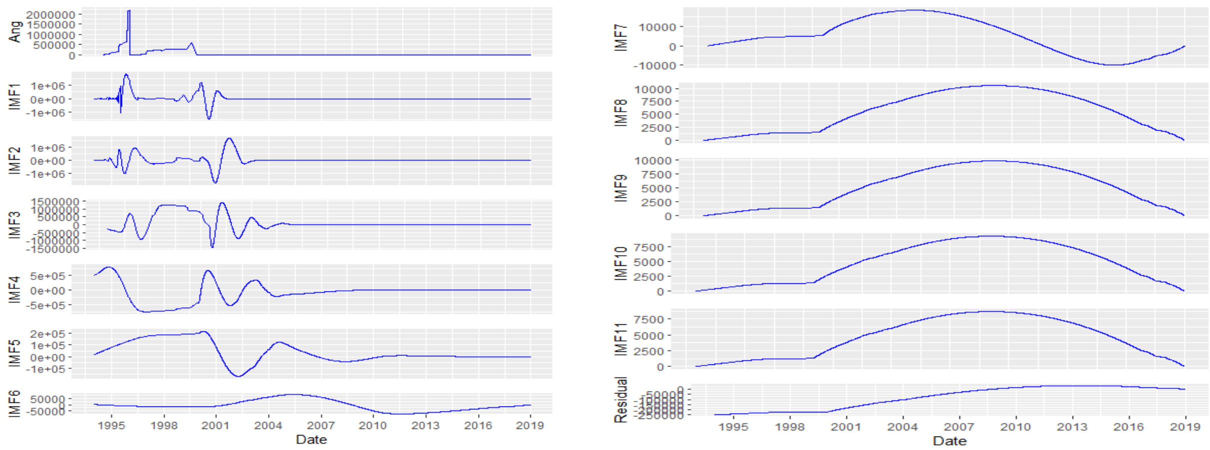


Figure 4.4 The residue comparison plots

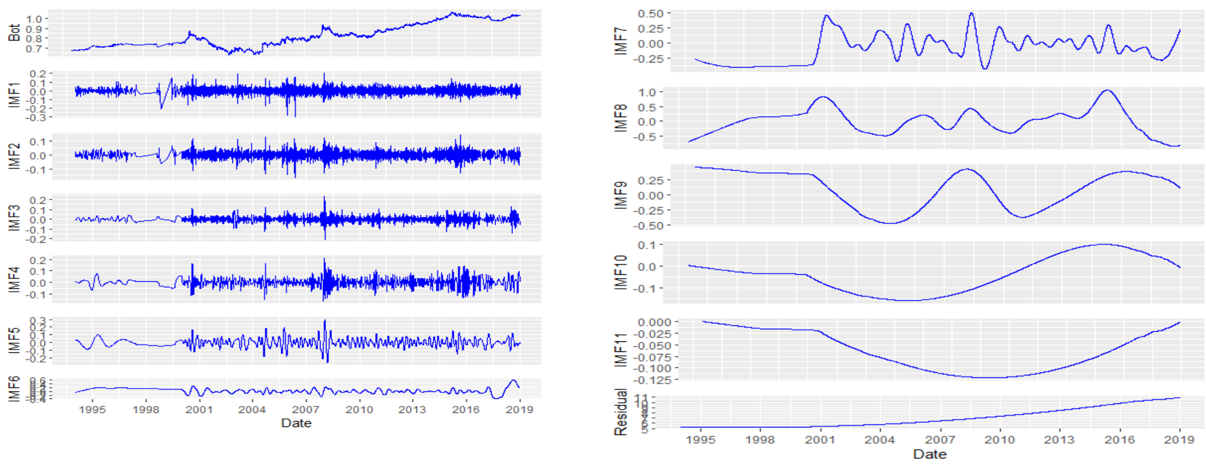
4.5 Conclusion

The purpose of the study was to investigate the similarities in exchange rate markets in SADC using EMD and EEMD. The real exchange rate data of 15 SADC countries were decomposed into several independent intrinsic modes and residue using EMD and EEMD. We compared the performance of the two through comparison of the Pearson correlation coefficient of the IMFs from both methods. The EEMD performed better on the orthogonality of IMFs than EMD and proved to be more stable for the 15 countries. The IMFs and residue obtained from the EEMD method for the 15 countries were analysed for similarity in structure using the IMFs statistics and graph of the residue. We observed from the IMF statistics that the residue is the dominant mode in almost all cases; the Pearson correlation coefficient between the residue and the observed series were very strong and variances of the residues explain a greater proportion of the total volatility in the observed data series, except for Angola. This implies that SADC shares common economic ties which can facilitate common monetary policy in a currency union. This indicates that exchange markets in SADC are driven mostly by fundamentals, but not exchange rate markets' short-term fluctuations. The plots of the residues showed similarity in at least 12 countries excluding Angola, Comoros and Seychelles, again with Angola and Comoros being obvious outliers, but showing some form of orientation toward the SADC market structure.

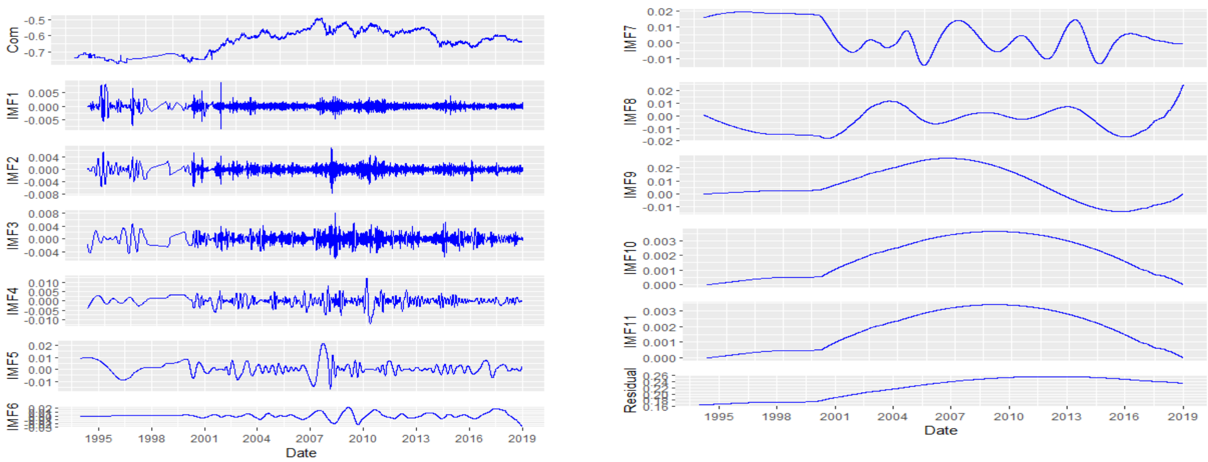
By these findings, we see some hope that SADC can form a monetary union and recommend gradual formation by expanding the existing CMA. This would accommodate those new countries orientating toward CMA countries taking into consideration other economics agents necessary for optimum currency area such as business cycle synchronisation and macroeconomic convergence.



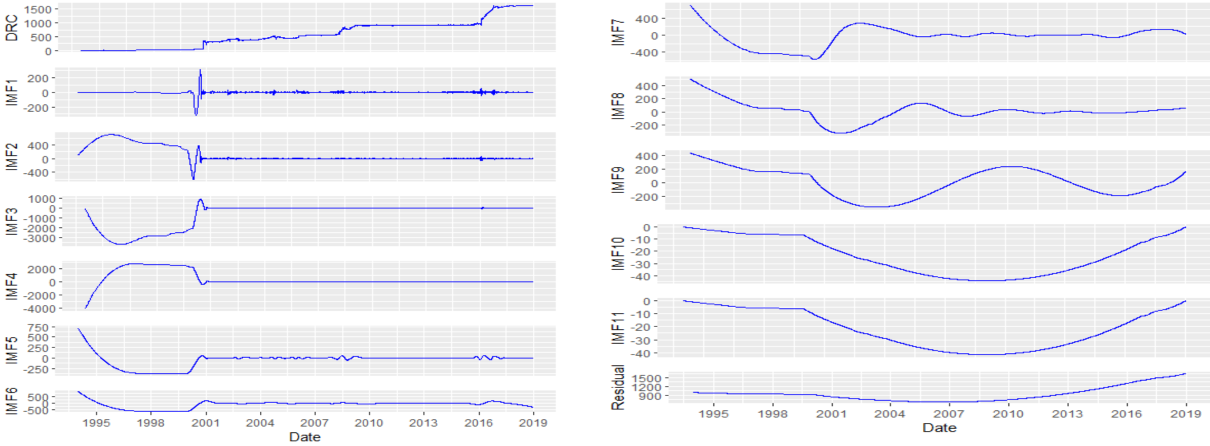
Angola



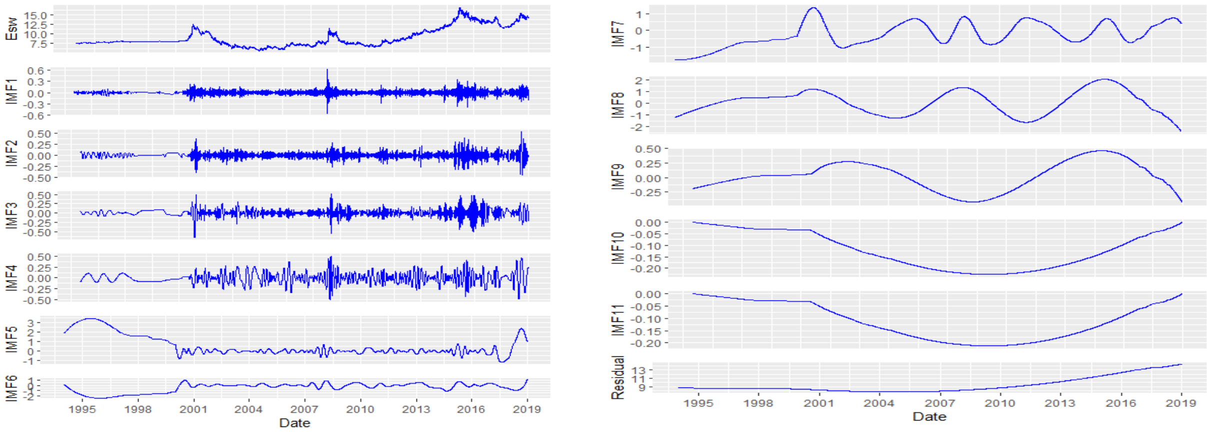
Botswana



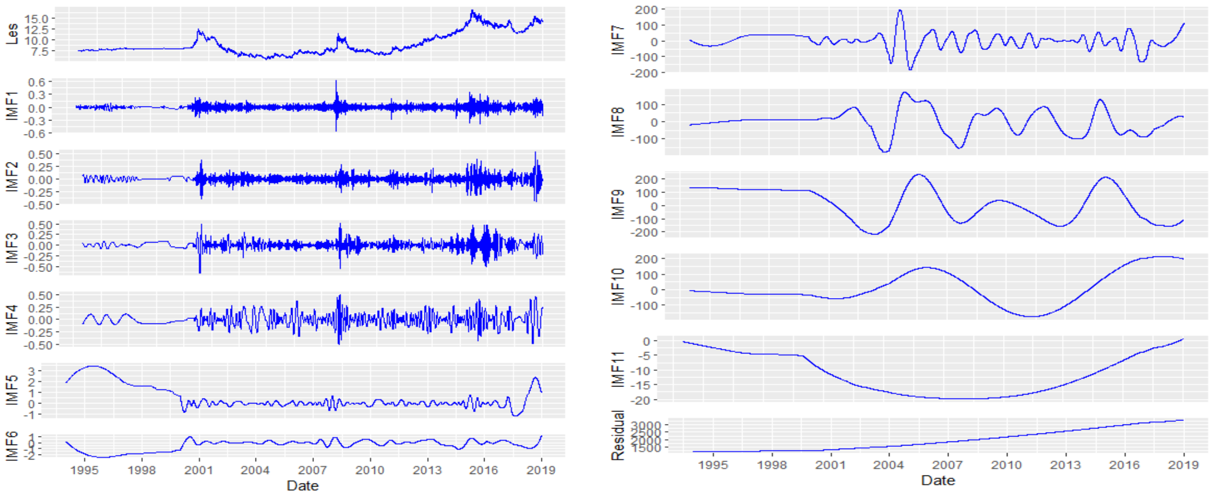
Comoros



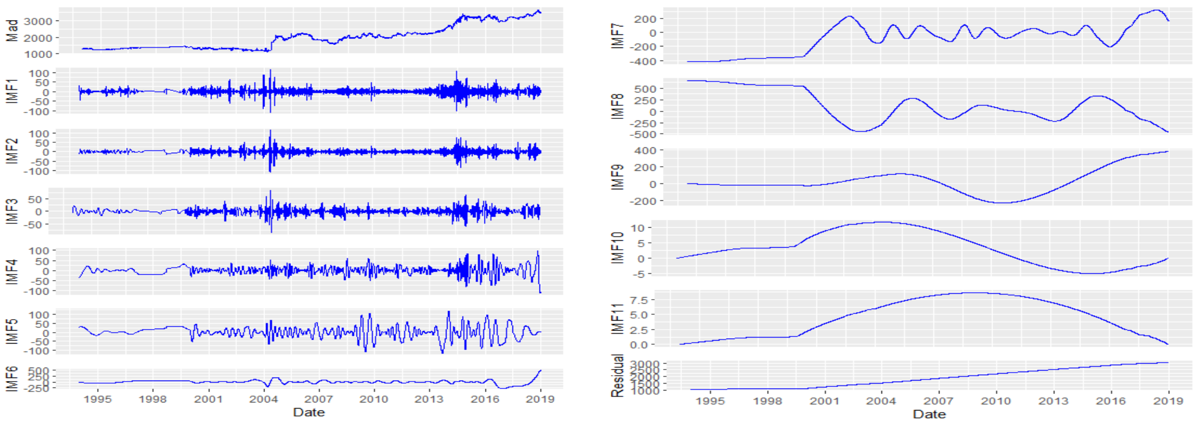
Demographic Republic of Congo



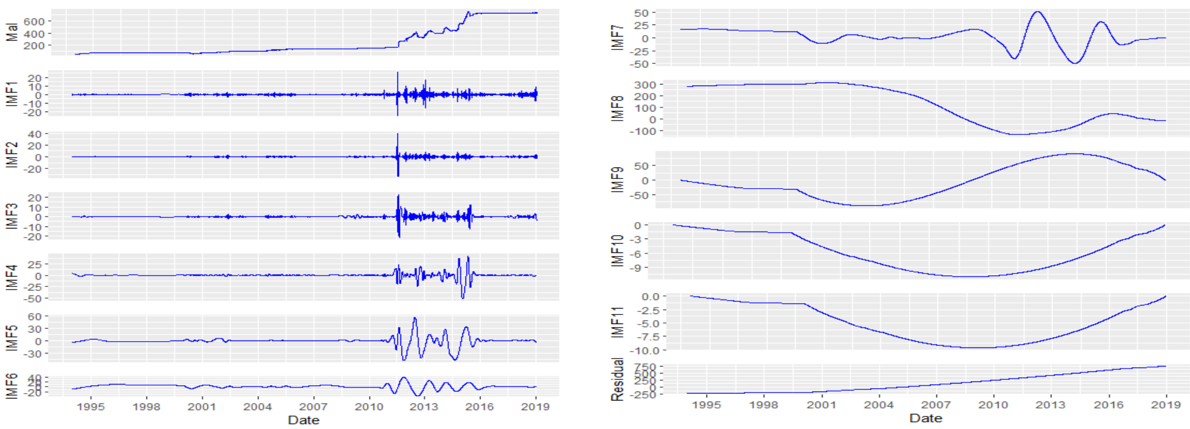
Eswatini

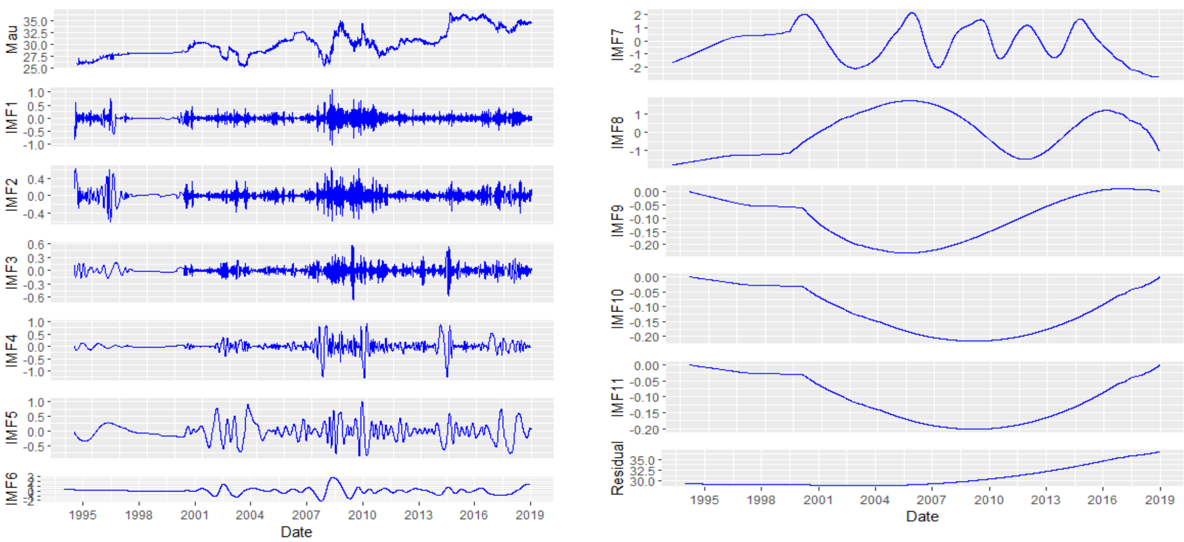


Lesotho

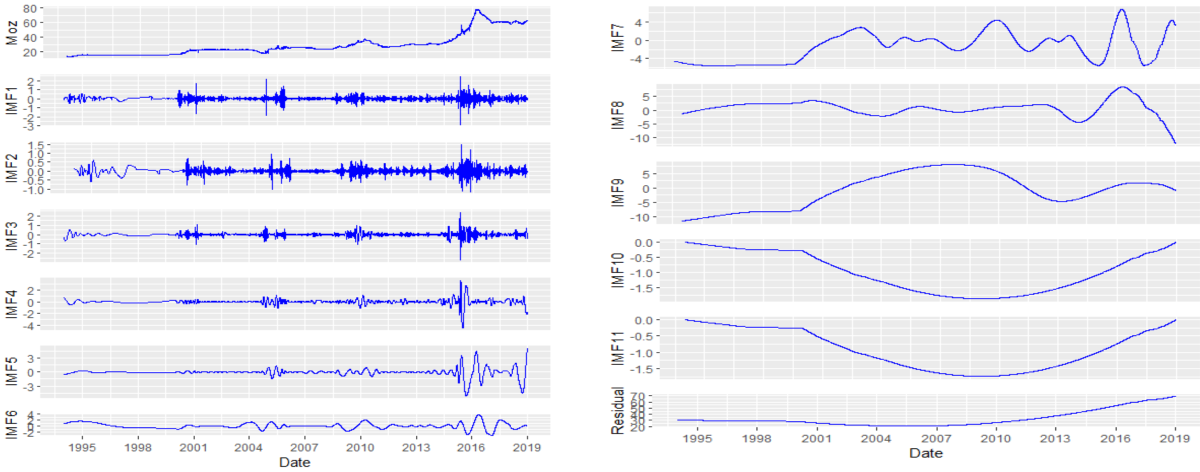


Malawi

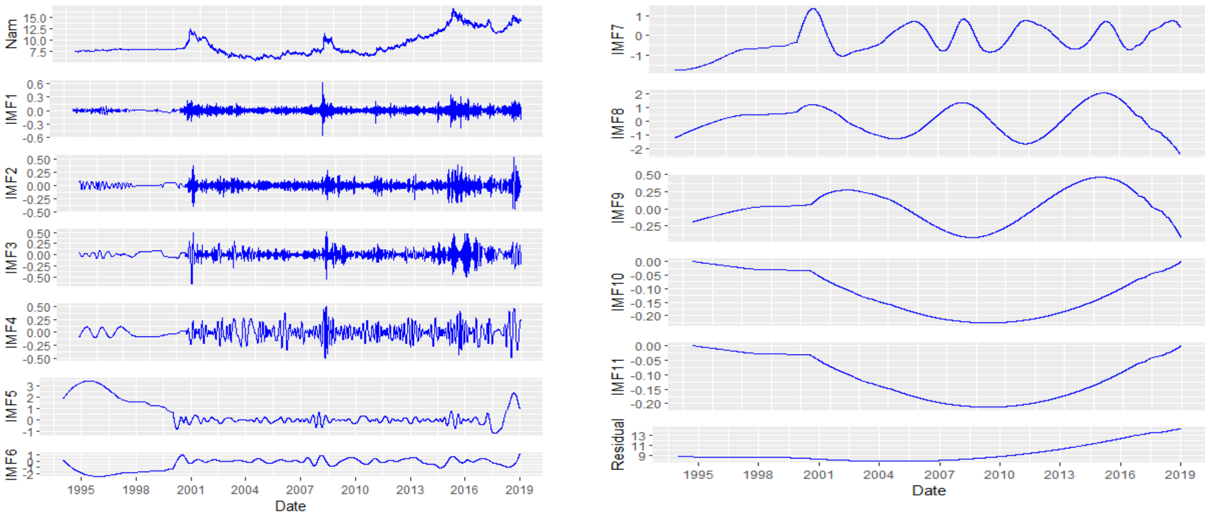




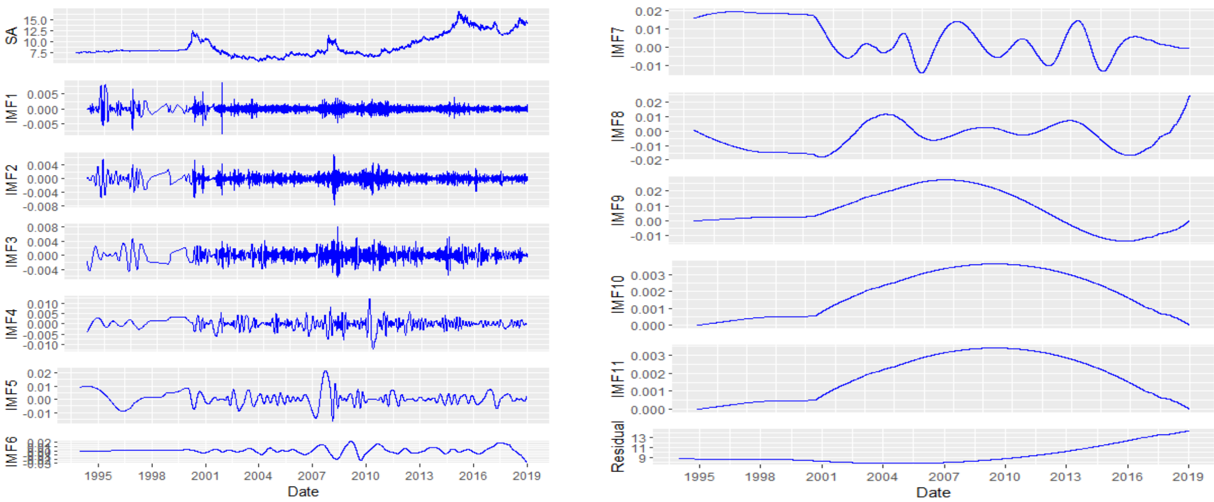
Madagascar



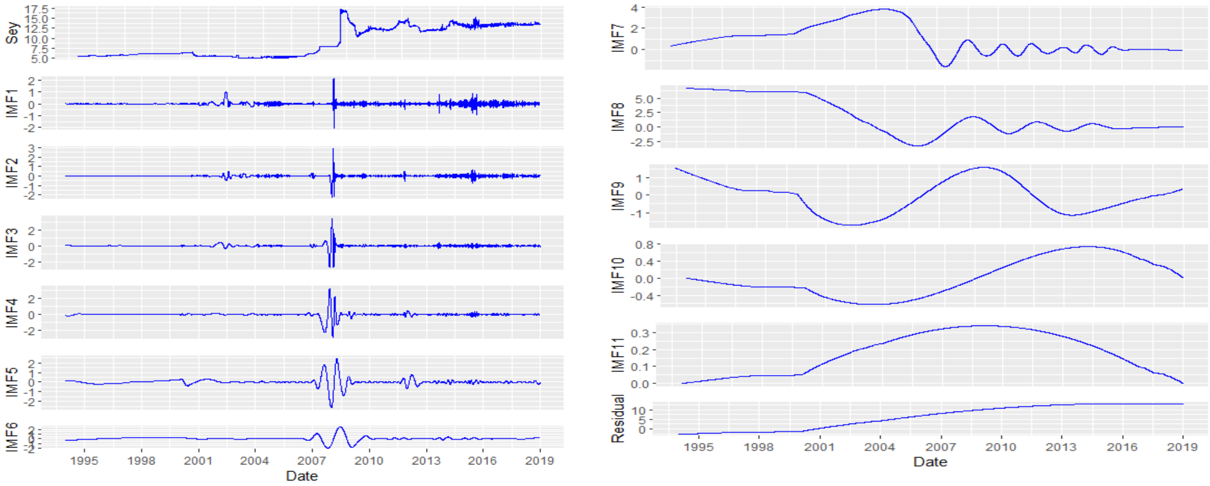
Mozambique



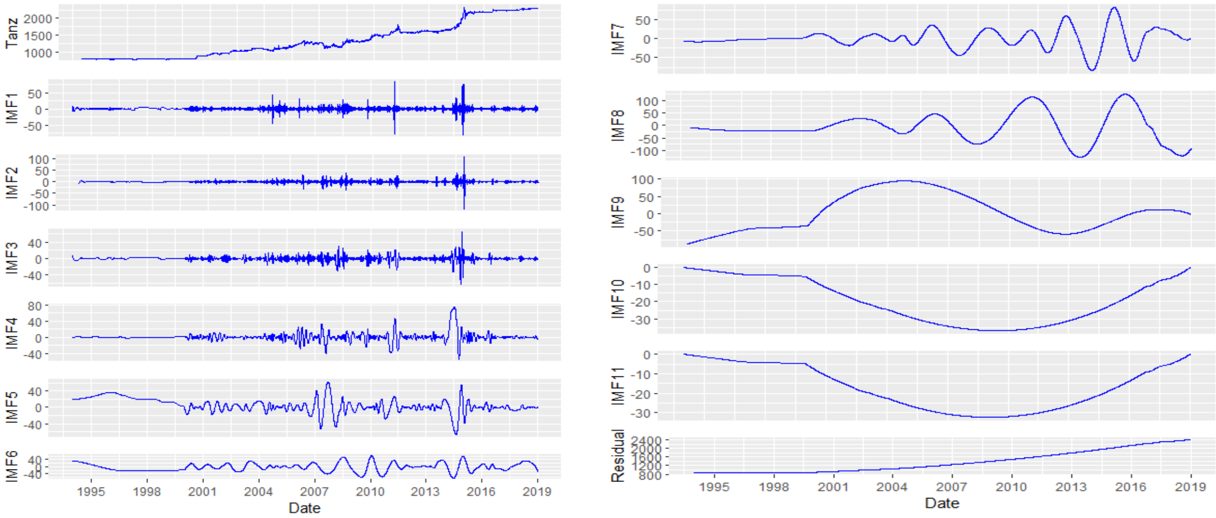
Namibia



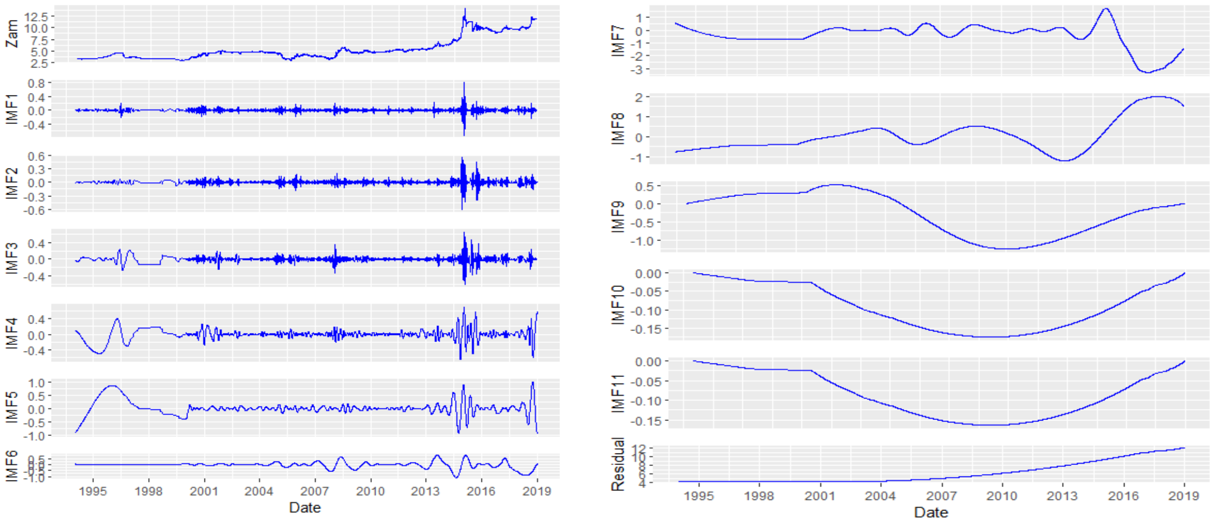
South Africa



Seychelles

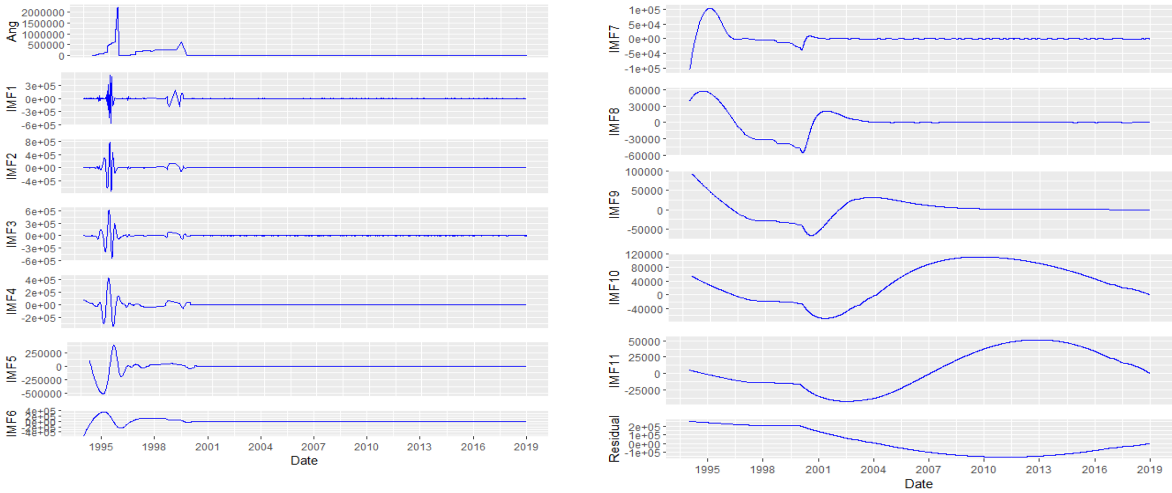


Tanzania



Zambia

Figure 4.5 Supplementary Plots of IMFs from EMD of Other countries



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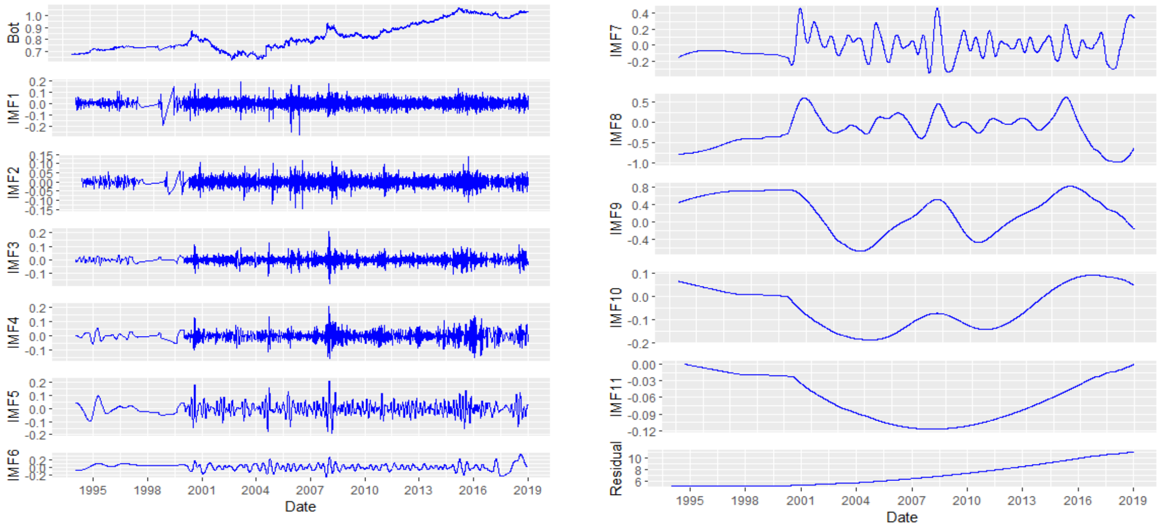

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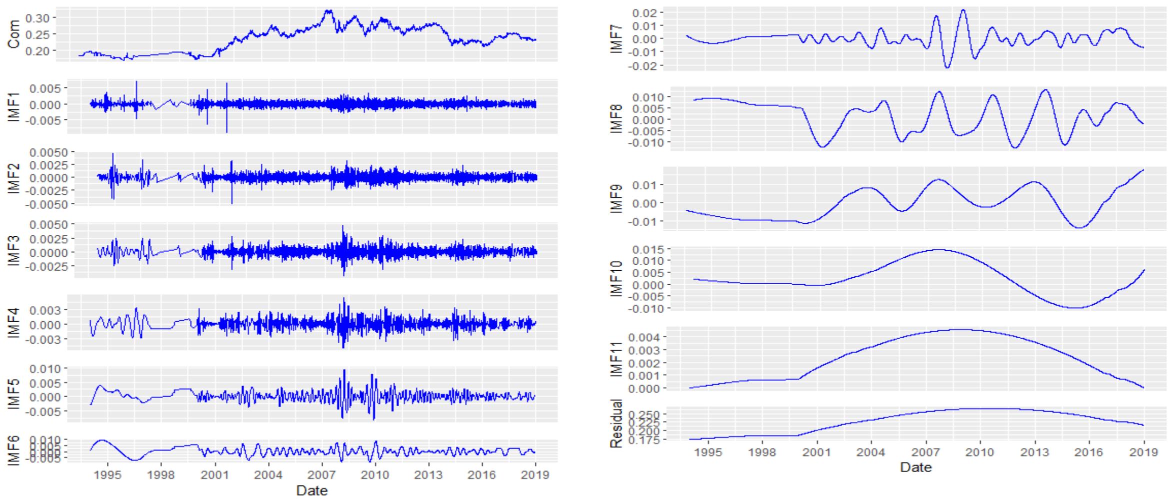

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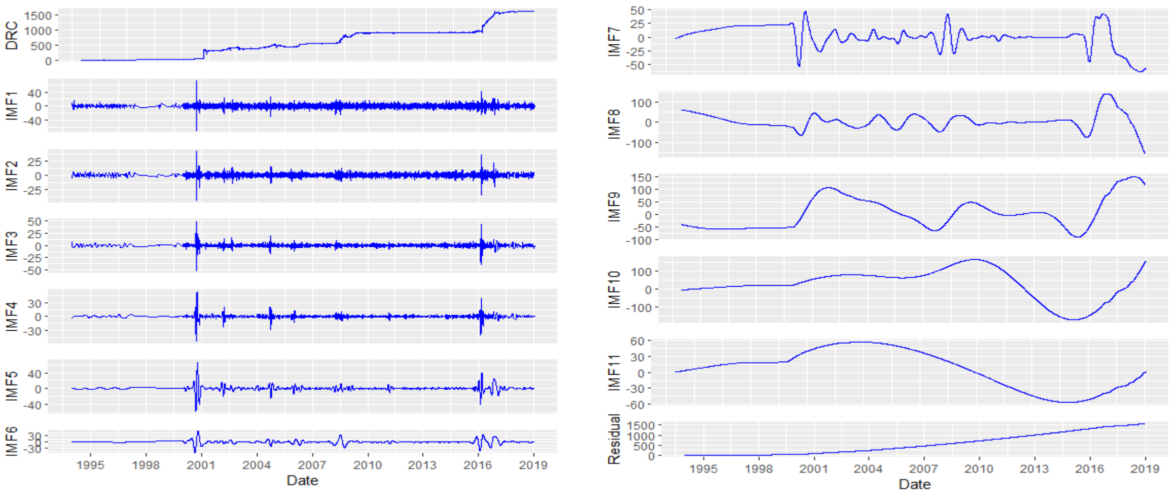

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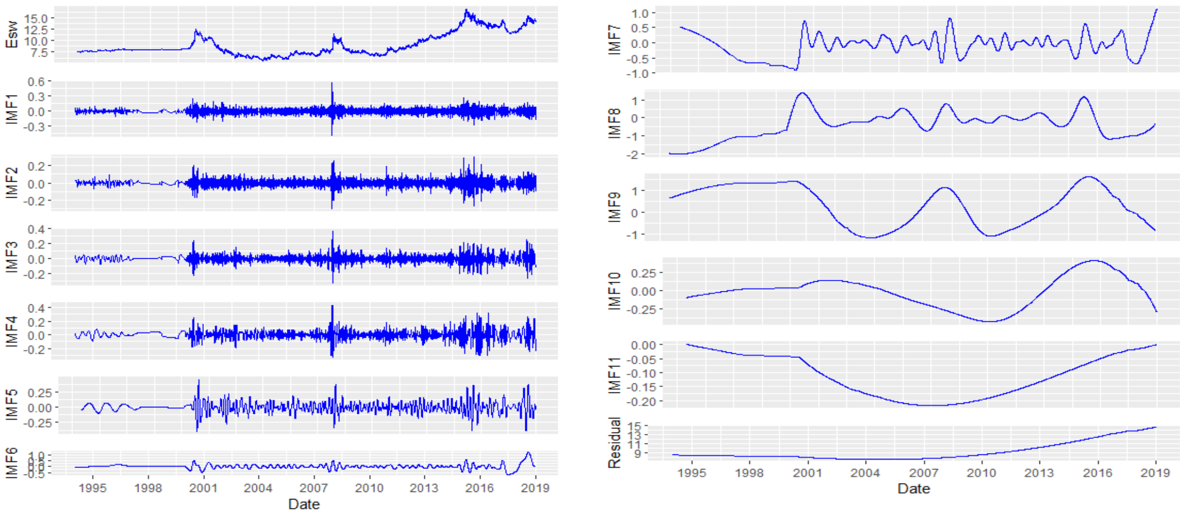

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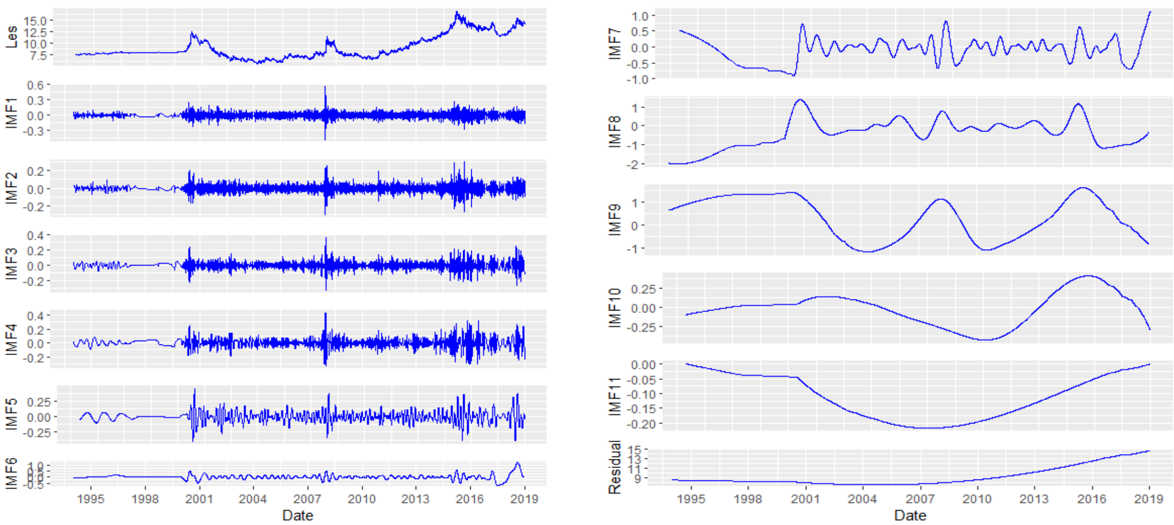

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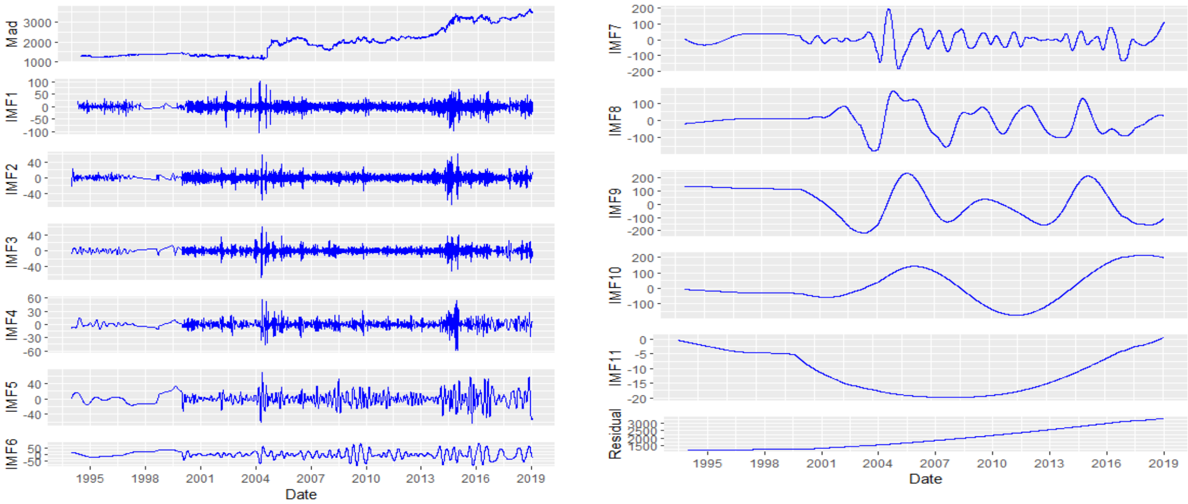

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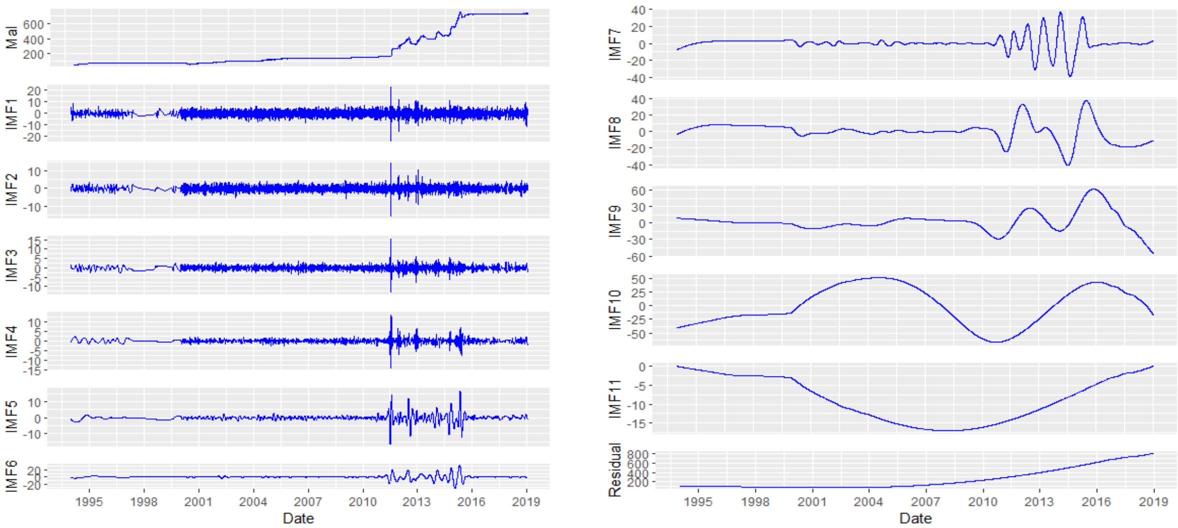

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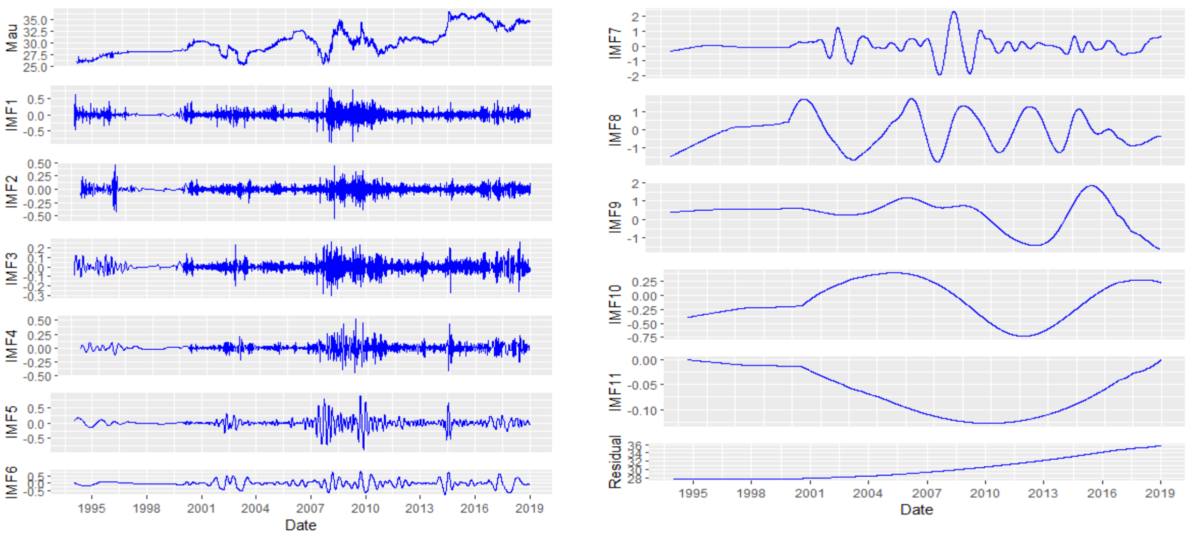

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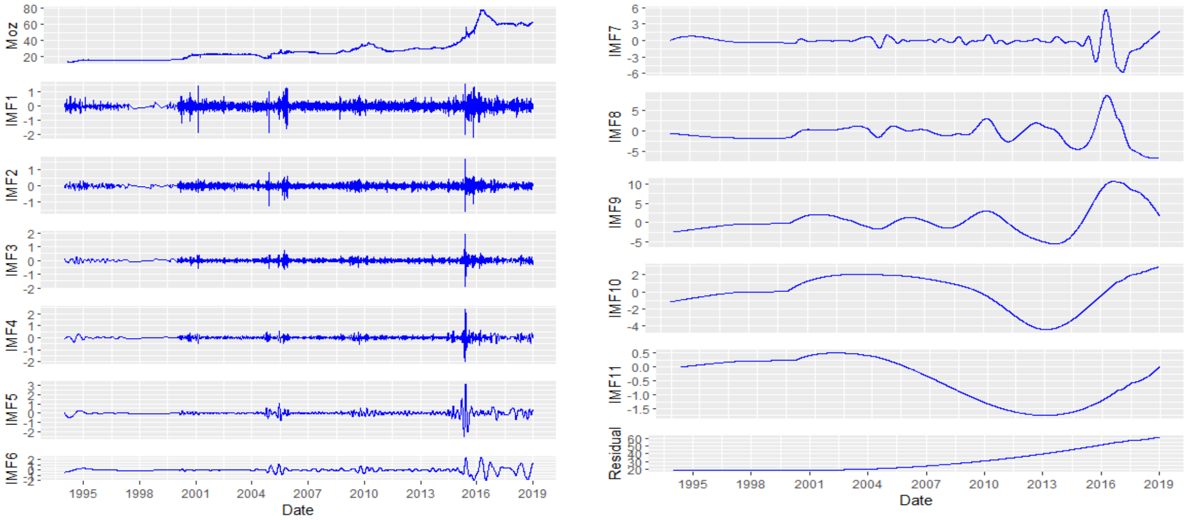

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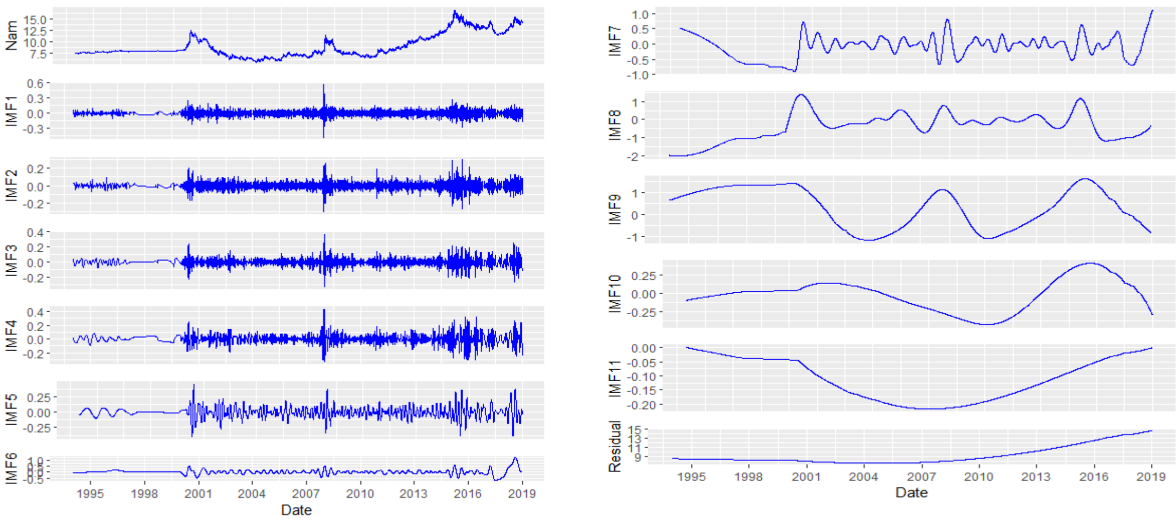

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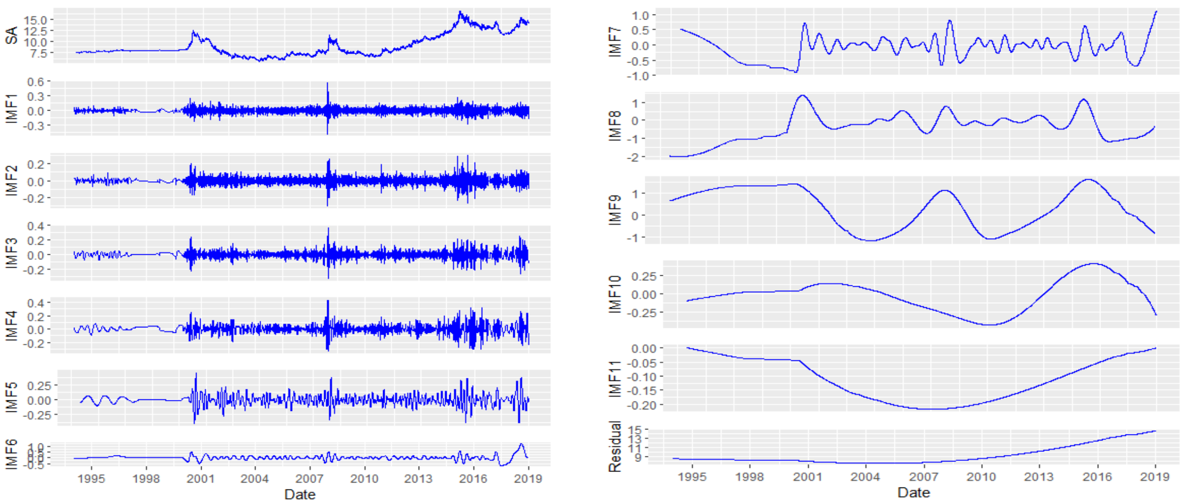

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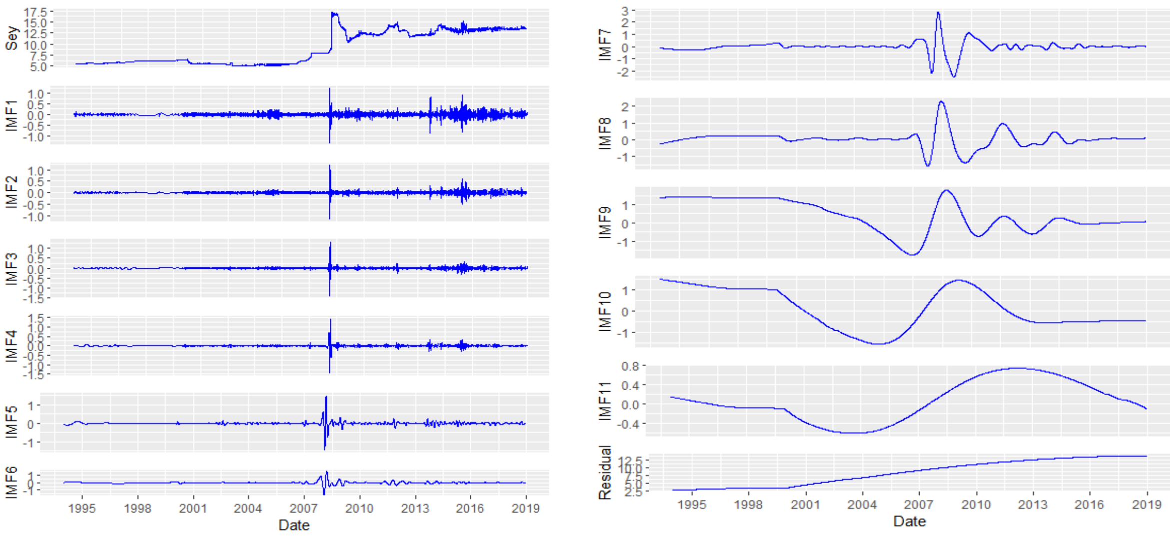

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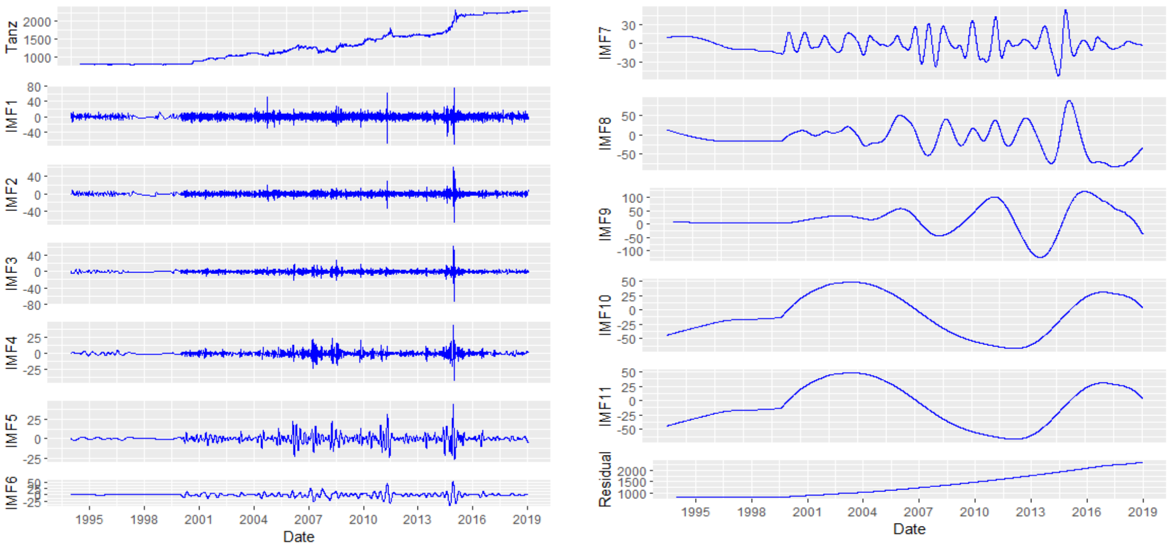

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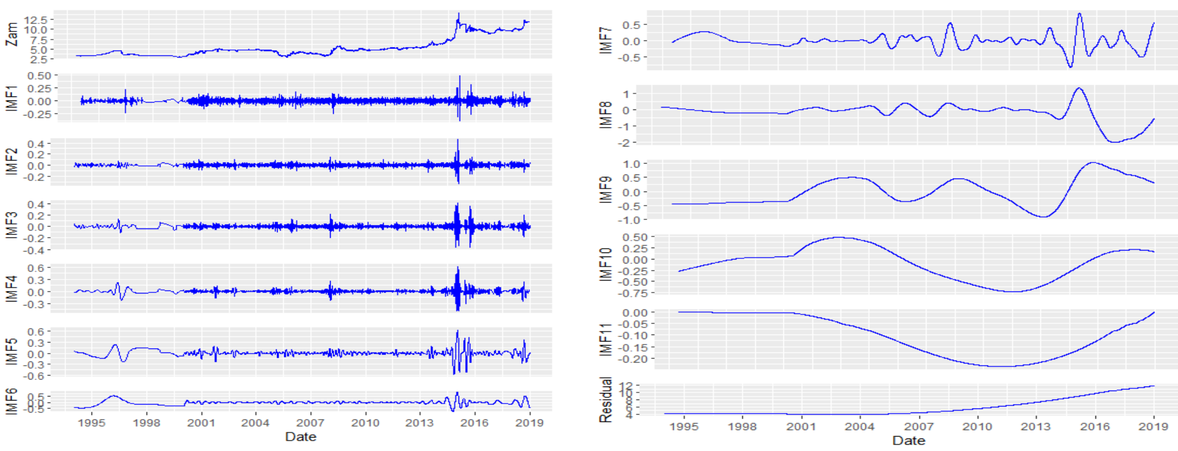


Zambia

Figure 4.6 Supplementary Plots of IMFS from EEMD of Other Countries

CHAPTER 5

Multifrequency Network for SADC Exchange Rate Markets using EEMD-based DCCA

Chapter Summary

We used the detrended cross-correlation analysis (DCCA) method based on ensemble empirical mode decomposition (EEMD) to study the dynamic interdependence structure of daily domestic currency to US dollar exchange rates of 15 Southern African Development Community (SADC) exchange rate markets. We first decomposed all series into intrinsic mode functions using EEMD and reconstructed the series into three frequency modes: high-, medium- and low frequency, and residue. The DCCA method was used to analyse the cross-correlation between the various frequencies, residues and original series. These were meant to address the non-linearity and non-stationarity in observed exchange rate data. Finally, we formed a correlation network from the cross-correlation coefficients in all cases which revealed rich information that would not have been obtained from the original series. We observed that similarities between the nature of cross-correlation between high-frequency series mimic the original series and the significant cross-correlation among the long-term trend of most SADC countries exchange rate markets. The innovation of this chapter is to combine EEMD with DCCA to study the multifrequency cross-correlations of exchange rate markets, which can provide policymakers a deeper understanding of the dynamics of exchange rate markets toward the formation of currency unions.

5.1 Introduction

The Article 3 of the Africa Union Constitution adapted from the Article II of the disbanded Organisation for African Unity Charter has a bigger agenda for economic and financial integration with the goal of introducing a common currency for Africa by the year 2021. To developing regions, economic and financial integration are seen as a panacea to harnessing resources and capabilities of individual countries toward economic development (Kreinin and Plummer, 2002). This is corroborated by Jefferis (2007) who classified benefits of economic and financial integration into four main areas. Firstly, it provides an “agency of restraint” that will reduce the ability of governments to pursue irresponsible and destabilising macroeconomic policies. Secondly, economic integration acts as a bulwark against currency speculation and contagion effects that could add to exchange rate volatility. Thirdly, it supports the exploitation of economies of scale in the financial sector, with accompanying efficiency benefits. Lastly, it exploits the traditional optimum currency area (OCA) benefits, i.e., the potential gains to trade from reduced transaction costs and exchange rate uncertainty, a net of potential losses resulting from reduced national policy autonomy and constrained ability to react and adjust to economic shocks.

The economic and financial integration process in Africa was envisioned to follow a two-way approach in which each economic community has been encouraged to form a monetary union with the intention to form Africa-wide financial integration with single currency (Alagidede, Tweneboah and Adam, 2008). This has caused several economic blocs vigorously pursuing currency union of which the Southern African Development Community (SADC) is no exception (Adam et al., 2021).

Southern African Development Community (SADC), the largest regional economic grouping in Africa, has vigorously pursued an integration agenda with the aim of becoming a monetary union with a common currency. The formation of SADC was aimed at promoting regional cooperation and integration, economic growth, socio-economic development, and durable peace and security among its member states. The SADC has over the years been successful in promoting regional peace and security and economic development for the betterment of the SADC region's most important resources — its people. These include the negotiation of vital government reforms and peaceful transition of political power in Lesotho; resolution of the border dispute between Zambia and the Democratic Republic of Congo; mobilisation of resources to address energy shortages that threaten regional development and economic integration; and many others.

Exchange rate market integration is a particular aspect of the broader issue of financial integration needed for coordination of policy indicators among member countries of a monetary union to achieve a stable monetary union as postulated by the Optimum Currency Area (OCA) hypothesis (Mundell, 1961; McKinnon, 1963; Kenen, 1969). Accordingly, exchange rate markets integration in various economic communities have been studied and cited as a key indicator for stable monetary union (Adam, Agyapong and Gyamfi, 2010; Coulibaly and Gnimassoun, 2013; Zehirun, Breitenbach and Kemegue, 2015; Zehirun, Breitenbach and Kemegue, 2016).

The readiness of SADC to form a monetary union has been studied within the OCA hypothesis and conclusions from these studies suggest that SADC as a whole is not ready yet (Tipoy, 2015; Kumo, 2011; Zerihun, Breitenbach and Kemegue, 2014). Rose (2008) posits that OCA convergence can be achieved ex-post than ex-ante, however, macro-economic coordination is required. Fritz and Mühlich (2010) corroborated this view and assert that uncoordinated macroeconomic policies in south-south economic integration have been a root cause of unsuccessful attempts towards monetary integration. The exchange rate has been cited as central to economic activity, as it affects and is being affected by all other policies, making policy coordination and harmonisation essential for the success of a common currency (Zehirun, Breitenbach and Kemegue, 2015).

In this regard, several studies have delved into the implications of exchange rate coordination on the possible monetary union in SADC area (Khamfula and Huizinga, 2004; Agbeyegbe, 2009; Zehirun, Breitenbach and Kemegue, 2015; Asongu, Nwachukwu and Tchamyou, 2015; Zehirun, Breitenbach, and Kemegue, 2016). Putting Rational Expectation Theory (RET) and the Efficient Market Hypothesis (EMH) under one umbrella and based on the assumptions of the EMH and RET, participants in the exchange rate market act rationally and homogeneously. This is because EMH suggests that market prices reflect all available information, news and events that come to the market are normally distributed leading to a lack of asymmetry of information (Fama, 1970). However, Shiller (2000) argues that most market participants are not smart, but rather follow 'trends and fashion' in their decision making. Therefore, the participants of the exchange rate market (speculators, central banks, dealers, individuals, etc.) are heterogeneous with different information, objective interest and investment behaviour as explained by the Heterogeneous Market Hypothesis (HMH) (Müller et al., 1993). Thus, the price and data generation of the exchange rate are mixed and noisy. In addition, the physical measurements of exchange rate data have been found to suffer from one or more of the following problems: short data span, non-stationarity, non-linearity, and long memory (Xu et al., 2016; Ferreira, Moore and Mukherjee, 2019), limiting its usage in research and practice. This implies that the use of symmetric models in analysing exchange rate data could lead to spurious results and conclusion.

The introduction of empirical mode decomposition (EMD) by Huang et al. (1998) presents a new way of analysing non-linear and non-stationary data. EMD method is intuitive, direct, posteriori and adaptive. EMD performs a time-adaptive decomposition of a complex signal into elementary, almost orthogonal components that do not overlap in frequency. By decomposing a time series into a small number of independent and concretely implicational intrinsic modes based on scale separation, EMD explains the generation of time-series data from an alternative perspective. This would be an improvement in the analysis of exchange rate market data over detrended fluctuation analysis (DFA) (Stošić, Stošić, Stošić, and Stanley, 2015; Ferreira, da Silva and de Santana, 2019) and wavelet transform (Owusu Junior, Adam and Tweneboah, 2017; Meng and Huang, 2019) employed in recent literature. As useful as EMD is, it suffers from the problem of mode-mixing. The ensemble empirical mode decomposition (EEMD) proposed by Wu and Huang (2009) corrects the issue of mode-mixing.

Exchange rate markets integration has been studied using several approaches, with the use of correlations and cointegration tests probably being the most common. As noted by Pereira et al. (2019), the evolution of methodology and data availability has led to multiple types of studies, with linear and non-linear methodologies, but also in different countries and regions. The commonest among these methods are the corelation based approaches such as dynamic correlation analysis (Engle, 2002), asymmetric dynamic correlation (Toyoshima, Tamakoshi, and Hamori, 2012; Tamakoshi and Hamori, 2013), cross-correlation function (Cheung and Ng, 1996; Nakajima and Hamori, 2012) and detrended cross-correlation analysis (DCCA) (Podobnik and Stanley, 2008). The DCCA has become the most extensively adopted methods to measure cross-correlation among non-stationary financial time series.

In this chapter, we propose an EEMD-based DCCA model to build a multifrequency network of exchange rate markets in SADC at different frequency scales. The DCCA is based on Zebende (2011) method to investigate the cross-correlation power laws between two simultaneous time series, called Detrended Cross-Correlation Analysis (DCCA). The EEMD-based DCCA model provides two innovations in examining exchange coordination over the existing studies. Firstly, it offers the opportunity to understand the extent of cross-correlation at different frequency scales. Secondly, the cross-correlation of the exchange rate series provide information on fundamental independence. The study contributes to the literature on exchange rate dependencies by introducing a new approach to the analysis of multifrequency interdependence. The findings allow us to gain new insight into the cross-correlations of exchange rate series.

The analysis of the high-, medium- and low frequencies together with the residue and the original series show that the observed series mimic the behaviour of the high frequency. The results from the residue, representing the deterministic trend, showed that SADC countries' long-run economic fundamentals are linked. These findings suggest the possibility of currency union formation in SADC, albeit policy direction, to address the difference in business cycle synchronisation. The innovation of this chapter is to combine EEMD with DCCA to study the multifrequency cross-correlations of exchange rate markets, which can provide policymakers a deeper understanding of the dynamics of exchange rate markets toward the formation of currency unions.

The rest of the chapter is structured as follows. Section 5.2 presents the literature review; Section 5.3 introduces the methods employed in the study and Section 5.4 describes the exchange rate data of SADC used in the study. Section 5.5 presents the results and analyses. Section 5.6 highlights the policy implications and concludes in Section 5.7.

5.2 Literature Review

Following the Rose (2008) proposition that a group of countries proposing to form a currency union need not meet the OCA convergence criteria ex-ante, but can be achieved ex-post with considerable macro-economic coordination, several studies have emerged to assess the readiness of various economic groupings from this perspective. According to Zehirun, Breitenbach and Kemegue (2015), exchange rate is key and linked to all economic activities, making its policy coordination and harmonisation essential for the success of a common currency. This corroborates the assertion of Inci and Lu (2004) that the exchange rate is sensitive to many economic factors such as money supply, inflation rates, economic growth rates, and trade variables in domestic and foreign economies.

Owing to this, a number of studies seeking to assess the readiness of the regional bloc have focused on the coordination of exchange rates across different blocs (see, Mai, Chen, Zou and Li 2018; Owusu Junior, Adam and Tweneboah, 2017; Abdalla, 2012; Baig, 2001; Baxter and Stockman, 1989; Ghosh, Gulde-Wolf and Wolf, 2002; Hsing, 2007; Lin, 2012; Orlov, 2009; Reboredo and Rivera-Castro, 2013). Mai, Chen, Zou and Li (2018), for example, employed a correlation network to analyse the exchange rate among Asian currencies. Owusu Junior, Adam and Tweneboah (2017), on the other hand, employed wavelet analysis to examine exchange rate coordination in the West African Monetary Zone. Within the SADC region, Khamfula and Huizinga (2004), Agbeyegbe (2009), Asongu, Nwachukwu and Tchamyou (2015), and Zehirun, Breitenbach and Kemegue (2014, 2015, 2016) have examined the extent of exchange coordination and implication for monetary union. Zehirun, Breitenbach and Kemegue (2014, 2015, 2016) employed panel cointegration, unit root and pool mean group to examine the coordination of exchange rates in SADC and made interesting conclusions about the readiness of SADC for the single currency. Similarly, the GARCH framework and other frequency invariant methods have been utilized to understand the extent of exchange rate independence in the SADC area (see, Khamfula and Huizinga, 2004; Asongu, Nwachukwu and Tchamyou, 2015). Unfortunately, these techniques are unable to reveal the dynamic structure of the data and cover the frequency domain of comovements associated with exchange rate markets. The nature of exchange rate data is such that using earlier methodologies is insufficient for a detailed understanding. We need to decompose the exchange-rate data into various frequencies to fully appreciate the dynamic interdependence of the data. Bailliu and King (2005) emphasised the need to use a model that can help economists to extract better high-frequency signals about the economy from apparently noisy exchange rate movements to provide a well-specified model of exchange rate movements over all time horizons.

5.3 Methodology

The methodology employed is such that the EEMD is combined with DCCA to produce cross-correlation coefficients at different frequencies and presented in a network form. The next subsections outline the EEMD and DCCA algorithms.

5.3.1 Ensemble empirical mode decomposition

The EEMD is an improvement of the EMD-based signal processing method to solve the easy mode mixing effect of EMD. The EMD is a dyadic filter bank in the frequency domain (Flandrin, Rilling and Goucalves, 2004). The goal of the empirical mode decomposition is to decompose the original data (non-stationary and non-linear data) into intrinsic mode functions (IMFs) and a residue. The EMD is a fully data-driven decomposition method and IMFs are derived directly from the signal itself. As indicated by Huang et al. (1998), an IMF must satisfy two criteria:

1. The number of extrema and the number of zero crossings must either be equal or differ at most by one.
2. At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

The first condition forces an IMF to be a narrow-band signal with no riding waves. The second condition ensures that the instantaneous frequency will not have fluctuations arising from an asymmetric waveform (Huang et al.,1998).

The IMFs are obtained through a process called the sifting process which uses local extrema to separate oscillations starting with the highest frequency. Given a time series $x(t), t = 1,2,3, \ldots.., M$, the process decomposes it into a finite number of functions, denoted as $IMF_k(t), k = 1, 2, 3, \ldots\ldots, n$ and a residue $r_n(t)$. The residue is the non-oscillating drift of the data. If the decomposed data consist of uniform scales in the frequency space, the EMD acts as a dyadic filter and the total number of IMFs is approximately equal to $n = log_2(M)$ (Flandrin, Rilling and Goucalves, 2004). At the end of the decomposition process, the original time series can be reconstructed as:

$$x(t) = \sum_{i=1}^{n} IMF_k(t) + r_n(t). \tag{5.1}$$

The EEMD makes the signal be continuous at different scales by the uniform distribution feature of the Gaussian white noise frequency. The noises are offset by multiple averaging processing to inhibit and even eliminate noise influence (Kim et al., 2014; Li et al., 2019). For a time series $x(t)$, the EEMD includes the following steps:

a. Generate a new signal of $y(t)$ by superposing to $x(t)$ a randomly generated white noise with an amplitude equal to a certain ratio of the standard deviation of $x(t)$.
b. Perform the EMD algorithm on $y(t)$ to obtain the IMFs.
c. Repeat steps 1 to 2 m times with different white noise to obtain an ensemble of IMFs $\{IMF_k^1(t), k = 1, 2, \dots, n\}, \{IMF_k^2(t), k = 1, 2, \dots, n\}, \dots, \{IMF_k^m(t), k = 1, 2, \dots, n\}$. 5.2
d. Calculate the average of IMFs $\{\overline{IMF_k(t)}, k = 1, 2, \dots, n\}$, where $\{\overline{IMF_k(t)} = 1/m \sum_i^m IMF_k^i(t)\}$. 5.3

The import of the process is that the observed data are a combination of true time series and noise and that the ensemble means of data with different noises are closer to the true time series. Therefore, the addition of white noise as an additional step to the steps in the EMD process may help to extract the true IMF by offsetting the noise through ensemble averaging (Chen and Pan, 2016).

5.3.2 Detrended Cross-Correlation Analysis (DCCA)

Podobnik and Stanley (2008) proposed the detrended cross-correlation analysis (DCCA) as a method for finding long-range cross-correlation properties in the non-stationary time series. The DCCA can analyse the noise effect by removing various types of trend in various box sizes compared to Pearson correlation, cross-correlation function, and dynamic correlation analysis (Piao and Fu, 2016; Horvatia et al., 2011; Shin et al., 2020).

Consider two time series $\{x_i\}$ and $\{y_i\}$ with $i = 1, 2, \dots, N$ equidistant observations. The first step is to accumulate the two variables $\{x_i\}$ and $\{y_i\}$, where

$$X_k = \sum_i^k x_i, \quad Y_k = \sum_i^k y_i \qquad (k = 1, 2, \dots, N). \tag{5.4}$$

Next, the whole sample is divided into N-n overlapping boxes of equal length n observations. For each box, a local trend ($\bar{x}_k$ *and* $\bar{y}_k$) is determined, using ordinary least squares. Subtracting "local trends" $\bar{X}_{k,i}$ from the accumulated data X_k and averaging the sum of squares obtains the local fluctuation $f_{DFA}^2(n,i)$ of the $i-th$ box with box-length $n+1$, where

$$f_{DFA}^2(n,i) = \frac{1}{n-1}\sum_{k=i}^{i+n-1}(X_k - \bar{X}_{k,i})^2. \tag{5.5}$$

To obtain the cross fluctuation of the two time series, we can calculate $f_{DCCA}^2(n,i)$ by replacing the square of X_k with the mutual product of X_k and Y_k where

$$f_{DCCA}^2(n,i) = \frac{1}{n-1}\sum_{k=i}^{i+n-1}(X_k - \bar{X}_{k,i})(Y_k - \bar{Y}_{k,i}). \tag{5.6}$$

Averaging all local fluctuations for $N-n$ overlapping boxes, the fluctuations $F_{DFA}^2(n)$ and $F_{DCCA}^2(n)$ are induced as in Equations 5.7 and 5.8, respectively.

$$F_{DFA}^{2}(n) = \frac{1}{N-n+1}\sum_{k=i}^{N-n+1} f_{DFA}^{2}(n,i) \quad 5.7$$

$$F_{DCCA}^{2}(n) = \frac{1}{n-1}\sum_{k=i}^{N-n+1} f_{DCCA}^{2}(n,i). \quad 5.8$$

The process is repeated for various n length boxes, allowing identification of the relationship between the DCCA fluctuation and n. The long-range cross-correlation $F_{DCCA}(n)$ is given by the power law: $F_{DCCA}(n)\sim n^{\lambda}$ with the λ parameter as the parameter of interest which quantifies the long-range power-law cross-correlations.
The DCCA method measures the covariation between series, but as indicated by Pereira et al. (2019), we use the correlation coefficient created by Zebende (2011) to better understand the degree of the relationship. The Zebende (2011) correlation coefficient is calculated as

$$\rho DCCA = \frac{F_{DCCA}^{2}}{F_{DFA\{x_i\}}F_{DFA\,\{y_i\}}}. \quad 5.9$$

where $F_{DFA\{x_i\}}$ and $F_{DFA\,\{y_i\}}$ are the DFA curves for time series variables x_i and y_i.
The correlation coefficient ranges $-1 \leq \rho DCCA \leq 1$; $\rho DCCA = 0$ indicates no cross-correlation, $\rho DCCA = 1$ means perfect cross-correlation, and $\rho DCCA = -1$ means complete anti-cross-correlation.

5.4 Data Description

The data employed for the study is daily real exchange rates of 15 out of 16 member countries of the SADC from 3rd January, 1994 to 7th January 2019, obtained from Thomson Reuters DataStream. Daily local currency per USD for Angola, Comoros, Botswana, Democratic Republic of Congo, Eswatini (formerly Swaziland), Lesotho, Madagascar, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Tanzania and Zambia were used. The choice of the study window and countries was informed by data availability and the period South Africa Joined SADC. Based on these inclusion criteria, Zimbabwe was excluded from the analysis. The real exchange rate used was nominal domestic currency per US.dollar (USD) multiplied by the domestic consumer price index divided by the US consumer price index. The choice of USD for this analysis is justified by the dominance of USD in international trade by these countries and the extent of dollarisation of most SADC countries. In spite of recent de-dollarisation in Angola, Mozambique and Zambia, the dollar remains dominant in international trade globally including SADC and provides a means of standardising units of pair of currencies (Corrales et al., 2016). The results was implement with R packages libeemd (Luukko et al., 2016) for EEMD and DCCA (Prass and Pumi, 2021). Figure 5.1 shows a graphical representation of the logarithm of exchange rates of SADC countries except for Zimbabwe. All exchange rates in the region trended upward except Angola. Angola has had periods of pegging the Angola Kwanza to the US dollar. The upward trend of all other countries shows that SADC exchange rates have over the period study depreciated against the US dollar. Generally, SADC countries are overvalued and as expected the equilibrium established overtime and therefore the upward trend is not surprising. Again, the trend movements are partially explained by the ‘peso problem’ proposition, which refers to biased expectations of the future exchange rates in small samples when there is uncertainty about when a future policy change will be implemented (Zhou, 2002).

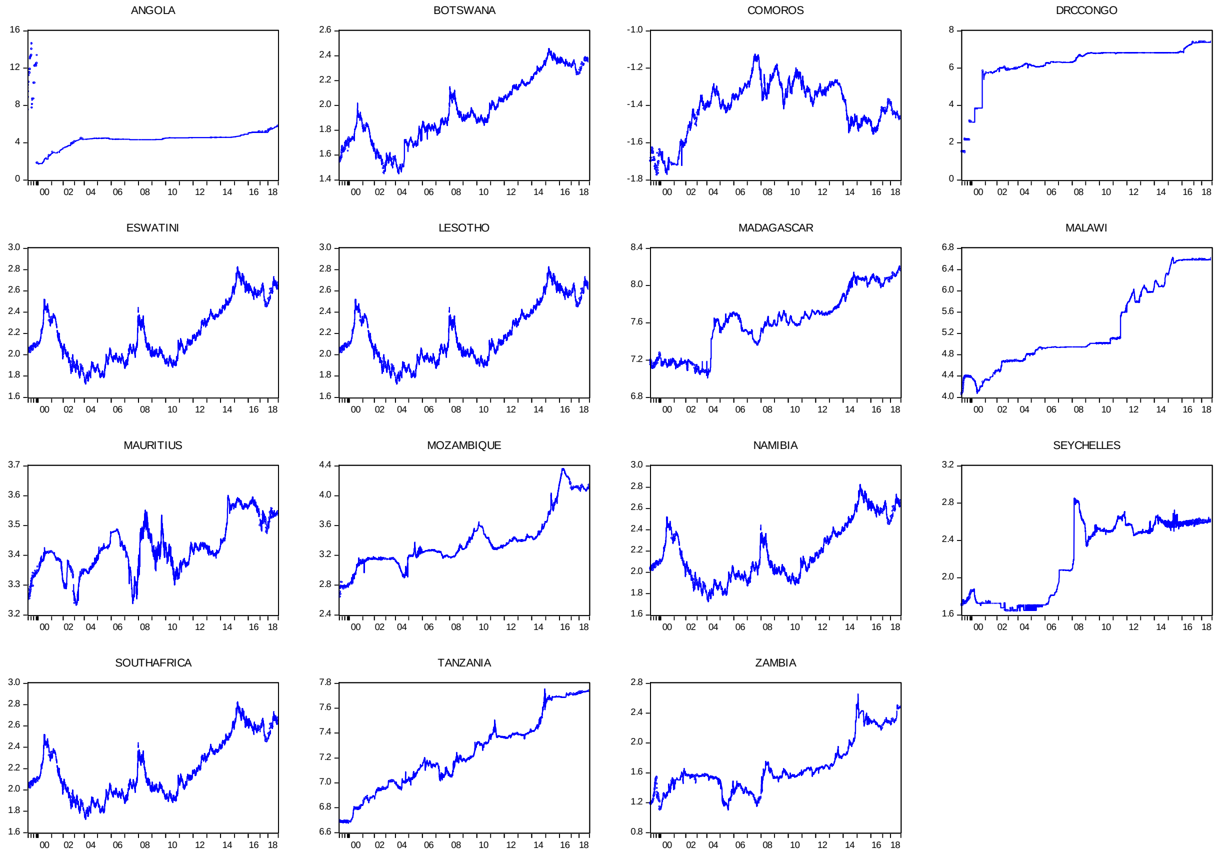


Figure 5.1 Log of the daily real exchange rate series of 15 countries of SADC

5.5 Results and Analysis

We began our analysis by decomposing the daily real domestic currency/USD exchange rate to obtain individual 11 IMFs for each country’s exchange rate and trend using EEMD. Figure

5.2 shows the trends of the IMFs and the residue obtained by EEMD for the case of Tanzania[1]. All IMFs extracted satisfied the necessary and sufficient conditions to be IMF as contained in Huang et al. (1998). The residue is the non-oscillating drift of the data, which is not affected by short-to-medium-term fluctuations, but by the structural changes in the data generation process. It, thus, represents the long-term trend of the data and for this study long trend behaviour of the exchange rate dictated by fundamentals of the economies.

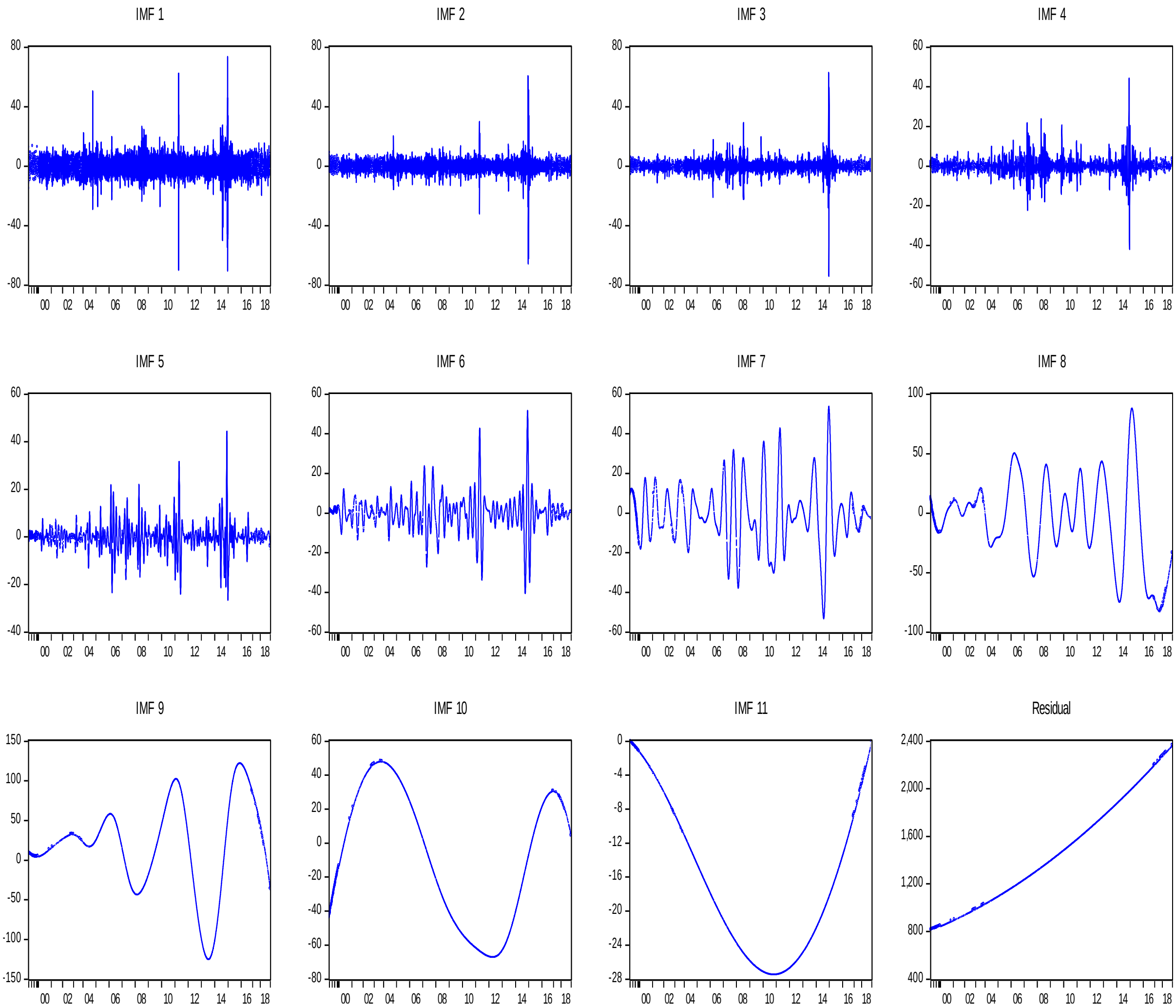


Figure 5.2: IMF of real exchange rate series of Tanzania obtained by EEMD

We proceed to analyse the characteristics of the IMFs obtained by decomposing the series with EEMD through their statistics. Table 5.1 presents the mean period of each IMF measured as the ratio of the total number of points to the number of peaks, Pearson product moment

[1] The IMFs of the remaining countries are available upon request to save space. In all, they look similar.

correlation between each IMF and the original data series, the variance percentage of each IMF to the original data series and to the sum of all IMFs and residue. The variance percentage of each IMF to the original data series explains the contribution of each IMF to the total volatility of the original data (Zhang et al., 2008). These characteristics provide detailed information about the exchange rate behaviours of the countries studied.

Table 5.1: Measures of IMFs and residues for SADC exchange rate markets derived through EEMD

Country	IMF 1	IMF 2	IMF 3	IMF 4	IMF 5	IMF 6	IMF 7	IMF 8	IMF 9	IMF 10	IMF 11	Residue
Angola												
μ	1.41	2.42	4.41	8.02	14.98	30.84	58.63	143.9	791.5	2374.1	2374.1	
ρ	0.00	0.14	0.41	0.37	0.17	0.51	0.56	0.25	0.09	-0.06	-0.05	0.26
σ_1^2	4.99%	14.72%	12.08%	7.96%	25.36%	20.83%	1.64%	1.83%	5.37%	40.25%	13.87%	144.78%
σ_2^2	4.99%	14.72%	12.08%	7.96%	25.36%	20.83%	1.64%	1.83%	5.37%	40.25%	13.87%	144.78%
Botswana												
μ	1.41	2.73	5.22	10.08	20.38	42.03	110.44	263.83	949.8	1187.25	4749	
ρ	0.01	0.02	0.02	0.04	0.04	0.12	0.14	-0.07	0.57	0.80	0.47	0.94
σ_1^2	0.03%	0.01%	0.02%	0.03%	0.06%	0.18%	0.61%	3.12%	4.79%	0.20%	0.03%	77.17%
σ_2^2	0.03%	0.01%	0.02%	0.03%	0.06%	0.18%	0.61%	3.12%	4.79%	0.20%	0.03%	77.14%
Comoros												
μ	1.39	2.78	5.23	10.01	20.64	45.66	105.53	215.86	431.2	678.42	949.8	
ρ	0.03	0.03	0.04	0.06	0.06	0.10	0.23	0.28	0.65	0.48	0.80	0.85
σ_1^2	0.05%	0.03%	0.05%	0.10%	0.29%	0.69%	3.66%	4.09%	5.45%	5.41%	0.16%	48.32%
σ_2^2	0.05%	0.03%	0.05%	0.10%	0.29%	0.69%	3.65%	4.09%	5.45%	5.41%	0.16%	48.29%
DRC												
μ	1.39	2.49	4.68	8.91	19.15	42.78	94.98	237.45	593.63	1187.25	2374.5	
ρ	0.01	0.01	0.01	0.02	0.03	0.03	-0.27	0.18	0.28	-0.26	-0.69	0.94
σ_1^2	0.02%	0.01%	0.01%	0.01%	0.03%	0.05%	0.22%	1.01%	2.05%	5.82%	0.99%	133.16%
σ_2^2	0.02%	0.01%	0.01%	0.01%	0.03%	0.05%	0.22%	1.00%	2.04%	5.82%	0.99%	133.08%
Eswatini												
μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1586	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22	0.18	0.07	0.61	0.63	0.77	0.87
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%
σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%
Lesotho												

μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1586	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22	0.18	0.07	0.61	0.63	0.77	0.87
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%
σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%
Madagascar												
μ	1.39	2.62	4.85	10	20.83	47.97	110.44	226.14	678.43	1187.25	4749	
ρ	0.02	0.02	0.02	0.03	0.06	0.11	0.08	0.02	0.16	0.46	0.42	0.94
σ_1^2	0.04%	0.01%	0.01%	0.02%	0.06%	0.18%	0.58%	1.31%	3.21%	2.87%	0.01%	86.37%
σ_2^2	0.04%	0.01%	0.01%	0.02%	0.06%	0.18%	0.58%	1.31%	3.21%	2.87%	0.01%	86.33%
Malawi												
μ	1.39	2.49	4.73	8.59	19	37.99	91.33	197.88	593.63	1583	4749	
ρ	0.00	0.00	0.01	0.02	0.04	0.02	0.05	-0.09	0.39	0.18	0.58	0.97
σ_1^2	0.01%	0.00%	0.00%	0.00%	0.01%	0.07%	0.19%	0.31%	0.78%	2.69%	0.05%	90.95%
σ_2^2	0.01%	0.00%	0.00%	0.00%	0.01%	0.07%	0.19%	0.31%	0.77%	2.69%	0.05%	90.89%
Mauritius												
μ	1.38	2.66	5.09	10.19	20.92	54.58	163.75	365.31	678.4	1583	4749	
ρ	0.04	0.04	0.04	0.06	0.11	0.18	0.32	0.37	0.20	0.10	0.10	0.74
σ_1^2	0.17%	0.06%	0.04%	0.11%	0.42%	0.84%	5.69%	12.71%	10.89%	1.98%	0.02%	83.27%
σ_2^2	0.17%	0.06%	0.04%	0.11%	0.42%	0.84%	5.69%	12.70%	10.88%	1.98%	0.02%	83.22%
Mozambique												
μ	1.36	2.58	4.82	9.52	20.74	40.94	98.94	296.81	678.4	2374.1	2374.1	
ρ	0.01	0.01	0.01	0.02	0.03	0.00	-0.06	0.14	0.74	0.00	-0.33	0.88
σ_1^2	0.03%	0.01%	0.01%	0.02%	0.05%	0.14%	0.78%	3.33%	7.20%	2.38%	0.33%	83.03%
σ_2^2	0.03%	0.01%	0.01%	0.02%	0.05%	0.14%	0.78%	3.33%	7.19%	2.38%	0.33%	82.98%
Namibia												
μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1583	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22	0.18	0.07	0.61	0.63	0.77	0.87
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%

σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%
Seychelles												
μ	1.37	2.57	4.81	9.37	19.07	43.97	98.94	215.86	593.63	1583	2374.5	
ρ	0.03	0.02	0.03	0.04	0.04	0.05	0.12	0.17	-0.01	0.31	0.84	0.87
σ_1^2	0.07%	0.03%	0.04%	0.06%	0.11%	0.26%	2.45%	2.27%	4.92%	5.38%	1.57%	78.57%
σ_2^2	0.07%	0.03%	0.04%	0.06%	0.11%	0.26%	2.44%	2.27%	4.92%	5.38%	1.57%	78.51%
South Africa												
μ	1.41	2.79	5.12	10.41	20.04	42.4	105.53	249.95	949.8	1583	4749	
ρ	0.02	0.03	0.04	0.06	0.06	0.22	0.18	0.07	0.61	0.63	0.77	0.87
σ_1^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.39%	11.11%	0.88%	0.07%	57.99%
σ_2^2	0.04%	0.03%	0.04%	0.08%	0.14%	0.66%	1.26%	5.38%	11.10%	0.88%	0.07%	57.96%
Tanzania												
μ	1.4	2.5	4.88	9.52	19.62	39.91	105.53	226.14	593.63	1583	4749	
ρ	0.01	0.01	0.02	0.03	0.02	0.03	0.00	-0.27	0.26	-0.11	-0.16	0.98
σ_1^2	0.02%	0.01%	0.01%	0.01%	0.02%	0.04%	0.13%	0.68%	1.65%	0.77%	0.04%	104.69%
σ_2^2	0.02%	0.01%	0.01%	0.01%	0.02%	0.04%	0.13%	0.68%	1.65%	0.77%	0.04%	104.63%
Zambia												
μ	1.39	2.6	5.12	9.89	21.3	42.4	103.24	226.14	474.9	678.43	949.8	
ρ	0.02	0.02	0.02	0.03	0.07	0.14	0.05	-0.35	0.55	0.13	0.10	0.91
σ_1^2	0.03%	0.02%	0.03%	0.07%	0.17%	0.47%	0.99%	6.67%	4.49%	2.88%	0.12%	103.79%
σ_2^2	0.03%	0.02%	0.03%	0.07%	0.17%	0.47%	0.99%	6.67%	4.48%	2.88%	0.12%	103.74%

μ = mean period (days), ρ = Pearson correlation coefficient, σ_1^2 = variance as % of observed, σ_2^2 = variance as % of the sum of all IMFs and Residue

The correlation coefficients and variances of the IMFs and residue, show that the residue which measures deterministic long-term behaviour is the dominant mode in all cases, except for Angola. The means show the average frequency of each IMF. The mean and amplitude of the IMFs were categorized into various frequencies; high frequency (sum of IMFs 1-5), medium frequency (sum of IMFs 6-8) and long frequency (sum of IMFs 9-11) using cluster analysis. The high, medium and long frequencies have mean time-frequency of less than 30 days, between a month and 12 months, and more than 12 months, respectively. Table 5.2 presents the cluster of IMFs with high (period of 1-15 days), medium (up to 144 days), and low frequencies (up to 2374 days). The descriptive statistics of the low, medium, high frequencies and the residue presented in Table 5.2 corroborate with the descriptive statistics of the individual IMFs. The Pearson product moment and Kendall tau-b correlations between each frequency and the original data series, the variance percentage of each frequency in the original data series and the sum of all frequencies and residues indicate that residue is the dominant mode in all cases.

To understand the dynamic relationship at various frequencies for all countries related to the observed exchange rates, we calculate cross-correlation coefficients of daily real exchange rate series, high-, medium-, and low-frequency components of 15 exchange rate market in SADC at 5 different time scale of $n = 10, n = 20, n = 40, n = 80, n = 160$ and $n = 240$. The average cross-correlation coefficient, $\rho DCCA_{ij}$, between country i and j is given as

$$\rho DCCA_{ij} = \frac{1}{5}\sum_{1}^{5} \rho\, DCCA_{ij,n}, \qquad (5.10)$$

where $\rho DCCA_{ij,n}$ is the cross-correlation coefficient between countries i and j at time scale of n.

For brevity, the cross-correlation coefficients between the respective original data, low, medium, high frequencies, and the residue are presented in correlation network form. The correlation network formed based on $\rho DCCA$ at the various frequencies and the original are presented in Figures 5.3 (a-f). In Figures 5.3(a) and (d), only Eswatini, Lesotho, Namibia and South Africa were found to be linked indicated by the size of the edge of the network. The similarity and resemblance of the correlation network are surprising as the Pearson correlation showed a weak relationship. However, Pearson correlation is deficient in analysing series with noise compared to the DCCA (Piao and Fu, 2016; Horvatia et al., 2011; Shin et al., 2020).

Table 5.2: Descriptive statistics of the reconstructed series and the residue for SADC exchange rate markets derived through EEMD

	Pearson correlation coefficient				Kendall tau-b				Variance as % of observed				variance as % of the sum of all IMFs and Residue			
Country	HFRQ	MFRQ	LFRQ	RESID	HFRQ	MFRQ	LFRQ	RESID	HFRQ	MFRQ	LFRQ	RESID	HFRQ	MFRQ	LFRQ	RESID
Angola	0.49	0.56	-0.04	0.26	0.00	-0.14	0.15	-0.11	62.87	35.96	104.92	144.79	62.87	35.95	104.91	144.78
Botswana	0.06	0.02	0.63	0.94	0.04	0.08	0.46	0.74	0.22	4.53	7.14	77.17	0.22	4.53	7.14	77.14
Comoros	0.09	0.32	0.72	0.85	0.06	0.22	0.46	0.60	0.75	11.13	16.82	48.32	0.75	11.13	16.81	48.29
Dr Congo	0.03	0.05	-0.23	0.94	0.03	0.02	-0.26	0.88	0.10	1.74	15.47	133.16	0.10	1.74	15.46	133.08
Eswatini	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
Les Otho	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
Madagascar	0.06	0.08	0.41	0.94	0.08	0.10	0.18	0.74	0.20	2.51	7.47	86.37	0.20	2.51	7.46	86.33
Malawi	0.03	-0.02	0.35	0.97	0.02	-0.04	0.01	0.86	0.04	0.80	4.67	90.95	0.04	0.80	4.66	90.89
Mauritius	0.12	0.45	0.19	0.74	0.08	0.36	0.13	0.49	1.10	24.87	18.00	83.27	1.10	24.86	17.99	83.22
Mozambique	0.03	0.09	0.45	0.88	0.02	-0.04	0.01	0.79	0.15	5.40	16.04	83.03	0.15	5.39	16.03	82.98
Namibia	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
S. Africa	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
Seychelles	0.06	0.20	0.40	0.87	0.07	0.16	0.23	0.60	0.41	5.78	19.09	78.57	0.41	5.78	19.08	78.51
Tanzania	0.04	-0.21	0.11	0.98	0.03	-0.14	-0.02	0.92	0.10	1.05	3.57	104.69	0.10	1.05	3.57	104.63
Zambia	0.07	-0.24	0.40	0.91	0.05	-0.06	0.09	0.57	0.43	9.61	12.44	103.79	0.43	9.61	12.43	103.74

HFRQ=High Frequency series, MFRQ=Medium Frequency series, LFRQ=Low Frequency series

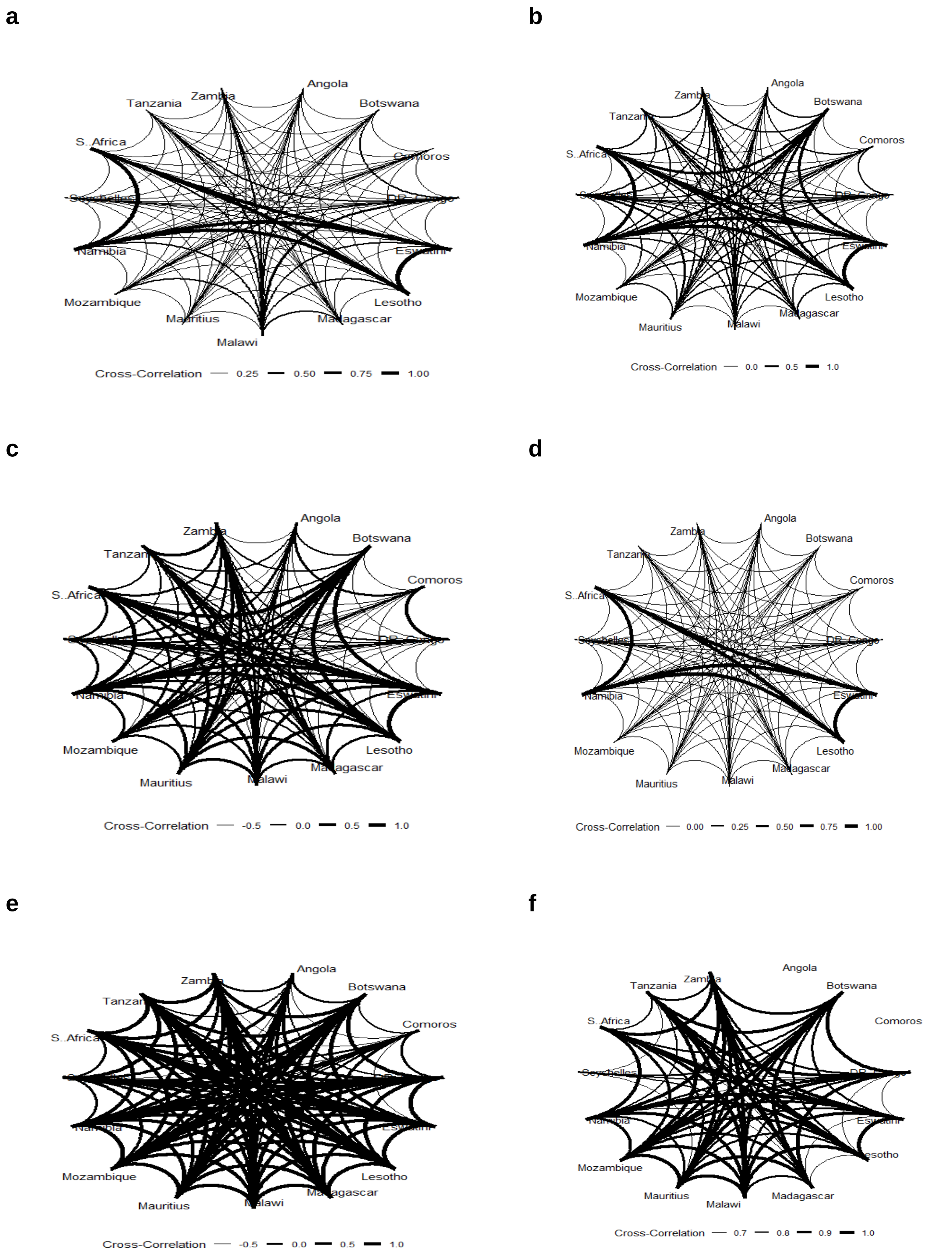


Figure 5.3: Network Plots of Various Frequencies: *(a) The DCCA cross-correlation of high-frequency series. (b) The DCCA cross-correlation of medium frequency series. (c) The DCCA cross-correlation of low-frequency series. (d) The DCCA cross-correlation of the original series. (e) The DCCA cross-correlation of residue series (f) The significant DCCA cross-correlation of residue series based on Podobnik and Stanley(2008)*

Again, the four countries have been members of the Common Monetary Area (CMA) since 1975, in which currencies of Eswatini, Lesotho and Namibia are issued at par with South African Rand (Masha, Wang, Shirono and Harris, 2007, Adam et al., 2021).
In Figures 5.3 (b) and (c), new countries such as Botswana and Mauritius emerged as having a strong correlation with Eswatini, Lesotho, Namibia and South Africa in the medium frequency, an indication of business cycle synchronisation. The emergence of Botswana and Mauritius could also be explained by their economic performance which satisfies the criteria for monetary union (Jefferis, 2007). Botswana, in particular, has had historic ties with South Africa as its currency once pegged to the South African Rand. This finding partially contradicts Nzimande and Ngalawa (2016) that only CMA countries are candidates for currency union within SADC.

The extent of correlation became clearer where all the noises were removed, the correlation network formed from the DCCA cross-correlation presented in Figure 5.3 (e) showed stronger linkage among most of the countries. We also observed Angola showing anticorrelation between most of the SADC countries.

To better understand the dynamics of linkages of the deterministic trend, we formed a network by the values of the significance of Podobnik and Stanley (2008), linking the edges of $\rho DCCA \geq 0.66$ for the residue as indicated in Figure 5.3(f). We observed a very strong cross-correlation among most SADC countries except Angola and Comoros. The findings show that there is a high cross-correlation between the long-term economic fundamentals of the included countries. This finding is consistent with Redda and Muzindusti (2017) that the exchange rate markets in SADC are driven mostly by long-term fundamentals, which, in turn, are most likely rooted in macroeconomic economic fundamentals. Similarly, Mpofu (2016) observed that money supply, foreign reserve, and output systematically affect exchange rate movement in South Africa. It is also consistent with the findings of some other developing economies, that government consumption and investment significantly drive the exchange movement in Sub-Saharan Africa (Ibhagui, 2017). The observed exchange rate differences in SADC, which are caused by short-term dynamics, are driven by noise from such activities as speculation, short-term policies, and timing of the response to external shocks. This finding contravenes Jefferis (2007) and Hanohan and Lane (2000) who argued that speculative activities are unlikely because SADC currencies are 'below the radar screen' of international speculators, reducing exposure to contagion problems. This departure may be as result of the robustness of this study as against prior studies. Significantly, separation of the exchange rate series into short-, medium-, and long-terms delineate the influence of noise. Toward a currency union, the economies require to be harmonised as stipulated in the expanded optimum currency decalogue to minimise short-term deviations. (Edwards, 2006; Bayoumi and Eichengreen, 1997; De Grauwe, 2001; Tavlas, 1993).

5.6 Policy Implications of findings

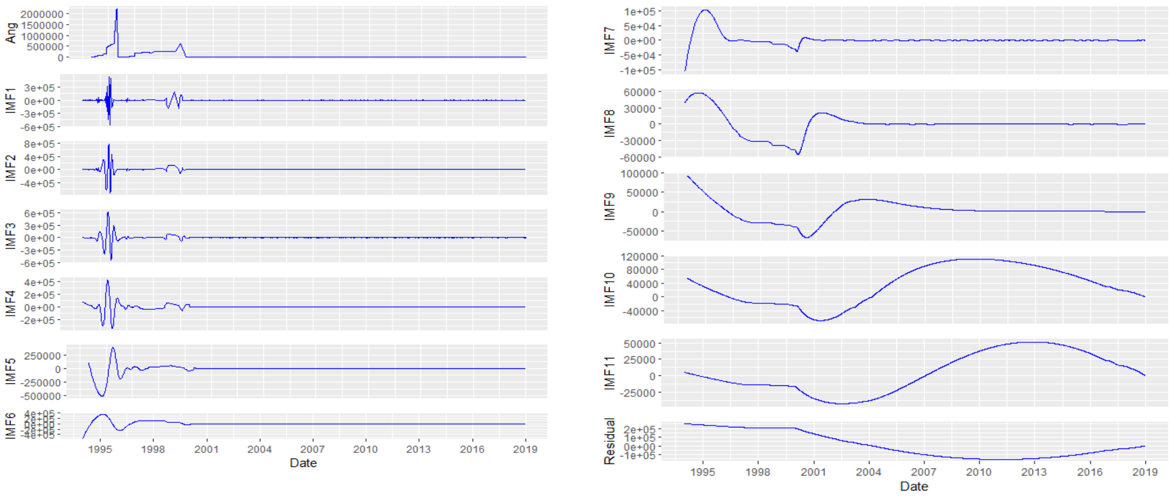
Through our empirical analysis, we have highlighted the dynamic interdependence of exchange rate markets in the SADC region. We also examined the important fundamental behaviours of the currencies of the bloc. Based on the objective of this chapter, we then examine the implications of our findings on policy formulation in the region.

The search for the extent of exchange rate market coordination among SADC countries is still ongoing with mixed results. The intrinsic components have been generally taken for granted in the analysis of exchange rate markets. However, the exchange rate has been found to be chaotic and noisy, thereby influencing its modelling (Bildirici and Sonüstün, 2019). The cross-correlation of the high-frequency component of the exchange rate markets mimics that of the original network. This is a manifestation that the speculative activity across the region is symmetrical and indicates the behaviour of the actual exchange rate. The cross-correlation network of medium-and low-frequencies shows that Botswana and Mauritius can easily join the CMA of Eswatini, Lesotho, Namibia and South Africa as there is some level of business synchronisation among these countries.

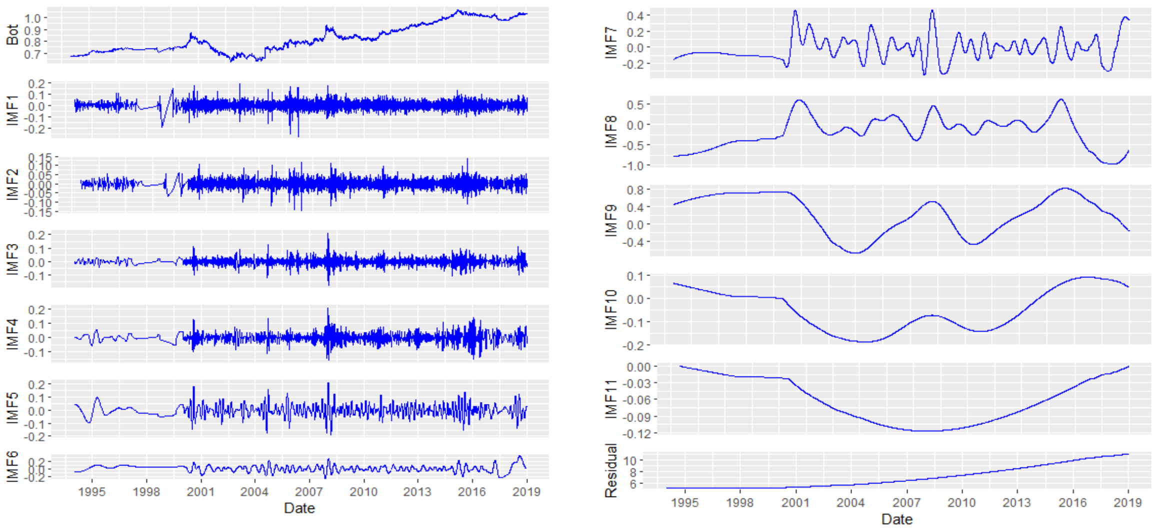
The significant cross-correlation of long-term trend proxy by the residue gives a wider understanding of the possibility of forming a currency union in SADC excluding Angola, Comoros and Zimbabwe. This can be done by expanding the CMA to include the remaining countries. In simple terms, our findings suggest that there is an emerging possibility of SADC as a whole to become a currency area, since CMA countries, Botswana and Mauritius appear to exhibit synchronisation of business cycles, thus their symmetric shocks could be addressed by a single monetary policy (Hitaj, Shapiro, Kolerus and Zdzienicka, 2013, Nzimande and Ngalawa, 2016). This could be piecewise extended to include all SADC countries, except Angola, Zimbabwe and Comoros because of evidence of coordination of their long-term fundamentals with policy direction to synchronize business cycles post ante. Figure 5.4 shows similarity in the structure of about 12 out of 15 countries included in the study with CMA countries being stronger.

5.7 Conclusion

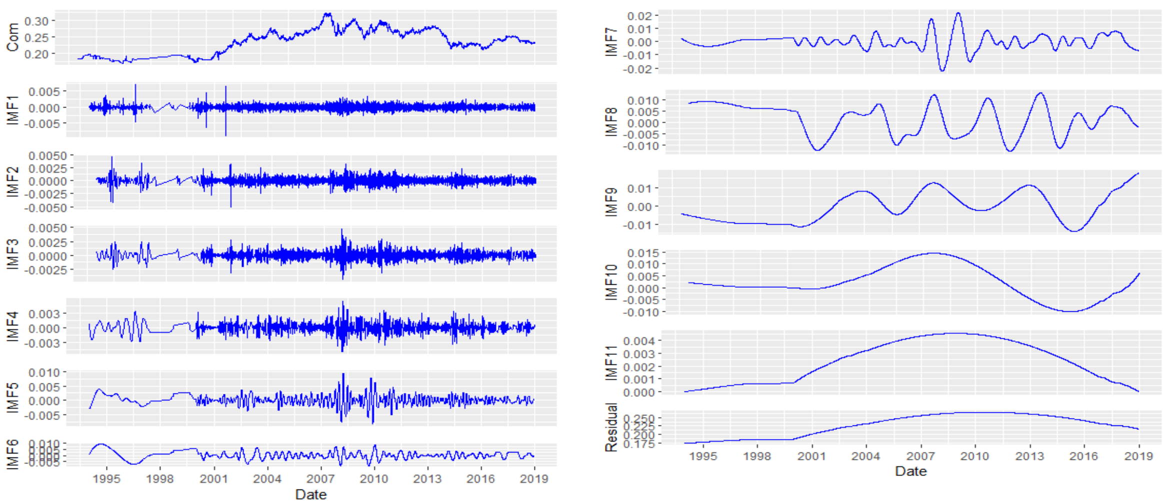
The difference in the behaviour of exchange rate participants makes the exchange rate modelling difficult because of the potential to be chaotic and non-stationary. This chapter used the EEMD-based DCCA method to study the multifrequency cross-correlation of daily exchange rate markets of 15 countries. We first used the EEMD method to decompose the exchange rate series into IMFs and then clustered them into a high-, medium- and low frequency and residue series. The DCCA cross-correlation coefficients of the reconstructed series were used to form a correlation network. The correlation network of the high frequency and the original showed the resemblance of their linkages. In both cases, Eswatini, Lesotho, Namibia and South Africa were found to be cross-correlated. The correlation network of medium-, and high-frequencies and residue showed an increased correlation with increasing frequency of the series and the long-term trend of exchange rates of SADC countries are stronger. Comparing the original series with the levels of decomposed series, the sources of deviation of the exchange rate markets have been identified as the high-frequency component which is linked to speculation activities, short-term policies, and timing of the response to external shocks. Therefore, the EEMD-based DCCA method can help obtain more internal characteristics and detailed information on exchange rate cross-correlation, which will help policymakers make a more accurate analysis of exchange rate dynamics. The innovation of this chapter is to combine EEMD with DCCA to study the multifrequency cross-correlations of exchange rate markets, which can provide policymakers with a new methodology for understanding of the essential characteristics and internal structures of exchange rate markets of SADC.



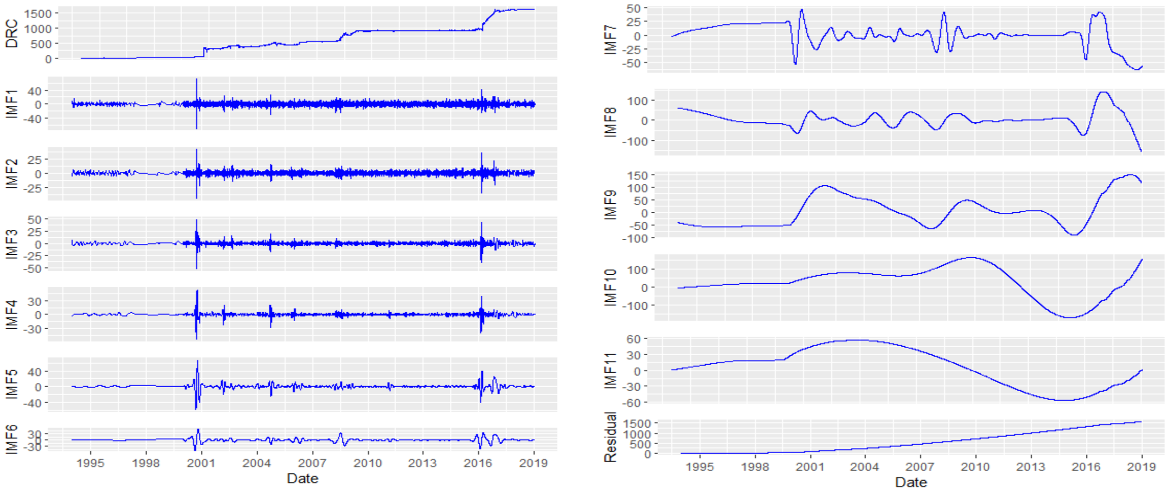
Angola



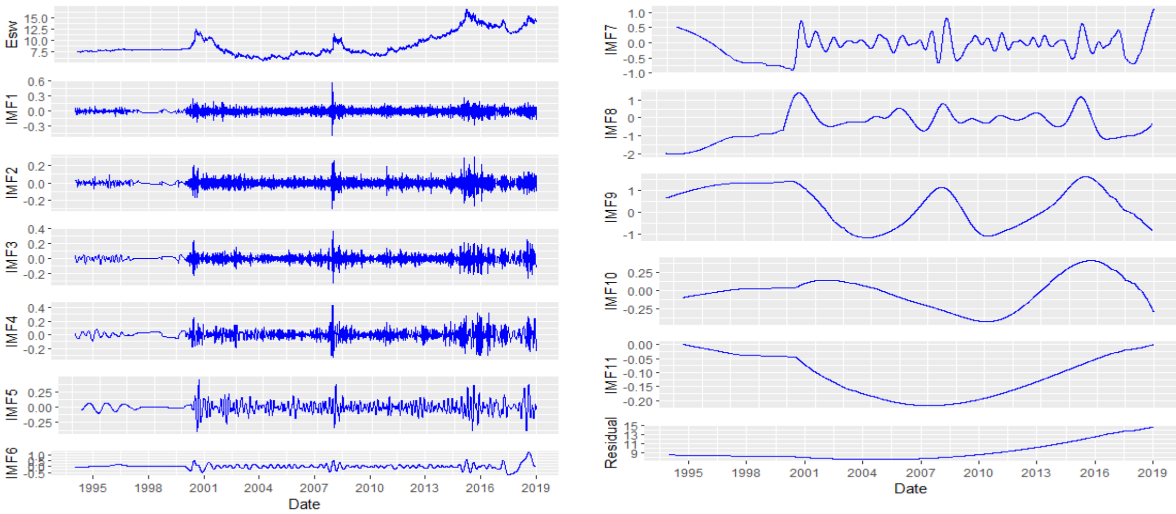
Botswana



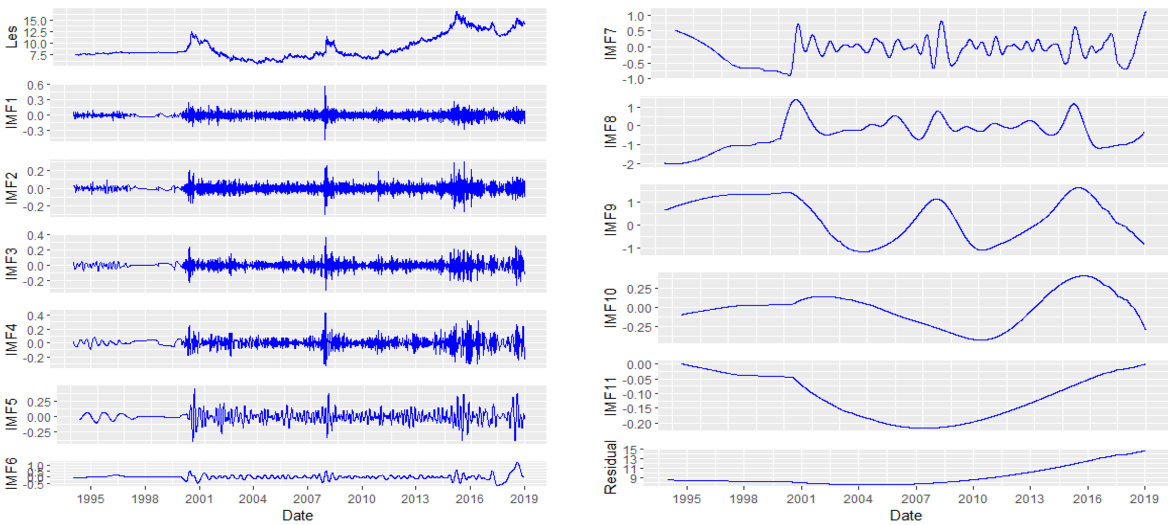
Comoros



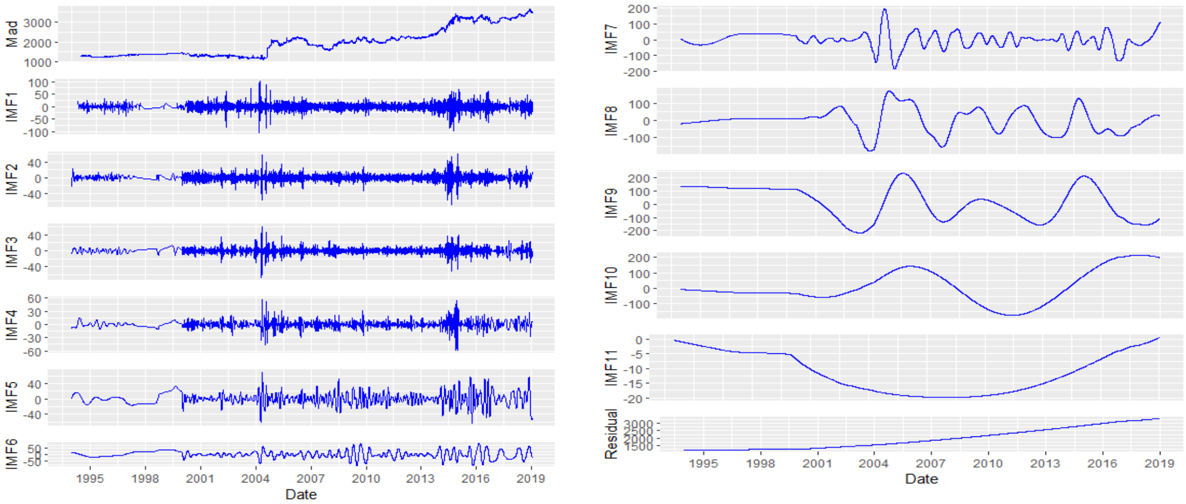
Democratic Republic Congo



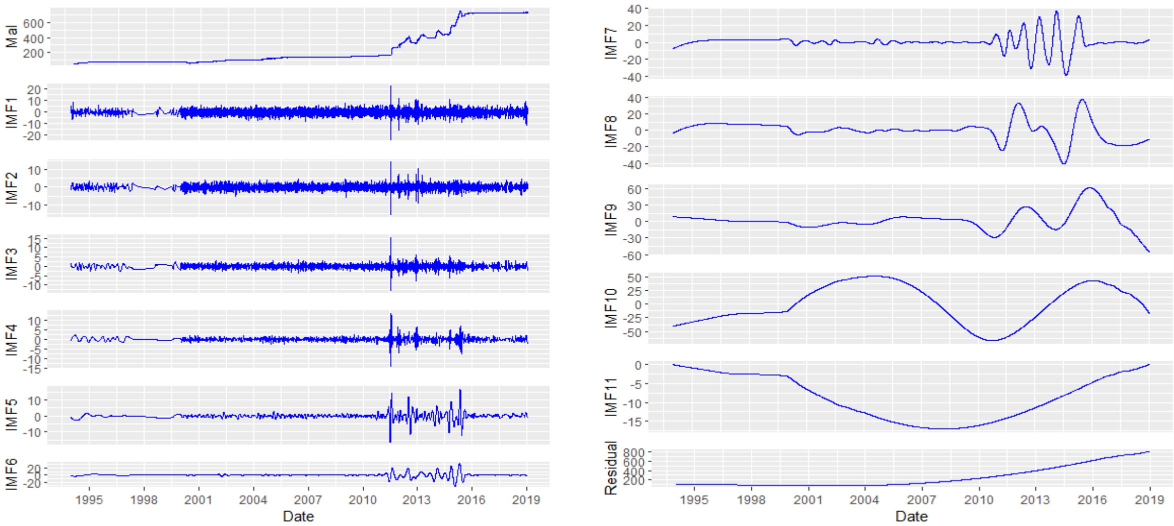
Eswatini



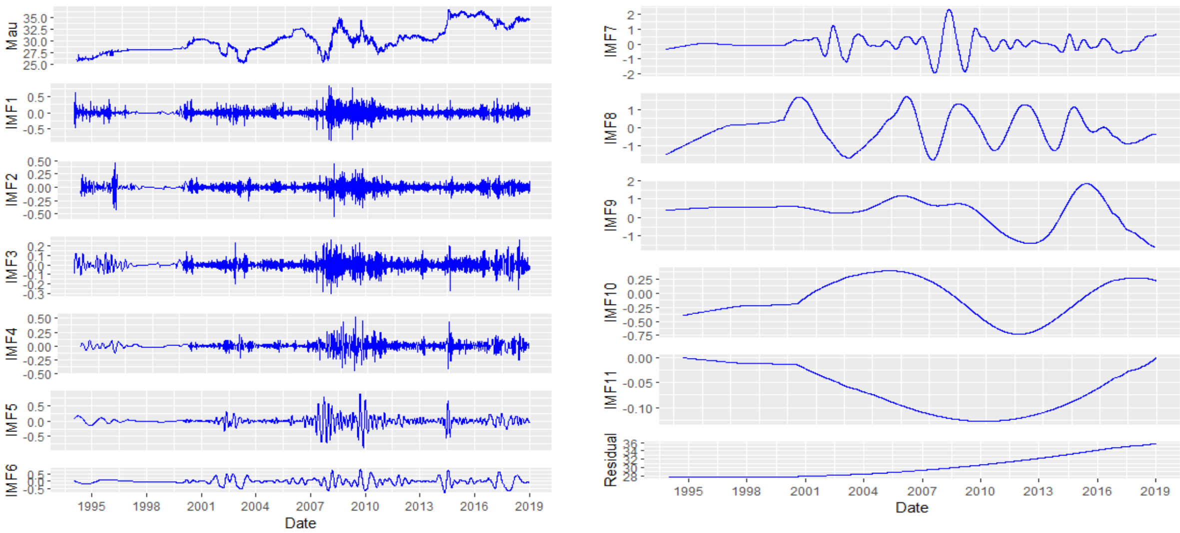
Lesotho



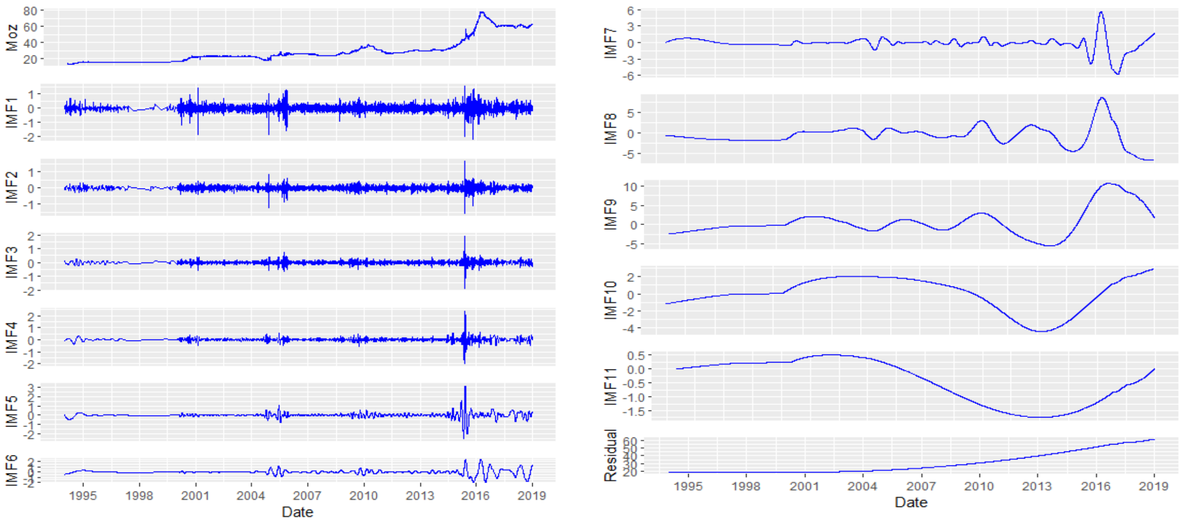
Madagascar



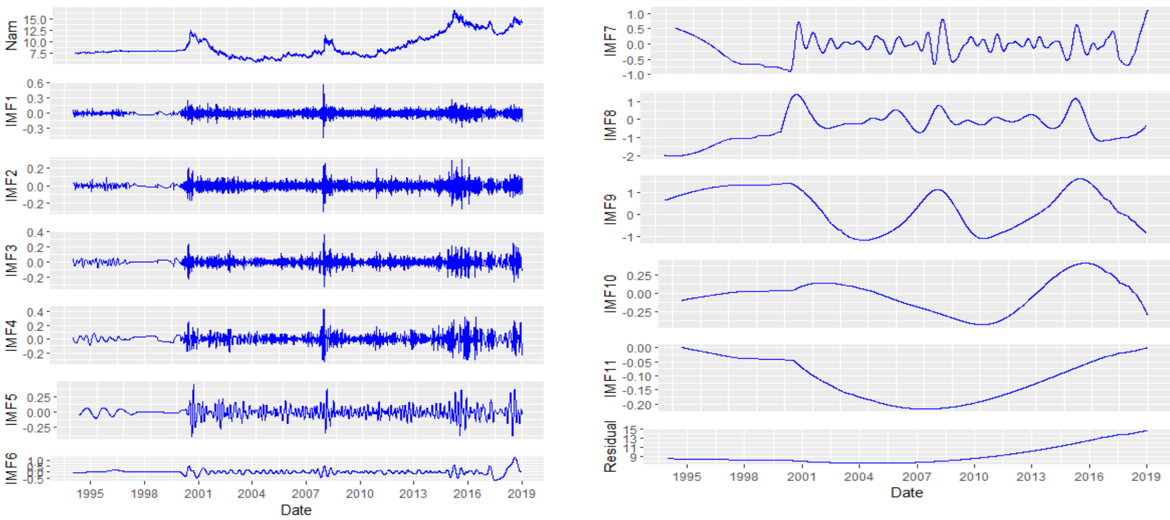
Malawi



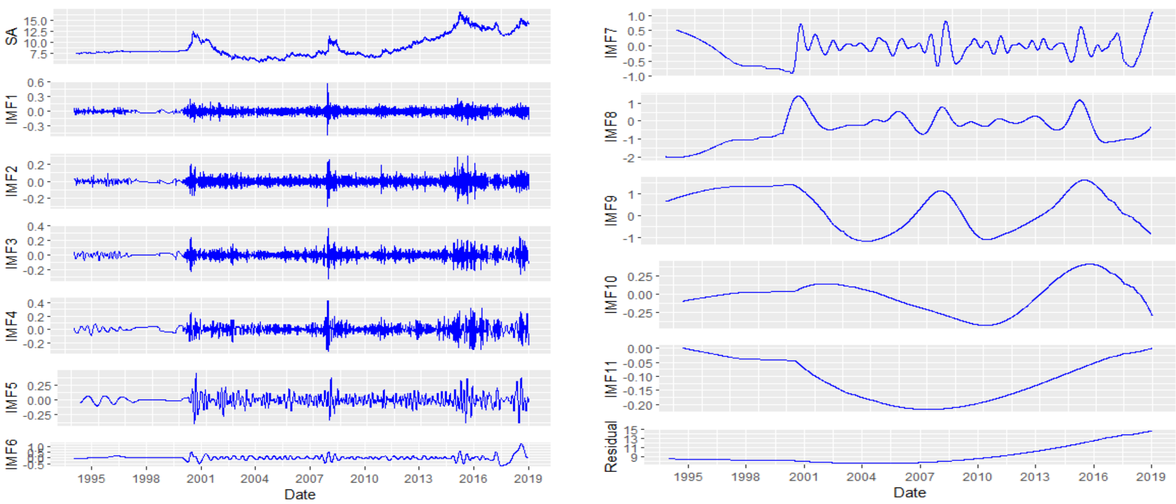
Mauritius



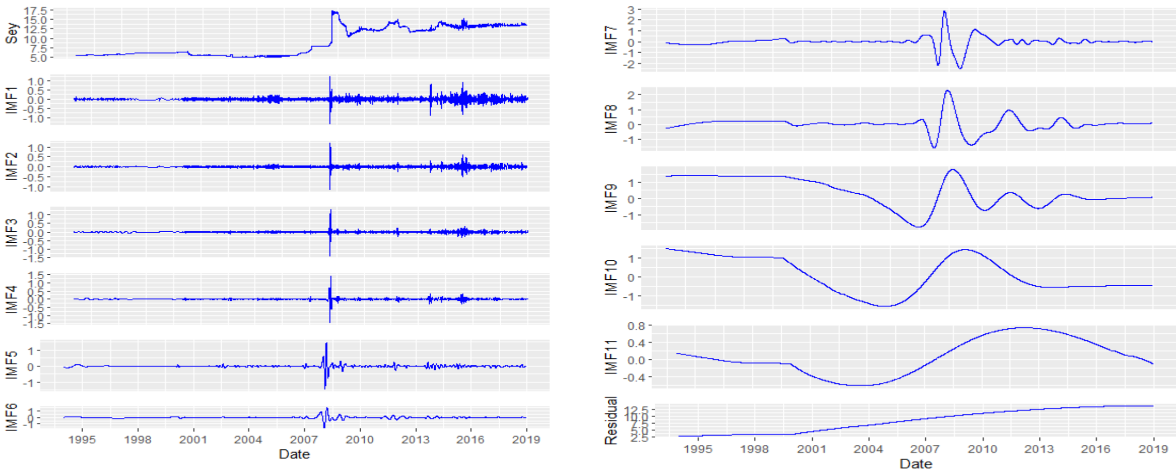
Mozambique



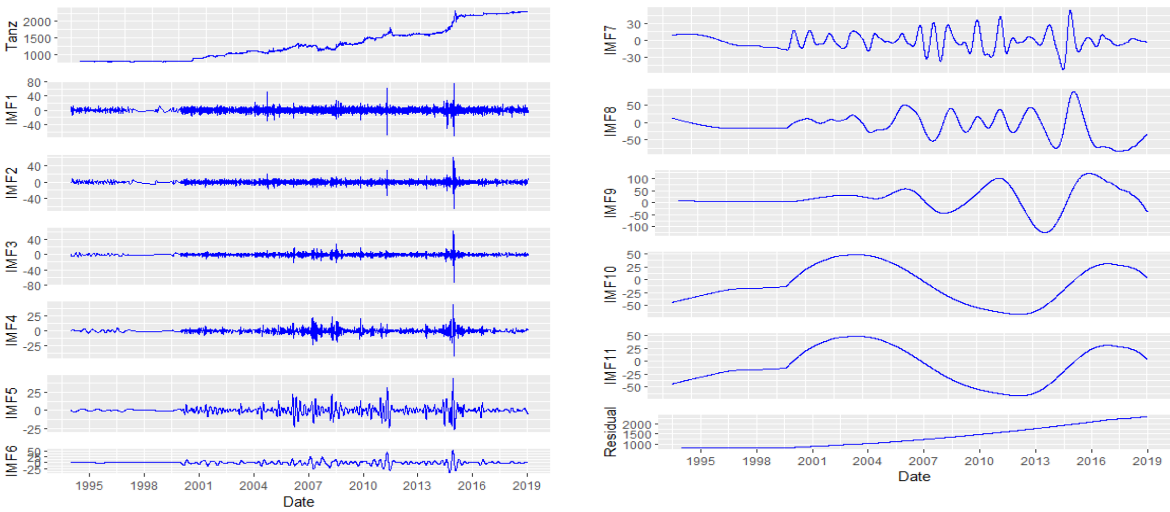
Namibia



South Africa



Seychelles



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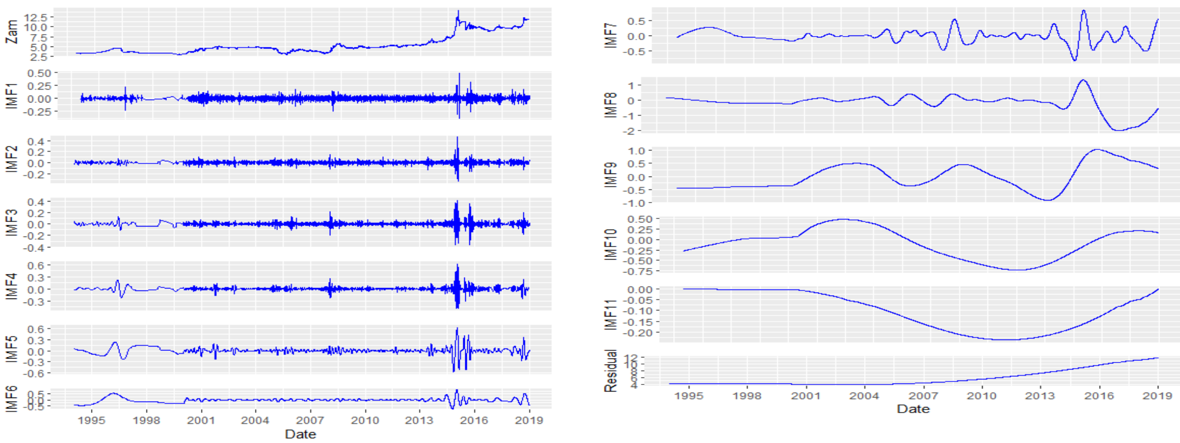

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Figure 5.4: Supplementary Plot of IMFs from EEMD

CHAPTER 6

A New EEMD-Effective Transfer Entropy-based Methodology for Exchange Rate Market Information Transmission in SADC

Chapter Summary

The desire to form monetary unions among regional blocs in Africa has necessitated the need to assess the degree of financial systems' interdependencies in Africa economic blocs for their suitability to have a harmonised economic policy and eventual monetary unions. In this regard, SADC has pursued policies to harmonise and integrate its financial system as a precursor to its intended monetary union. However, the ensuing risk among exchange rates of economies in SADC is presumed to rise during severe uncertainties. This study examines the degree of asymmetry and non-linear directional causality between exchange rates in SADC in the frequency-domain. We employ both the Ensemble Empirical Mode Decomposition (EEMD) and the Rényi effective transfer entropy techniques to investigate the multi-scale information that might be disregarded, and further quantify the directional flow of information. Analysis of the study is presented for high-, medium-, and low-frequencies, representing short-, medium-, and long-terms respectively, in addition to the residue (fundamental feature), achieving four frequency-domains. We find a mixture of asymmetric and non-linear bi-directional and unidirectional causality between exchange rates in SADC for the sampled period. The study reveals a significant positive information flow in the high frequency, but negative flow in the medium and low frequencies. In addition, we gauge a bi-directional significant negative information flow within all the 15 economies for the residue. This suggests a higher risk of uncertainties in exchange rates of SADC. Our findings for low probability events at multi-scales have implications for the direction of the future of the SADC monetary union. This calls for further sustained policy harmonisation in the region.

6.1 Introduction

The desire to integrate economies of Africa regional blocs as enshrined in Article 3 of the Africa Union Constitution has necessitated the need to ensure coordination of macroeconomic factors. The anticipated benefits of such economic integration are well-research and documented (Adam et al., 2021a).

The exchange rate market integration has been cited as a key indicator for stable economic integration because of its pass-through effect on other financial markets (Adam et al., 2021b).

The rising uncertainty in the global economic development makes the comovement of exchange rates important for the formation of monetary unions. The SADC formed in 1999 for decades has pursued policies to harmonise and integrate the financial system with the intention to form a monetary union. This hinges on the perceived benefits of membership in a monetary union. Thus, stronger exchange rate, low exchange risk and price stability.

Several studies have attempted to depict and determine precisely the linear and non-linear linkages between exchange rate markets aiming to form a monetary union as the understanding of the interconnectivity are important for the conduct of monetary policy and how to deal with activities of market participants. Among related studies, the linkage of the exchange rate in West Africa Monetary Zones (Alagidede, Tweneboah and Adam, 2008; Adu, Litsios and Baimbridge, 2019; Owusu Junior, Adam and Tweneboah, 2017) has been severally investigated. Within the SADC bloc, several studies have attempted to model the interaction of exchange rate markets (Adam et al., 2021a; 2021b; Agbeyegbe, 2008; Khamfula and Huizinga, 2004; Zehirun, Breitenbach and Kemegue, 2015; Zehirun, Breitenbach and Kemegue, 2016). Except for Adam et al. (2021a; 2021b), the identified studies seldom studied the comovement of the exchange rate from a multi-scale perspective. The exchange rate market is a complex system with varying participants, varying objectives, investment preferences and motives as depicted by the Heterogeneous Market Hypothesis (Adam et al., 2021a; 2021b; Zehirun, Breitenbach and Kemegue, 2016; Muller et al, 1993; Dacorogna et al., 1998). Thus, exchange rate data could be noisy, non-stationary, non-linear and mixed (Xu, Shang and Lin, 2016; Ferreira, Moore and Mukherjee, 2019). These intrinsic complexities of exchange rate data question the appropriateness of the use of static models in exchange rate studies.

Accordingly, recent studies on coordination of exchange rate markets have relied on models that can extract better high-frequency signals about the exchange rate to deal with its apparent noisy behaviour to provide better understanding (Adam et al., 2021a; 2021b; Owusu Junior, Adam and Tweneboah, 2017; Khuntia and Pattanayak, 2020; Meng and Huang, 2019; Qureshi and Aftab, 2020). This is aimed at analysing exchange rate markets from the time-frequency domain perspective instead of the traditional time-domain viewpoint. The Fourier and wavelet transform approaches of studying the time-frequency domain has been widely used in this regard (Asafo-Adjei et al., 2020; Mariani et al., 2020; Owusu Junior, Tweneboah and Adam, 2019). Huang et al. (1998) noted that Fourier-based approaches are not data-adaptive, unable to capture the time-varying characteristics of the neural signal and only designed for the frequency analysis of stationary time series. The Wavelet transform is however counterintuitive in its interpretation and nonadaptive.

The empirical mode decomposition (EMD) by Huang et al. (1998) provides a new perspective of analysing non-linear and non-stationary data in the time-frequency domain based on the direct extraction of signal energy associated with various intrinsic time scales. The EMD is adaptive, fully posterior, and physically meaningful which makes it superior to its alternatives (Huang et al., 1998; Liu et al., 2021). In addition, because the EMD process is completely based on the local time scales of time series, with no prior basis, the extracted oscillations reflect the time series accurately. However, the conventional EMD suffers from the problem of mode-mixing which is corrected by the ensemble empirical mode decomposition (EEMD) proposed by Wu and Huang (2009).

The extant literature on exchange rate markets comovements has relied on spillover index, cross-correlation, the Granger causality test, the vector autoregressive model, and the generalised autoregressive conditional heteroskedasticity model. The many limitations associated with the applications of these econometric models emanating from the need to satisfy stationarity and distributional properties hampers their applications. The transfer entropy proposed by Schreiber (2000) come in handy to quantify information transfer or information flow between variables in a system with no assumptions about the distribution or intercorrelation of the original variables. The transfer entropy is effective in identifying linear and non-linear relationships between variables; asymmetric and built upon transition probability; practical in all systems and does not require prior specification model (Montalto, Faes and Marinazzo, 2014; Mao and Shang, 2017).

In this chapter, we propose EEMD-Effective Transfer Entropy (EEMD-ETE)-based methodology to analyse exchange rate markets information transmission. The effective transfer entropy is based on Renyi Transfer Entropy (Renyi, 1970). The EEMD-ETE methodology provides two novelties in studying exchange rates comovement compared to previous studies. Firstly, it examines exchange rate series from the perspective of information transmission and quantification to capture information spillover and interactions among different markets, which provides useful information on the spillover direction between variables (Ji et al., 2019). Secondly, financial time series often exhibit different characteristics at different time frequencies, and the relations between different variables vary widely across time scales (Geng, Ji and Fan, 2017; Sun et al., 2020). The utilization of EEMD-ETE, therefore, offers the opportunity to understand the extent of information transmission at different frequency scales. Lastly, it delineates the influence of noise from the quantification of information flow across the exchange rate markets.

We find a mixture of asymmetric and non-linear bi-directional and unidirectional causality between exchange rates in SADC for the sampled period. The study reveals a significant negative information flow in the medium (medium-term) and low frequencies (long-term), but a more positive flow in the high frequency (short-term). However, from the fundamental feature represented by the residue, we gauge a bi-directional significant negative information flow within all the 15 economies. This suggests a higher risk of uncertainties in exchange rates of SADC.

The rest of the chapter is structured as follows. Section 6.2 introduces the methods employed in the study, Section 6.3 describes the exchange rate data of SADC used in the study and Section 6.4 presents the results and analysis. Section 6.5 highlights the policy implications and conclusion.

6.2 Methodology

We initially present the EEMD technique, followed by the transfer entropy. Thus, the outcome generated from the EEMD is used as input data for the effective transfer entropy estimations.

6.2.1 Ensemble empirical mode decomposition

The EEMD is an improvement of the EMD-based signal processing method to solve the easy mode mixing effect of EMD. The EMD is a dyadic filter bank in the frequency domain (Flandrin, Rilling and Goucalves, 2004). The goal of the empirical mode decomposition is to decompose the original data (non-stationary and non-linear data) into IMFs and a residue. The EMD is a fully data-driven decomposition method and IMFs are derived directly from the signal itself. As indicated by Huang et al. (1998). An IMF must satisfy two criteria:

1. The number of extrema and the number of zero crossings must either be equal or differ at most by one.
2. At any point, the mean value of the envelope is defined by the local maxima and the envelope defined by the local minima is zero.

The first condition in criterion 1 forces an IMF to be a narrow-band signal with no riding waves. The second condition in criterion 2 ensures that the instantaneous frequency will not have fluctuations arising from an asymmetric waveform (Huang et al.,1998).

The IMFs are obtained through a process called the sifting process which uses local extrema to separate oscillations starting with the highest frequency. Given a time series $x(t), t = 1,2,3, \ldots, M,$ the process decomposes it into a finite number of functions, denoted as $IMF_k(t), k = 1, 2, 3, \ldots, n$ and a residue $r_n(t)$. The residue is the non-oscillating drift of the data. If the decomposed data consist of uniform scales in the frequency space, the EMD acts as a dyadic filter and the total number of IMFs is approximately equal to $n = log_2(N)$ (Flandrin, Rilling and Goncalves, 2004). At the end of the decomposition process, the original time series can be reconstructed as:

$$x(t) = \sum_{i=1}^{m} IMF_k(t) + r_m(t). \quad (6.1)$$

The EEMD makes the signal be of continuity at different scales by the uniform distribution feature of the Gaussian white noise frequency. The noises are offset by multiple averaging processing to inhibit and even eliminate noise influence (Kim et al., 2014; Li et al., 2019). For a time series $x(t)$, the EEMD includes the following steps:

a. Generate a new signal of $y(t)$ by superposing to $x(t)$ a randomly generated white noise with an amplitude equal to a certain ratio of the standard deviation of $x(t)$.
b. Perform the EMD algorithm on $y(t)$ to obtain the IMFs.
c. Repeat steps 1 to 2 for m times with different white noise to obtain an ensemble of IMFs $\{IMF_k^1(t), k = 1, 2, \ldots, n\}$, $\{IMF_k^2(t), k = 1, 2, \ldots, n\}$,…, $\{IMF_k^m(t), k = 1, 2, \ldots, n\}$. (6.2)
d. Calculate the average of IMFs $\{\overline{IMF_k(t)}, k = 1, 2, \ldots, n\}$, where $\{\overline{IMF_k(t)} = 1/m \sum_i^m IMF_k^i(t)\}$. (6.3)

The import of the process is that the observed data are a combination of true time series and noise and that the ensemble means of data with different noises are closer to the true time series. Therefore, the addition of white noise as an additional step to the steps in the EMD process may help to extract the true IMF by offsetting the noise through ensemble averaging (Chen and Pan, 2016).

6.2.2 Measuring information flows using Rényi transfer entropy

Before we discuss the Rényi transfer entropy, we present the concept of Shannon entropy as a measure of uncertainty upon which transfer entropy is embedded in information theory (Behredt et al., 2019; Adam, 2020). We consider a probability distribution of diverse results of a given experiment p_j. Following Hartley (1928), each symbol's average information is specified as:

$$H = \sum_{j=1}^{n} P_j log_2 \left(\frac{1}{P_j}\right) \text{ bits} \quad (6.4)$$

where n is number of distinct symbols concerning the probabilities P_j.

The concept of entropy, later referred to as Shannon entropy, was introduced in 1948 by Shannon (1948). It proffers that, for a discrete random variable (J) with probability distribution ($P(j)$), the average number of bits needed to optimally encode independent draws (Behredt et al., 2019) can be presented as

$$H_J = -\sum_{j=1}^{n} P(j) log_2 P(j). \tag{6.5}$$

Shannon entropy is connected with the concept of Kullback-Leibler distance (Kullback and Leibler, 1951) to assess the information flow between two-time series with the concept of Markov processes. We present I and J as two discrete random variables with corresponding marginal probabilities of $P(i)$ and $P(j)$, joint probability $P(i, j)$, with dynamic structures in line with a stationary Markov process of order k (Process I) and I (process J). The Markov property implies that the probability to observe I at time $t + 1$ in state i conditional on the k prior observations is $p(i_{t+1}|i_t, \dots, i_{t-k+1}) = p(i_{t+1}|i_t, \dots, i_{t-k})$. To encode the observation in $t + 1$, the average bits number required once the ex-ante k values are known can be written as

$$h_j(k) = -\sum_i P(i_{t+1}, i_t^{(k)}) log P\left(i_{t+1} \middle| i_t^{(k)}\right). \tag{6.6}$$

where $i_t^{(k)} = (i_t, \dots, i_{t-k+1})$ (correspondingly for process *J*). In a bivariate perspective as well as relying on the Kullback-Leibler distance (Kullback and Leibler, 1951), information transmission from process *J* to process *I* is measured by quantifying the deviation from the generalised Markov property $P\left(i_{t+1} \middle| i_t^{(k)}\right) = P\left(i_{t+1} \middle| i_t^{(k)}, j_t^{(I)}\right)$. The Shannon transfer entropy can thus be presented as:

$$T_{J \to I}(k, l) = \sum P(i_{t+1}, i_t^{(k)}, j_t^{(I)}) log \frac{P\left(i_{t+1} \middle| i_t^{(k)}, j_t^{(I)}\right)}{P\left(i_{t+1} \middle| i_t^{(k)}\right)}. \tag{6.7}$$

where $T_{J \to I}$ is the measure of the information flow from J to I. Analogously, $T_{I \to J}$, as a measure for the information flow from I to J, can be derived. Calculating the differential between the two can reveal the prevalent direction of information flow between $T_{J \to I}$ and $T_{I \to J}$.

Based on the Shannon entropy so far discussed, we present the Rényi Transfer Entropy (Rényi, 1970) which is contingent on a weighting factor q and can be calculated as

$$H_J^q = \frac{1}{1-q} log \sum_j P^q(j) \tag{6.8}$$

with $q > 0$. For $q \to 1$, Rényi entropy converges to Shannon entropy. For $0 < q < 1$, thus, low probability events receive more weight, while for $q > 1$ the weights benefit outcomes j with a higher original probability. As a result, Rényi entropy permits to accentuate diverse distribution areas, depending on factor q (Behredt et al., 2019; Adam, 2020).

Applying the escort distribution (Beck and Schögl, 1995) $\emptyset_q(j) = \frac{p^q(j)}{\sum_j p^q(j)}$ with $q > 0$ to normalize the weighted distributions, Rényi transfer entropy (Rényi, 1970) was derived as:

$$RT_{J\rightarrow I}(k,l) = \frac{1}{1-q} P(i_{t+1}, i_t^{(k)}, j_t^{(l)}) log \frac{\sum_i \emptyset_q\left(i_t^{(k)}\right) P^q\left(i_{t+1}\middle|i_t^{(k)}\right)}{\sum_{i,j} \emptyset_q\left(i_t^{(k)}, j_t^{(l)}\right) P^q\left(i_{t+1}\middle|i_t^{(k)}, j_t^{(l)}\right)}, \qquad (6.9)$$

It is worth noting that the Rényi transfer entropy calculation can have negative results. Knowing the history of J, in this case, indicates considerably more uncertainty than knowing the history of I alone would indicate.

The transfer entropy estimates are biased in small samples (Marschinski and Kantz, 2002). The correction of the bias is possible and can be used to calculate the effective transfer entropy as:

$$ETE_{J\rightarrow I}(k,l) = T_{J\rightarrow I}(k,l) - T_{Jshuffled\rightarrow I}(k,l), \qquad (6.10)$$

where $T_{Jshuffled\rightarrow I}(k,l)$ depicts the transfer entropy using a shuffled version of the time series J; that is, randomly drawing values from the observed time series J and realigning them to generate a new time series, which destroys the time series dependencies of J, not forgetting the statistical dependencies between J and I. This enjoins $T_{Jshuffled\rightarrow I}(k,l)$ to come together to zero with increasing sample size, and any nonzero value of $T_{Jshuffled\rightarrow I}(k,l)$ is due to small sample effects. To produce a bias-corrected effective transfer entropy estimate, repeated shuffling and the average of the shuffled transfer entropy estimates overall replications serve as an estimator for the small sample bias, which is removed from the Shannon or Rényi transfer entropy estimate.

Relying on a Markov block bootstrap, the statistical significance of the transfer entropy estimates, as given by Eq. 6.10, can be assessed (Dimpfl and Peter, 2014). This preserves the dependencies within the variables *J* and *I*, but eliminates the statistical dependencies between them contrary to shuffling. Repeated estimation of transfer entropy then provides the distribution of the estimates under the null hypothesis of no information flow. The associated *p*-value is given by $1 - \hat{q}_T$, where $\hat{q}_T$ denotes the quantile of the simulated distribution that is determined by the respective transfer entropy estimate (Adam, 2020).

6.3 Data description

We utilized daily real exchange rates of 15 out of 16 member countries of the SADC from 3rd January, 1994 to 7th January 2019, obtained from Thomson Reuters DataStream in this study. Specifically, daily data for local currency per USD for Angola, Botswana, Comoros, Democratic Republic of Congo, Eswatini (formerly Swaziland), Lesotho, Madagascar, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Tanzania and Zambia were employed. The study period and countries were informed by data availability and the period South Africa formed part of SADC. As a result of this criteria, Zimbabwe was expunged from the analysis. The real exchange rate was measured based on nominal domestic currency per US.dollar (USD) multiplied by the domestic consumer price index divided by the US consumer price index. The justification for using USD as a proxy currency is due to the dominance of the USD in international trade by these countries and the extent of dollarisation of most SADC economies. In spite of recent de-dollarisation in Angola, Mozambique and Zambia, the dollar remains dominant in international trade globally including SADC and provides a means of standardising units of pair of currencies (Corrales et al., 2016). The study was executed with R packages libeemd (Luukko, Helske and Räsänen, 2016) for EEMD and RTransferEntropy for ETE (Adam, 2020).

From Figure 6.1, the graphical presentation of the logarithm of exchange rates of SADC economies is shown, apart from Zimbabwe. Almost all the exchange rates trend upwards except Angola. The trajectory behaviour is explained by periods of pegging of the Angola Kwanza to the US dollar. The upward trend of remaining other countries shows that SADC exchange rates have over the period depreciated against the US dollar. It is not quite surprising to experience this upward trend because, generally, SADC countries are overvalued, and as expected, equilibrium is established overtime. Again, there is a biased expectation of the future exchange rates in small samples in times of uncertainty about when a future policy change will be implemented (Zhou, 2002). This partially explains the ‘peso problem’ proposition for the trend movements.

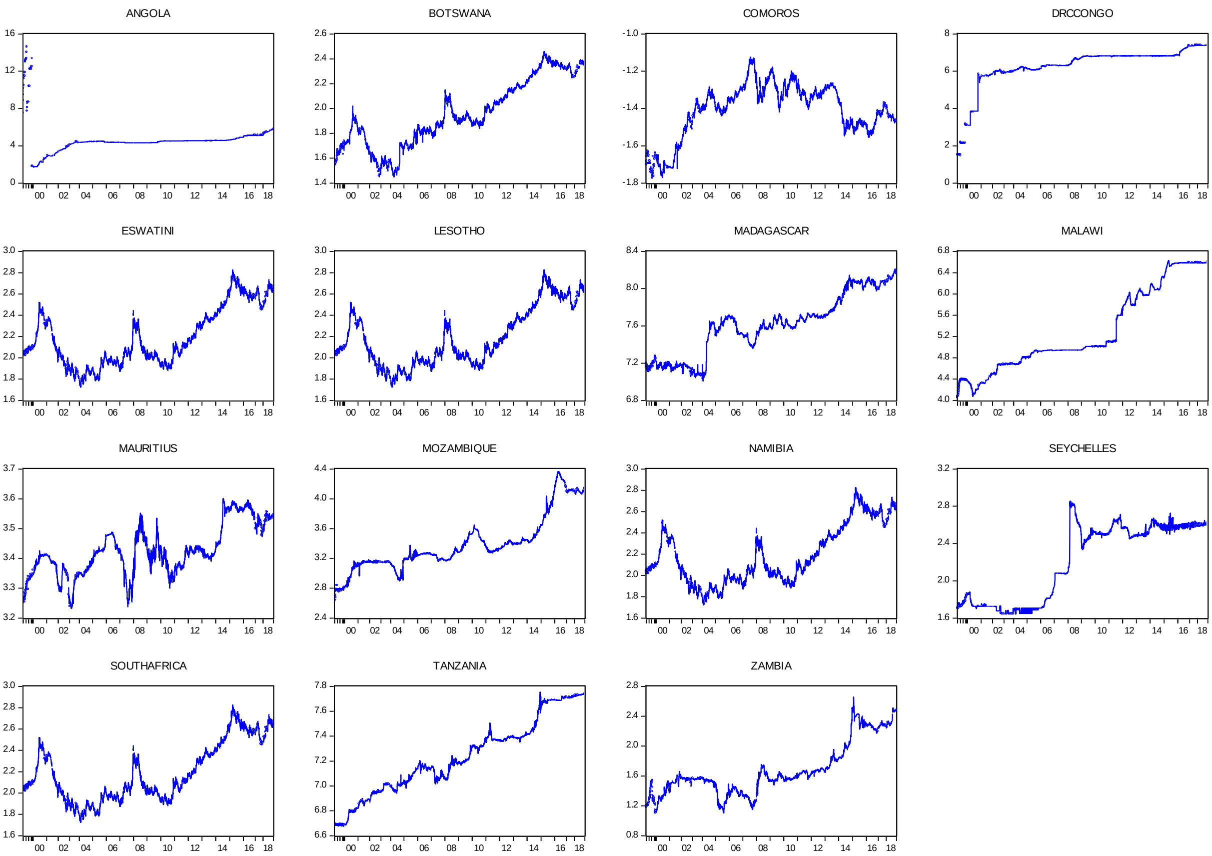


Figure 6.1: Log of the daily real exchange rate of 15 countries of SADC

6.4 Results and Analysis

The analysis of this study was structured to follow three processes. First, we decomposed the daily real domestic currency/USD exchange rate to obtain individual 11 IMFs for each country's exchange rate and trend using EEMD. We examined the properties of all IMFs extracted and were found to satisfy the necessary and sufficient conditions to be IMFs as prescribed by Huang et al. (1998). The residue is the non-oscillating drift of the data, which is not affected by short-to- medium-terms fluctuations, but by the structural changes in the data generation process. It, thus, represents the long-term trend of the data and for this study long trend behaviour of the exchange rate dictated by fundamentals of the economies. Second, the mean and amplitude of the IMFs of each country were classified into various frequencies; high frequency (sum of IMFs 1-5), medium frequency (sum of IMFs 6-8) and low frequency (sum of IMFs 9-11) using cluster analysis. The high-, medium-, and low-frequencies have mean time-frequency of less than 30, between a month and 12 months and more than 12 months, respectively.

Table 6.1 presents the clusters of IMFs into high frequency (period of 1-15 days), medium frequency (up to 144 days), low frequency (up to 2374 days) in addition to the residue. The

Pearson product-moment and Kendall tau-b correlations between each frequency and the original data series, the variance percentage of each frequency in the original data series and the sum of all frequencies and residue indicate that the residue is the dominant mode in all cases.

Last, to understand the flow of information at various frequencies for all countries related to the observed exchange rates, we further examine the Rényi effective transfer entropy of the logarithm of daily real exchange rate series, high-, medium-, low-frequencies, and residue components of 15 exchange rate market in SADC at q = 0.3 to account for low probability events.

We present the bi-directional EEMD-ETE estimates in addition to the 95% confidence bounds between exchange rates in SADC at various frequencies. The frequencies indicate the importance of multi-scales in financial time series. Thus, the dynamics of exchange rates do not occur immediately, but at several investment horizons as provided by the heterogeneous market hypothesis (Adam et al., 2021b; Muller et al., 1993).

The presence of a negative ETE implies that awareness of the exchange rate from a particular country suggests a higher risk coverage for the exchange rate in another country. A positive ETE indicates that the knowledge of the exchange rates reduces the risk of the exchange rate of a specific country. The knowledge in the tails is assigned a high weight for low values of q, resulting in a significant effective transfer entropy result in the current situation. For this reason, we set q from the Rényi effective transfer entropy to 0.3 to offer more weights to the tail events, which bears direct implications. The ETE decreases and even becomes negative as the weight is reduced.

Table 6.1: Descriptive statistics of the reconstructed series and the residue for SADC exchange rate markets derived through EEMD

	Pearson correlation coefficient				Kendall tau-b				Variance as % of observed				variance as % of the sum of all IMFs and Residue			
Country	HFRQ	MFRQ	LFRQ	RESID	HFRQ	MFRQ	LFRQ	RESID	HFRQ	MFRQ	LFRQ	RESID	HFRQ	MFRQ	LFRQ	RESID
Angola	0.49	0.56	-0.04	0.26	0.00	-0.14	0.15	-0.11	62.87	35.96	104.92	144.79	62.87	35.95	104.91	144.78
Botswana	0.06	0.02	0.63	0.94	0.04	0.08	0.46	0.74	0.22	4.53	7.14	77.17	0.22	4.53	7.14	77.14
Comoros	0.09	0.32	0.72	0.85	0.06	0.22	0.46	0.60	0.75	11.13	16.82	48.32	0.75	11.13	16.81	48.29
Dr Congo	0.03	0.05	-0.23	0.94	0.03	0.02	-0.26	0.88	0.10	1.74	15.47	133.16	0.10	1.74	15.46	133.08
Eswatini	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
Les Otho	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
Madagascar	0.06	0.08	0.41	0.94	0.08	0.10	0.18	0.74	0.20	2.51	7.47	86.37	0.20	2.51	7.46	86.33
Malawi	0.03	-0.02	0.35	0.97	0.02	-0.04	0.01	0.86	0.04	0.80	4.67	90.95	0.04	0.80	4.66	90.89
Mauritius	0.12	0.45	0.19	0.74	0.08	0.36	0.13	0.49	1.10	24.87	18.00	83.27	1.10	24.86	17.99	83.22
Mozambique	0.03	0.09	0.45	0.88	0.02	-0.04	0.01	0.79	0.15	5.40	16.04	83.03	0.15	5.39	16.03	82.98
Namibia	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
S. Africa	0.08	0.18	0.69	0.87	0.04	0.09	0.53	0.65	0.50	8.74	16.49	57.99	0.50	8.73	16.48	57.96
Seychelles	0.06	0.20	0.40	0.87	0.07	0.16	0.23	0.60	0.41	5.78	19.09	78.57	0.41	5.78	19.08	78.51
Tanzania	0.04	-0.21	0.11	0.98	0.03	-0.14	-0.02	0.92	0.10	1.05	3.57	104.69	0.10	1.05	3.57	104.63
Zambia	0.07	-0.24	0.40	0.91	0.05	-0.06	0.09	0.57	0.43	9.61	12.44	103.79	0.43	9.61	12.43	103.74

HFRQ=High Frequency series, MFRQ=Medium Frequency series, LFRQ=Low Frequency series, and RESID=Residue

The Rényi effective transfer entropy emphasises various sections of the involved probability density functions in a non-linear way. The Rényian transfer entropy is specifically used in this study to account for tail events associated with the dynamics of exchange rates movements within SADC. Since transfer entropy is a nonparametric estimate and has a higher likelihood of determining statistical interdependence between time series, we present the discussion between exchange rates of SADC following the concept of interdependencies. Doyle (1997) postulated that when there is interdependence, it results in increased economic relations among countries which promotes the forming of unions. Furthermore, Polachek (1980) argues that countries gain from interdependence, for example, the diverse advantages obtained from trading with other nations which most governments try to sustain. This may not be far from SADC which has pursued policies to harmonise and integrate the financial system with the intention to form monetary union. The purpose of the analysis is to ascertain whether SADC could form a reliable monetary union with stronger exchange rate, low exchange rate risk, and ensure price stability.

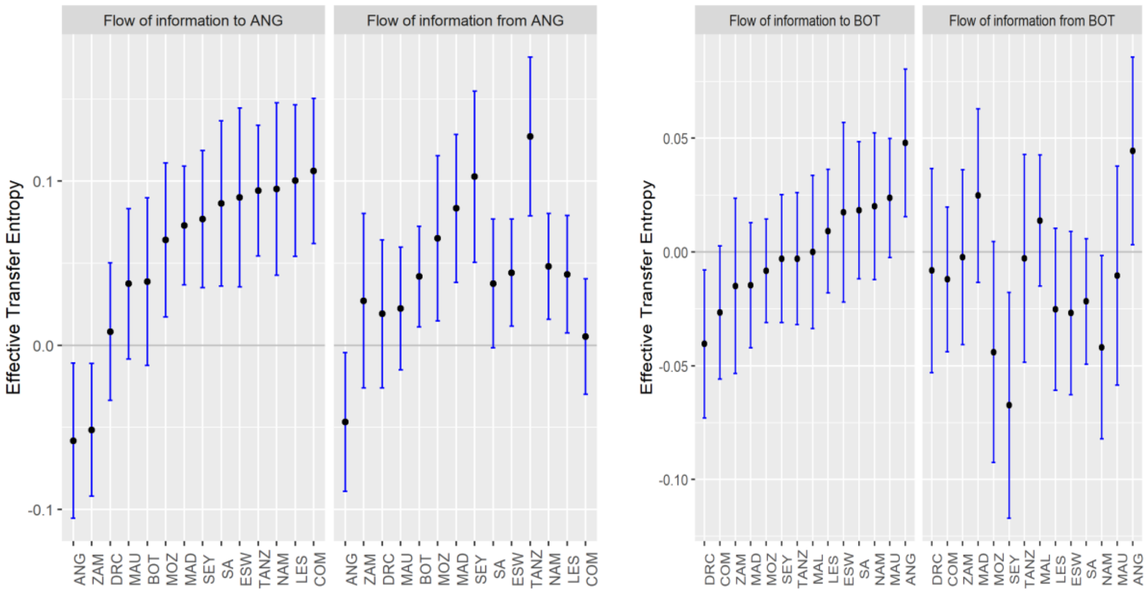
Analyses of the study are presented for fifteen SADC economies concerning exchange rate which is important for trade and investments. The decomposed returns series are presented for the high-, medium and low-frequencies, representing short-, medium-, and long-terms respectively. In addition, we present the residue which denotes the non-oscillating drift of the data, which is not affected by short-to- medium term fluctuations but by the structural changes in the data generation process. It, therefore, represents the trend behaviour of the exchange rate dictated by fundamentals of the economies. The final outputs are tail dependent and reveal the directional flow of information between exchange rates other than the ones shown by other statistical techniques which assume linearity and stationarity. Consequently, Mokoena, Gupta, and Van Eyden (2009) make it clear that SADC economies' exchange rates exhibit non-linear relationships when purchasing power parity was assessed.

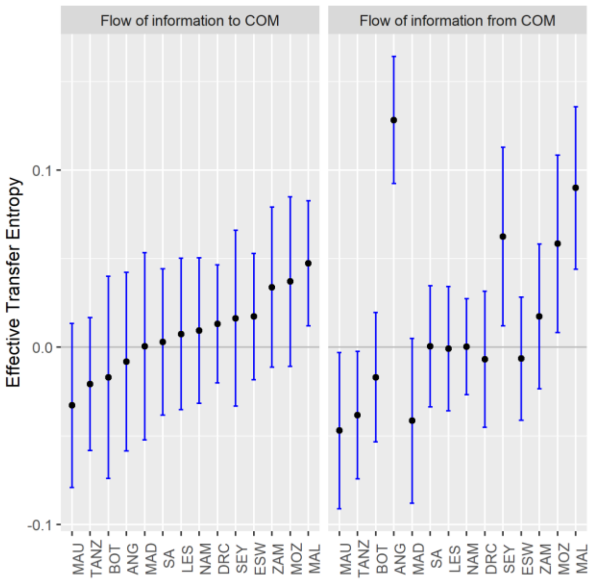
6.4.1 Exchange rates Information Transfer at High Frequency

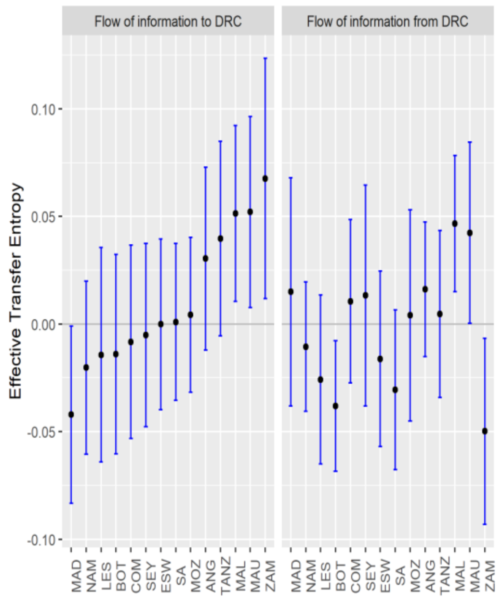
Figure 6.2 shows the information flow between exchange rates of SADC for high frequencies representing short-term horizon. It can be inferred that significant information flows between exchange rates within SADC are either positive or negative. However, there are more positive flows as compared to negative flows in the high frequency. Thus, the knowledge of the exchange rates from countries reduces the risk of the exchange rate of a specific country in the short-term. This is also true for information flow from a specific country's exchange rate to the remaining SADC exchange rates. These observations imply that quantification of information flow between exchange rates in SADC depicts less uncertainties in the short-term.

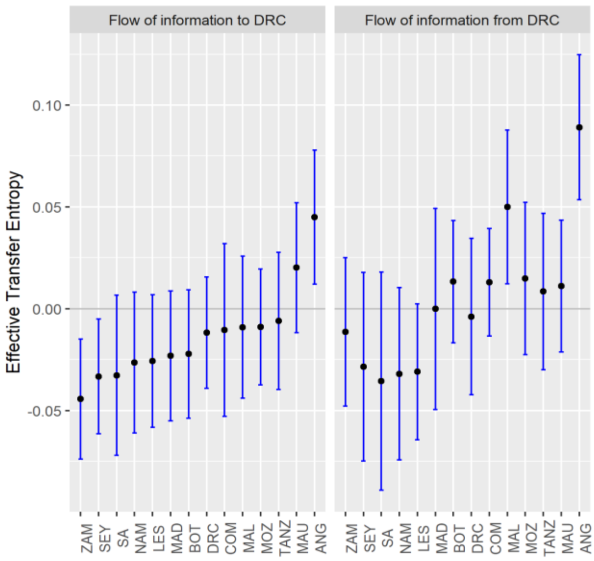
In other words, the knowledge of the history of one country's exchange rate illustrates considerably less uncertainty than knowing the history of only the remaining exchange rate(s).

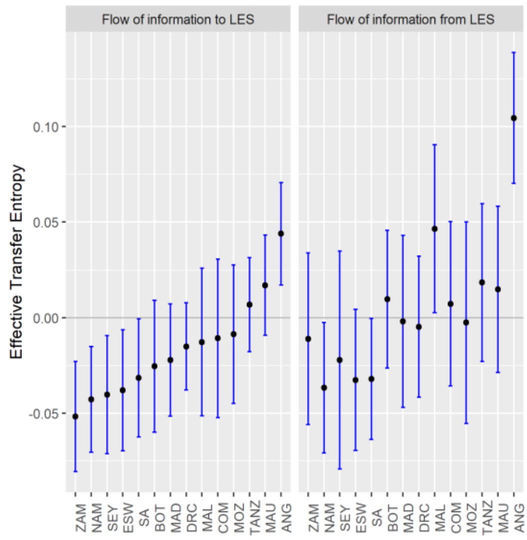
Specifically, we find a bi-causality positive significant information flow with countries such as Angola, Botswana, Comoros, Democratic Republic of Congo, Madagascar, Malawi and Mauritius. This suggests that trade and investment within Angola, Botswana, Comoros, Democratic Republic of Congo, Madagascar, Malawi and Mauritius with the remaining SADC economies would reduce exchange rate risk. Consequently, the presence of these economies in SADC with the quest of forming a reliable monetary union for stronger exchange rate, low exchange rate risk, and ensure price stability can suffice in the short-term. In this regard, economies of like nature may form a reliable monetary union with less shocks from a specific country's exchange rate. The findings for the short-term perspective corroborate the outcome of Zehirun, Breitenbach and Kemegue (2015) who found a weak positive comovement in exchange rates in SADC when Johansen's multivariate co-integration technique was employed.




Flow of information to COM
Flow of information from COM
Effective Transfer Entropy
0.1
0.0
-0.1
MAU TANZ BOT ANG MAD SA LES NAM DRC SEY ESW ZAM MOZ MAL
MAU TANZ BOT ANG MAD SA LES NAM DRC SEY ESW ZAM MOZ MAL


Flow of information to DRC
Flow of information from DRC
Effective Transfer Entropy
0.10
0.05
0.00
-0.05
-0.10
MAD NAM LES BOT COM SEY ESW SA MOZ ANG TANZ MAL MAU ZAM
MAD NAM LES BOT COM SEY ESW SA MOZ ANG TANZ MAL MAU ZAM


Flow of information to DRC
Flow of information from DRC
Effective Transfer Entropy
0.10
0.05
0.00
-0.05
ZAM SEY SA NAM LES MAD BOT DRC COM MAL MOZ TANZ MAU ANG
ZAM SEY SA NAM LES MAD BOT DRC COM MAL MOZ TANZ MAU ANG


Flow of information to LES
Flow of information from LES
Effective Transfer Entropy
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0.05
0.00
-0.05
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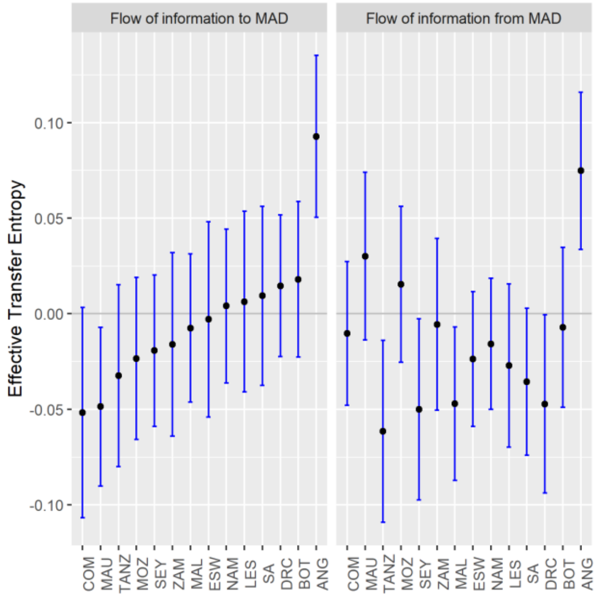
ZAM NAM SEY ESW SA BOT MAD DRC MAL COM MOZ TANZ MAU ANG


Flow of information to MAD
Flow of information from MAD
Effective Transfer Entropy
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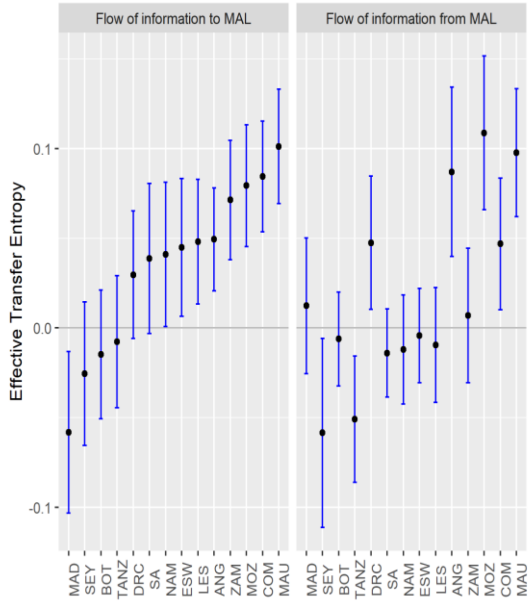
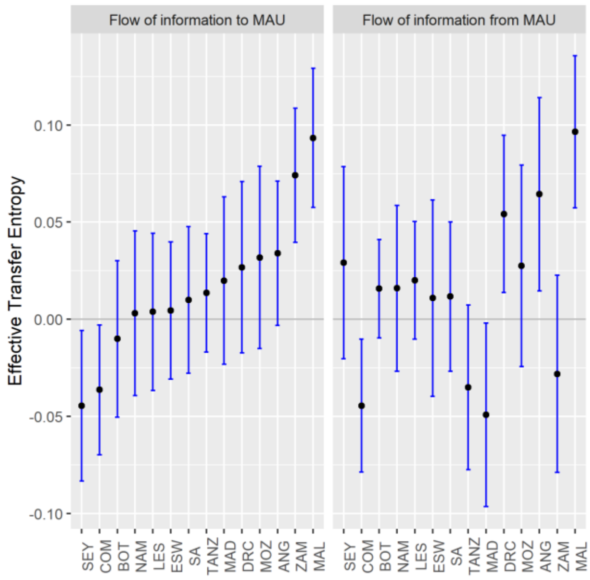
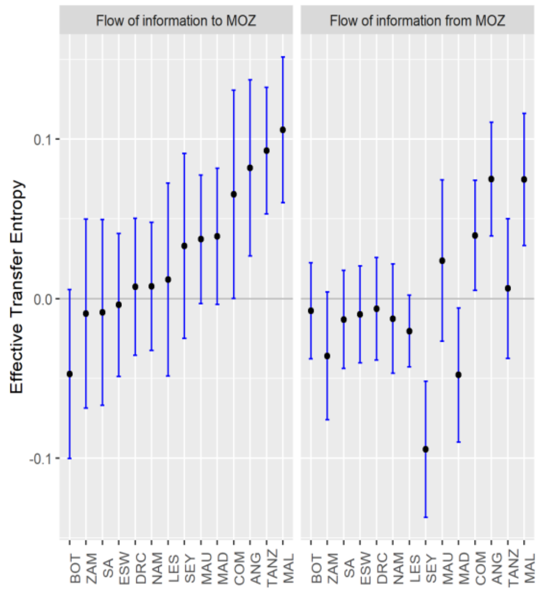
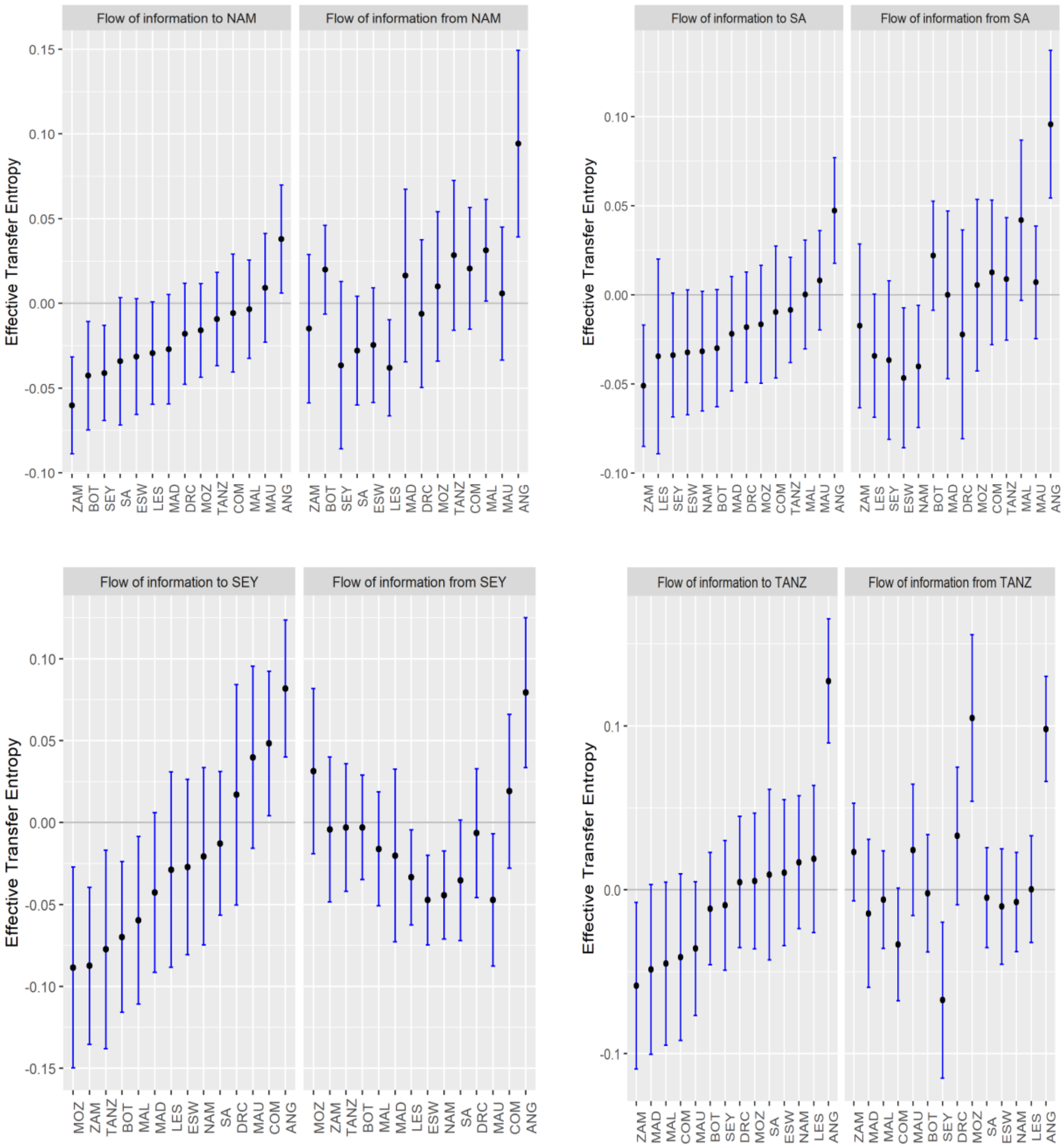
0.05
0.00
-0.05
-0.10
COM MAU TANZ MOZ SEY ZAM MAL ESW NAM LES SA DRC BOT ANG
COM MAU TANZ MOZ SEY ZAM MAL ESW NAM LES SA DRC BOT ANG


Flow of information to MAL
Flow of information from MAL
Effective Transfer Entropy
0.1
0.0
-0.1
MAD SEY BOT TANZ DRC SA NAM ESW LES ANG ZAM MOZ COM MAU
MAD SEY BOT TANZ DRC SA NAM ESW LES ANG ZAM MOZ COM MAU


Flow of information to MAU
Flow of information from MAU
Effective Transfer Entropy
0.10
0.05
0.00
-0.05
-0.10
SEY COM BOT NAM LES ESW SA TANZ MAD DRC MOZ ANG ZAM MAL
SEY COM BOT NAM LES ESW SA TANZ MAD DRC MOZ ANG ZAM MAL


Flow of information to MOZ
Flow of information from MOZ
Effective Transfer Entropy
0.1
0.0
-0.1
BOT ZAM SA ESW DRC NAM LES SEY MAU MAD COM ANG TANZ MAL
BOT ZAM SA ESW DRC NAM LES SEY MAU MAD COM ANG TANZ MAL


Flow of information to NAM
Flow of information from NAM
Flow of information to SA
Flow of information from SA
Flow of information to SEY
Flow of information from SEY
Flow of information to TANZ
Flow of information from TANZ
Effective Transfer Entropy
ZAM BOT SEY SA ESW LES MAD DRC MOZ TANZ COM MAL MAU ANG
ZAM BOT SEY SA ESW LES MAD DRC MOZ TANZ COM MAL MAU ANG
ZAM LES SEY ESW NAM BOT MAD DRC MOZ COM TANZ MAL MAU ANG
ZAM LES SEY ESW NAM BOT MAD DRC MOZ COM TANZ MAL MAU ANG
MOZ ZAM TANZ BOT MAL MAD LES ESW NAM SA DRC MAU COM ANG
MOZ ZAM TANZ BOT MAL MAD LES ESW NAM SA DRC MAU COM ANG
ZAM MAD MAL COM MAU BOT SEY DRC MOZ SA ESW NAM LES ANG
ZAM MAD MAL COM MAU BOT SEY DRC MOZ SA ESW NAM LES ANG

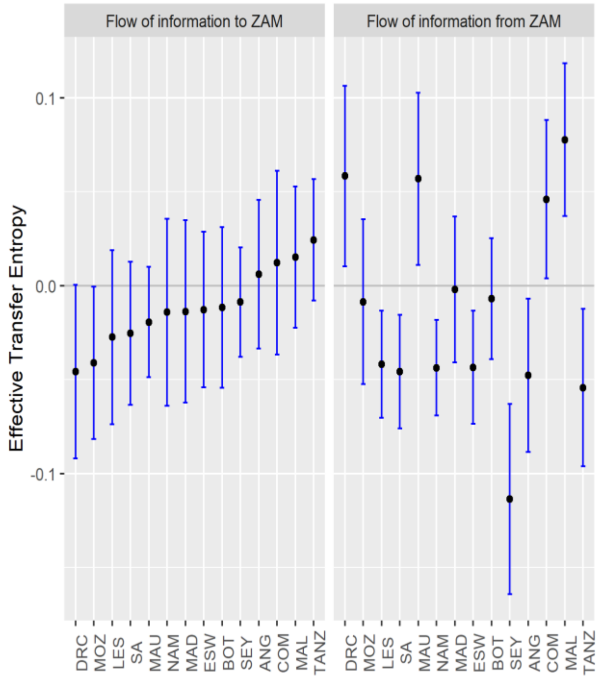


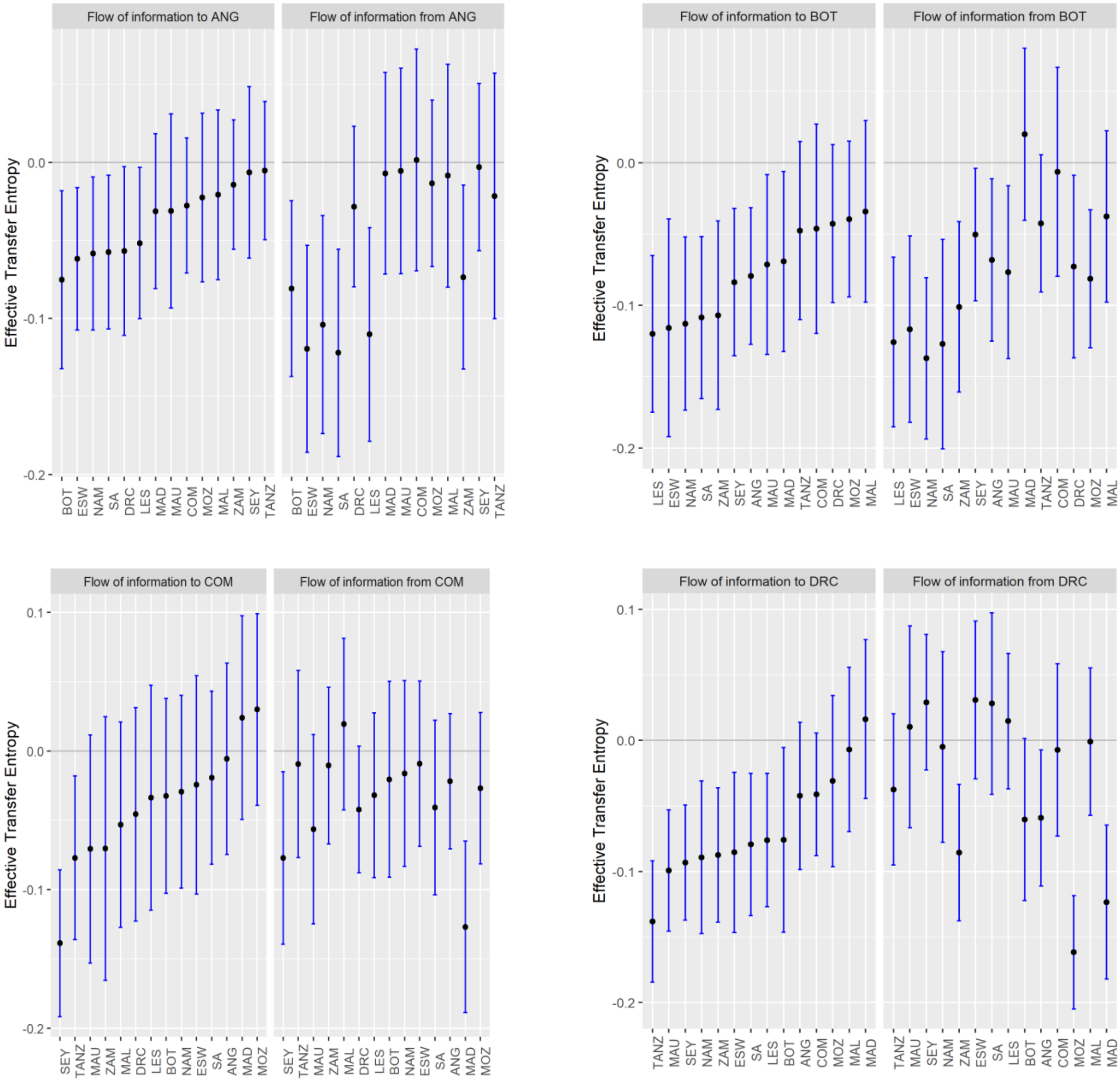
Figure 6.2 Information flow of high-frequency exchange rate series among SADC countries

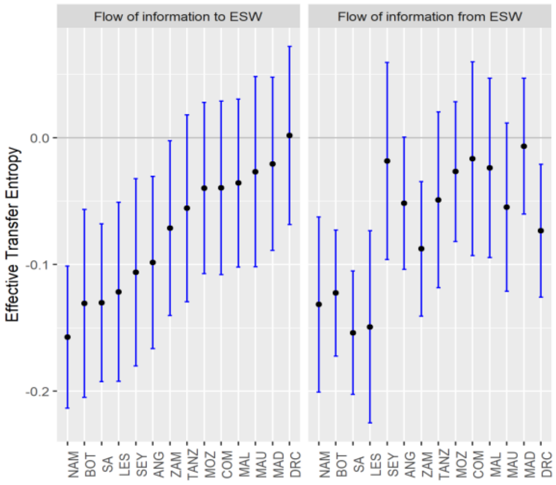
6.4.2 Exchange rates Information Transfer at Medium Frequency

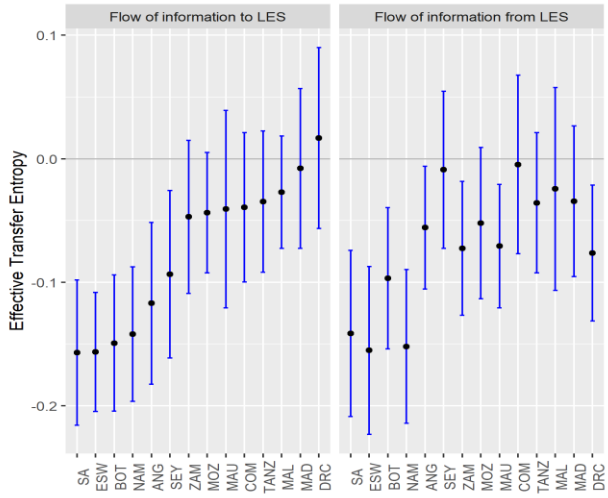
Figure 6.3 shows the information flow between exchange rates at SADC for medium frequencies representing a medium-term horizon. Overall, there is a more negative significant flow of information in the medium-term. Thus, the knowledge of the exchange rates from countries increases the risk of the exchange rate of a specific country. This is also true for information flow from a specific country's exchange rate to the remaining SADC exchange rates. These observations imply that quantification of information flow between exchange rates in SADC indicates more uncertainties in the medium-term. In other words, the knowledge of the history of one country's exchange rate illustrates considerably more uncertainty than knowing the history of only the remaining exchange rate(s) as compared to the high-frequency estimates.

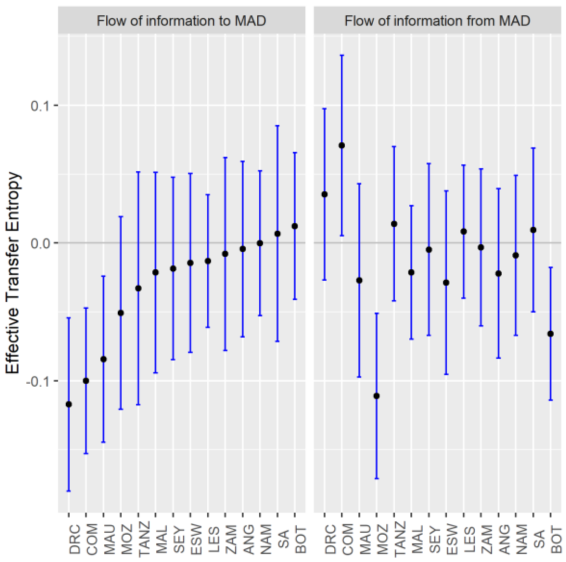
Specifically, negative significant information flow from the exchange rates of Botswana, Eswatini, Namibia, South Africa, Democratic Republic of Congo and Lesotho to Angola. The reverse is true, except for information flow from Angola to the Democratic Republic of Congo. That is, there is bi-directional causality between exchange rates of Botswana, Eswatini, Namibia, South Africa, Lesotho and Angola. Also, significant negative information flow from Lesotho, Eswatini, Namibia, South Africa, Zambia, Seychelles, Angola and Mauritius to Botswana, with the reverse indicating a similar outcome which exhibits bi-directional causality. This suggests that trade and investment within Lesotho, Eswatini, Namibia, South Africa, Zambia, Seychelles, Angola and Mauritius by Botswana would pose a higher risk for the exchange rate of Botswana, and vice-versa. Relatively, trade and investment by Angola indicate less negative information flow from other countries as compared to Botswana. Furthermore, trade and investment to and from Zambia within SADC exhibit a higher degree of uncertainty with countries such as Botswana, Lesotho, South Africa, Eswatini and the Democratic Republic of Congo. The outcome for trade and investment to and from SA with countries such as Lesotho, Namibia, Eswatini and Botswana is no exception. The exchange rate negative information flow from South Africa to other SADC economies concurs with the findings of Qabhobho, Wait and Le Roux (2020) when GARCH models were considered to assess exchange rate volatilities. Accordingly, the presence of these economies in SADC with the quest of forming a reliable monetary union may not auger well for stronger exchange rates, low exchange rate risk, and ensure price stability in the medium-term. In this regard, knowing the history of one country's exchange rate demonstrates considerably more uncertainty than knowing the history of only the remaining economies' exchange rates, especially with the bi-directional significant negative causality case.

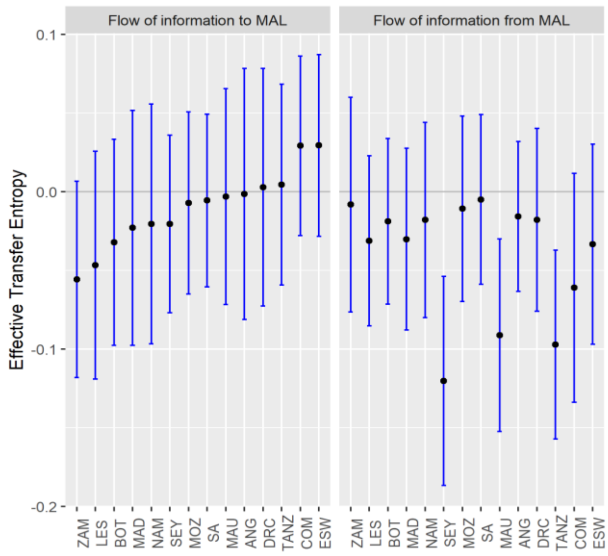
On the other hand, countries with insignificant unidirectional or preferably, bi-directional insignificant or positive significant information flows may experience stronger exchange rate, low exchange rate risk, and may result in price stability. For instance, the exchange rate dynamics of Tanzania depicts less risk of information sharing with the remaining SADC members in this study. In this regard, economies of like nature may form a reliable monetary union with fewer shocks from a specific country's exchange rate in the medium-term.

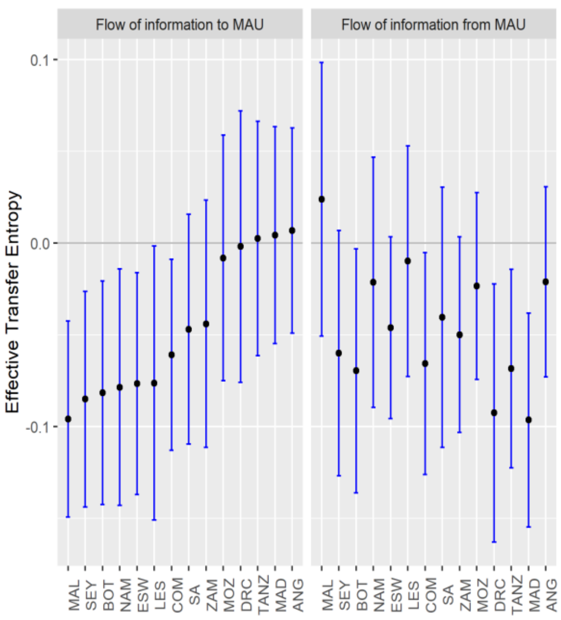


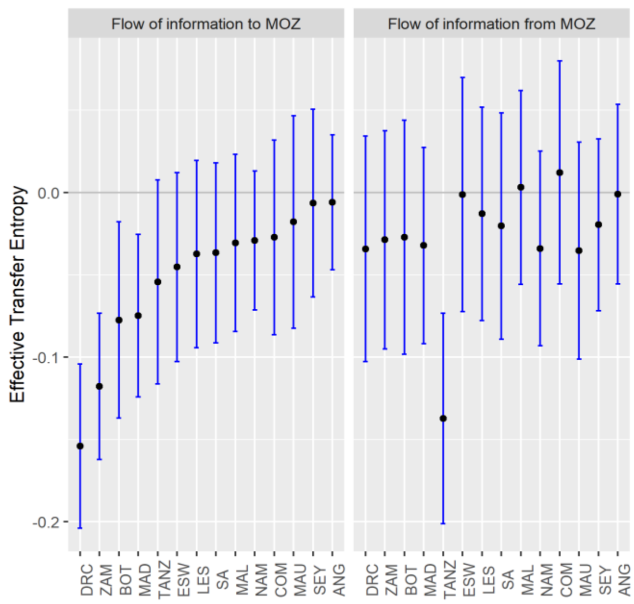

Flow of information to ESW
Flow of information from ESW
Effective Transfer Entropy
0.0
-0.1
-0.2
NAM BOT SA LES SEY ANG ZAM TANZ MOZ COM MAL MAU MAD DRC
NAM BOT SA LES SEY ANG ZAM TANZ MOZ COM MAL MAU MAD DRC


Flow of information to LES
Flow of information from LES
Effective Transfer Entropy
0.1
0.0
-0.1
-0.2
SA ESW BOT NAM ANG SEY MOZ ZAM MAU COM TANZ MAL MAD DRC
SA ESW BOT NAM ANG SEY ZAM MOZ MAU COM TANZ MAL MAD DRC


Flow of information to MAD
Flow of information from MAD
Effective Transfer Entropy
0.1
0.0
-0.1
DRC COM MAU MOZ TANZ MAL SEY ESW LES ZAM ANG NAM SA BOT
DRC COM MAU MOZ TANZ MAL SEY ESW LES ZAM ANG NAM SA BOT


Flow of information to MAL
Flow of information from MAL
Effective Transfer Entropy
0.1
0.0
-0.1
-0.2
ZAM LES BOT MAD NAM SEY MOZ SA MAU ANG DRC TANZ COM ESW
ZAM LES BOT MAD NAM SEY MOZ SA MAU ANG DRC TANZ COM ESW


Flow of information to MAU
Flow of information from MAU
Effective Transfer Entropy
0.1
0.0
-0.1
MAL SEY BOT NAM ESW LES COM SA ZAM MOZ DRC TANZ MAD ANG
MAL SEY BOT NAM ESW LES COM SA ZAM MOZ DRC TANZ MAD ANG


Flow of information to MOZ
Flow of information from MOZ
Effective Transfer Entropy
0.0
-0.1
-0.2
DRC ZAM BOT MAD TANZ ESW LES SA MAL NAM COM MAU SEY ANG
DRC ZAM BOT MAD TANZ ESW LES SA MAL NAM COM MAU SEY ANG

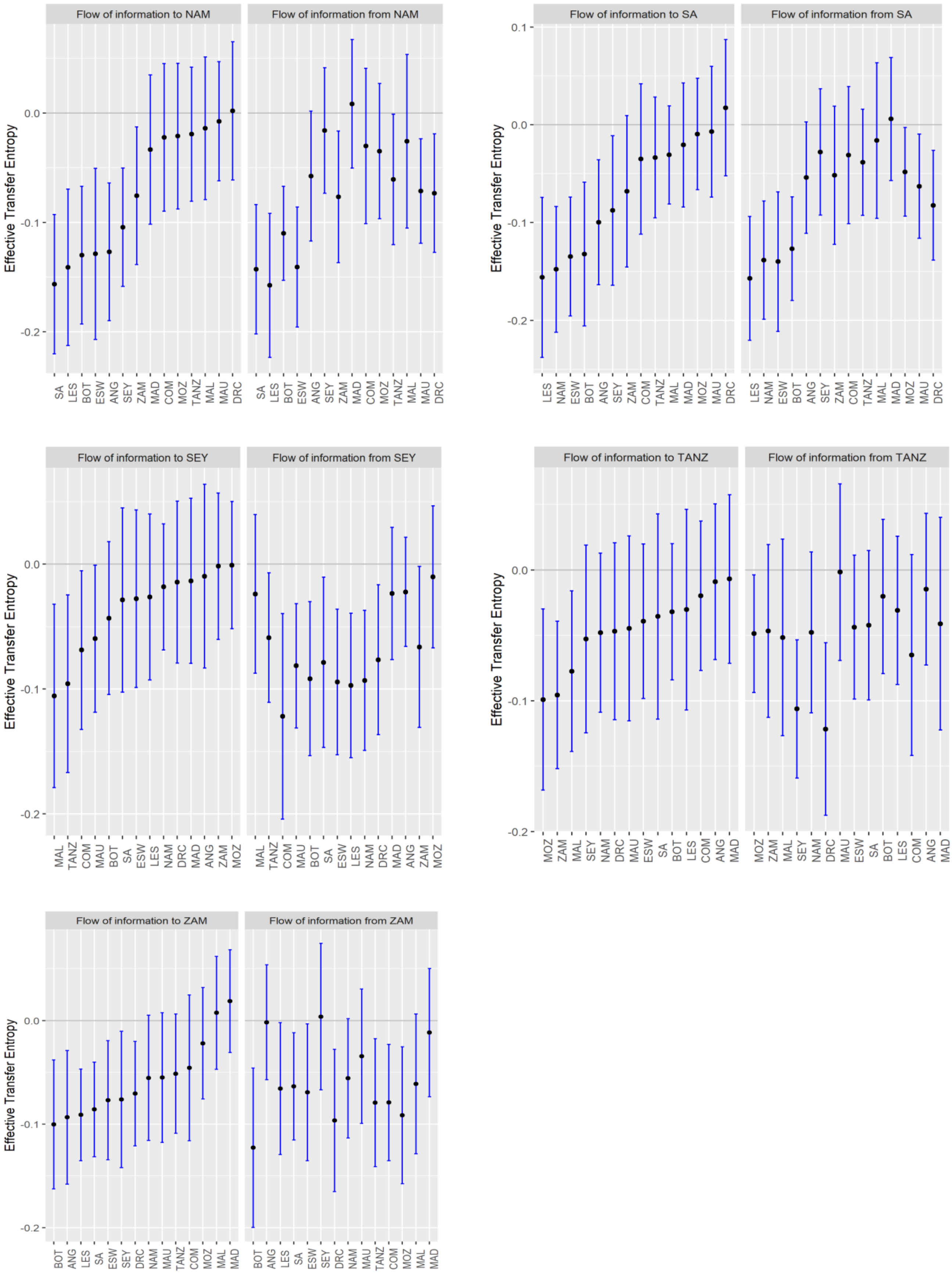


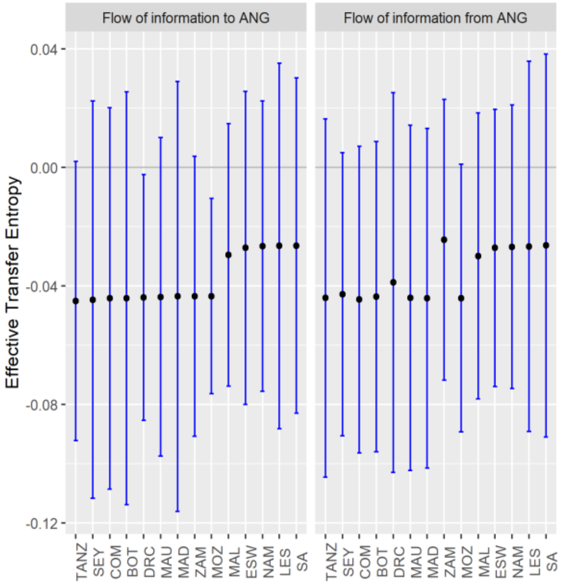
Figure 6.3 Information Flow between exchange rates at medium frequency series

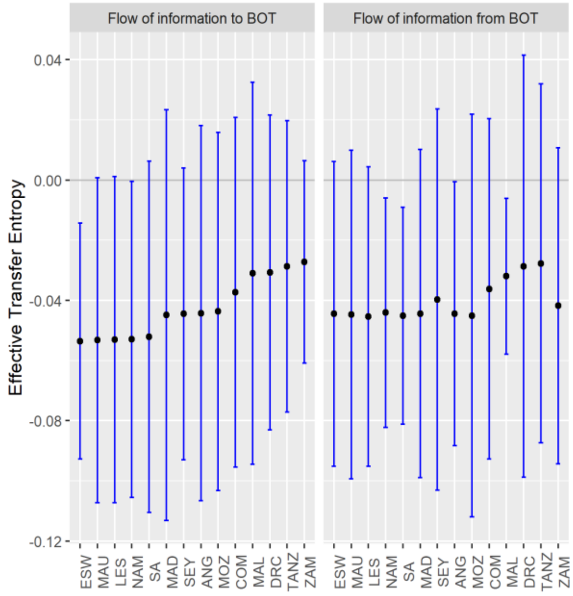
6.4.3 Exchange rates Information Transfer at Low Frequency

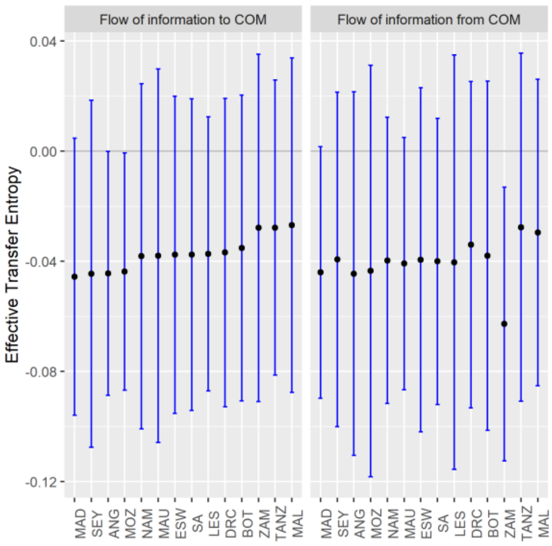
Figure 6.4 shows the information flow between exchange rates at SADC for low frequencies representing a long-term horizon. Throughout the low frequency, there are potentials for negative flows than positive flows between the exchange rates in SADC. Thus, the knowledge of the exchange rates from countries increases the risk of the exchange rate of a specific country. This is also true for information flow from a specific country's exchange rate to the remaining SADC exchange rates. These observations imply that quantification of information flow between exchange rates in SADC depicts more uncertainty.

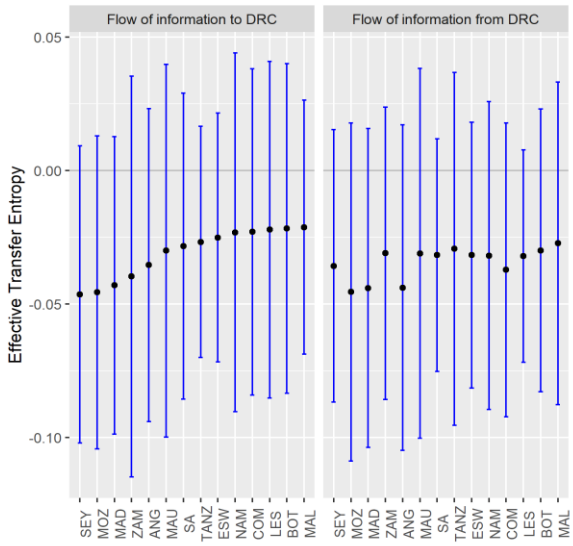
Specifically, negative significant information flow from the exchange rates of Democratic Republic of Congo to Angola; Eswatini to Botswana; South Africa, Lesotho, Namibia and Mauritius to Eswatini; Namibia, Eswatini, South Africa and Mauritius to Lesotho; Angola to Mozambique; South Africa, Eswatini, Lesotho and Mauritius to Namibia; Lesotho, Namibia, Eswatini and Mauritius to South Africa; Malawi and Tanzania to Seychelles; and South Africa, Lesotho, Eswatini, Malawi, Comoros and Botswana to Zambia. We found a bi-directional causality with the exchange rates of South Africa, Lesotho, Namibia, Mauritius and Eswatini. These economies exhibit negative significant information flow in exchange rates with respect to their possible combinations. Consequently, the inclusion of these economies as part of SADC with the quest of forming a reliable monetary union may not auger well for stronger exchange rate, low exchange rate risk, and ensure price stability. The negative bi-directional causality further signifies that the knowledge of the exchange rates among these countries increases the risk of the exchange rates within these regions. Thus, at low probability events, adverse fluctuations in exchange rates of SADC occur relative to favourable outcomes.

Accordingly, countries with a mixture of insignificant unidirectional/bi-directional flow or preferably, bi-directional positive information flow may experience stronger exchange rate, low exchange rate risk, which may result in price stability. For instance, the exchange rates of Madagascar, Tanzania and Malawi, to mention but a few, depict less uncertainties of information sharing. The outcome for Madagascar, Tanzania and Malawi supports the assertion made by Anoruo and Ahmad (2013) of monetary convergence in SADC. In this regard, economies of like nature may form a reliable monetary union with less shocks from a specific country's exchange rate.


Flow of information to ANG
Flow of information from ANG
Effective Transfer Entropy
0.04
0.00
-0.04
-0.08
-0.12
TANZ SEY COM BOT DRC MAU MAD ZAM MOZ MAL ESW NAM LES SA
TANZ SEY COM BOT DRC MAU MAD ZAM MOZ MAL ESW NAM LES SA


Flow of information to BOT
Flow of information from BOT
Effective Transfer Entropy
0.04
0.00
-0.04
-0.08
-0.12
ESW MAU LES NAM SA MAD SEY ANG MOZ COM MAL DRC TANZ ZAM
ESW MAU LES NAM SA MAD SEY ANG MOZ COM MAL DRC TANZ ZAM


Flow of information to COM
Flow of information from COM
Effective Transfer Entropy
0.04
0.00
-0.04
-0.08
-0.12
MAD SEY ANG MOZ NAM MAU ESW SA LES DRC BOT ZAM TANZ MAL
MAD SEY ANG MOZ NAM MAU ESW SA LES DRC BOT ZAM TANZ MAL


Flow of information to DRC
Flow of information from DRC
Effective Transfer Entropy
0.05
0.00
-0.05
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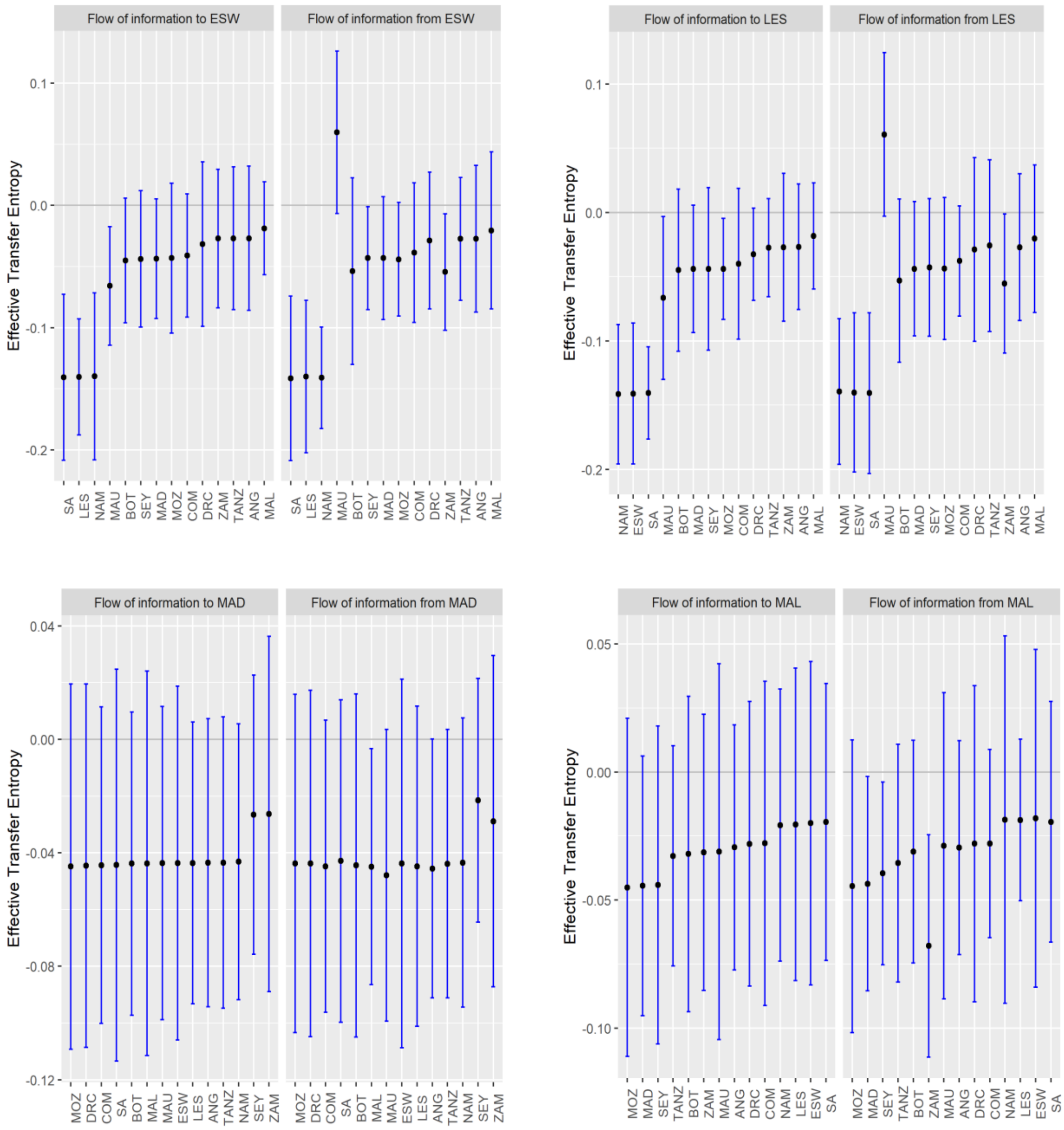
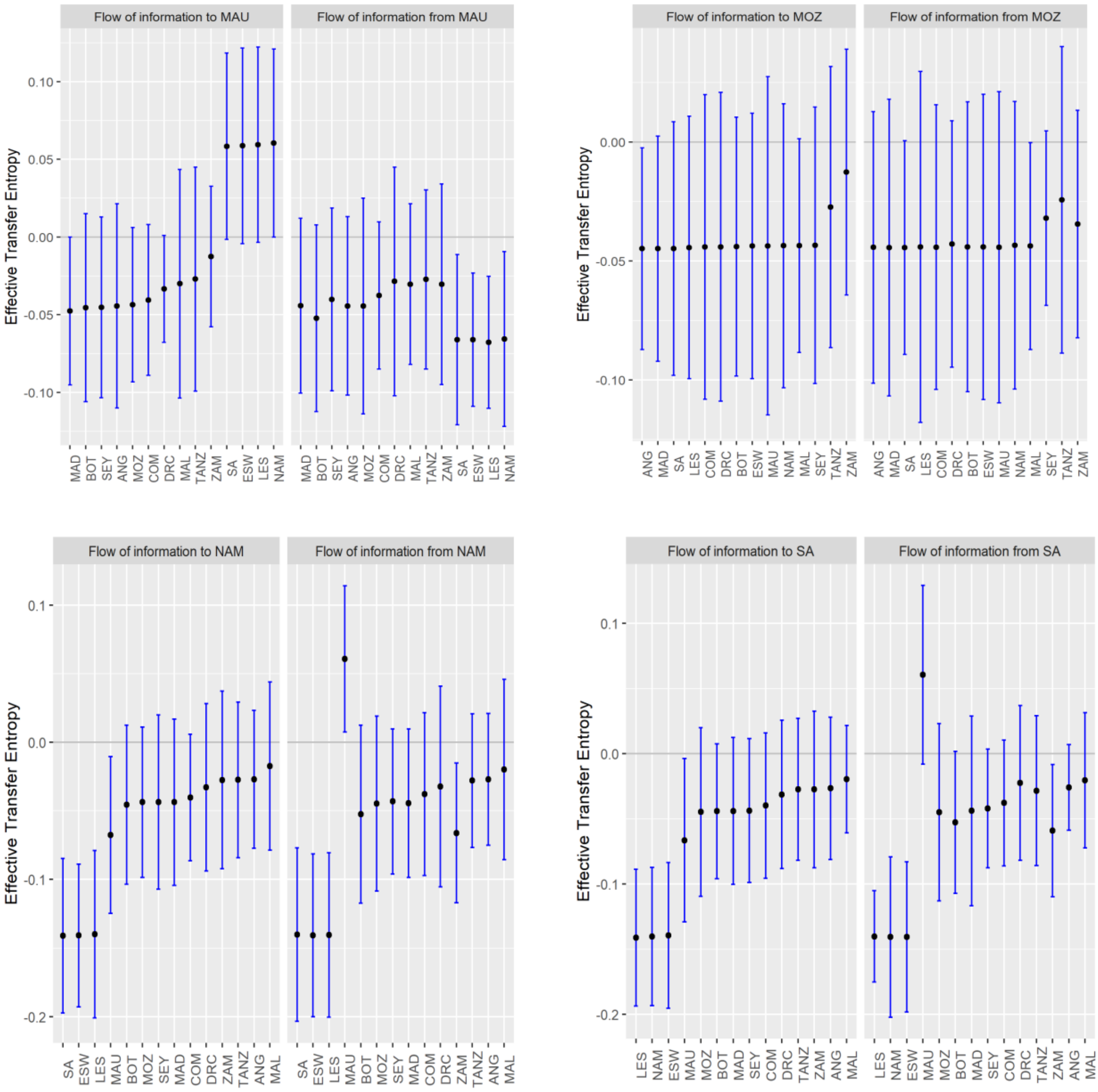
SEY MOZ MAD ZAM ANG MAU SA TANZ ESW NAM COM LES BOT MAL
SEY MOZ MAD ZAM ANG MAU SA TANZ ESW NAM COM LES BOT MAL


Flow of information to ESW
Flow of information from ESW
Effective Transfer Entropy
0.1
0.0
-0.1
-0.2
SA LES NAM MAU BOT SEY MAD MOZ COM DRC ZAM TANZ ANG MAL
SA LES NAM MAU BOT SEY MAD MOZ COM DRC ZAM TANZ ANG MAL
Flow of information to LES
Flow of information from LES
Effective Transfer Entropy
0.1
0.0
-0.1
-0.2
NAM ESW SA MAU BOT MAD SEY MOZ COM DRC TANZ ZAM ANG MAL
NAM ESW SA MAU BOT MAD SEY MOZ COM DRC TANZ ZAM ANG MAL
Flow of information to MAD
Flow of information from MAD
Effective Transfer Entropy
0.04
0.00
-0.04
-0.08
-0.12
MOZ DRC COM SA BOT MAL MAU ESW LES ANG TANZ NAM SEY ZAM
MOZ DRC COM SA BOT MAL MAU ESW LES ANG TANZ NAM SEY ZAM
Flow of information to MAL
Flow of information from MAL
Effective Transfer Entropy
0.05
0.00
-0.05
-0.10
MOZ MAD SEY TANZ BOT ZAM MAU ANG DRC COM NAM LES ESW SA
MOZ MAD SEY TANZ BOT ZAM MAU ANG DRC COM NAM LES ESW SA


Flow of information to MAU
Flow of information from MAU
Effective Transfer Entropy
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0.00
-0.05
-0.10
MAD BOT SEY ANG MOZ COM DRC MAL TANZ ZAM SA ESW LES NAM
MAD BOT SEY ANG MOZ COM DRC MAL TANZ ZAM SA ESW LES NAM
Flow of information to MOZ
Flow of information from MOZ
Effective Transfer Entropy
0.00
-0.05
-0.10
ANG MAD SA LES COM DRC BOT ESW MAU NAM MAL SEY TANZ ZAM
ANG MAD SA LES COM DRC BOT ESW MAU NAM MAL SEY TANZ ZAM
Flow of information to NAM
Flow of information from NAM
Effective Transfer Entropy
0.1
0.0
-0.1
-0.2
SA ESW LES MAU BOT MOZ SEY MAD COM DRC ZAM TANZ ANG MAL
SA ESW LES MAU BOT MOZ SEY MAD COM DRC ZAM TANZ ANG MAL
Flow of information to SA
Flow of information from SA
Effective Transfer Entropy
0.1
0.0
-0.1
-0.2
LES NAM ESW MAU MOZ BOT MAD SEY COM DRC TANZ ZAM ANG MAL
LES NAM ESW MAU MOZ BOT MAD SEY COM DRC TANZ ZAM ANG MAL

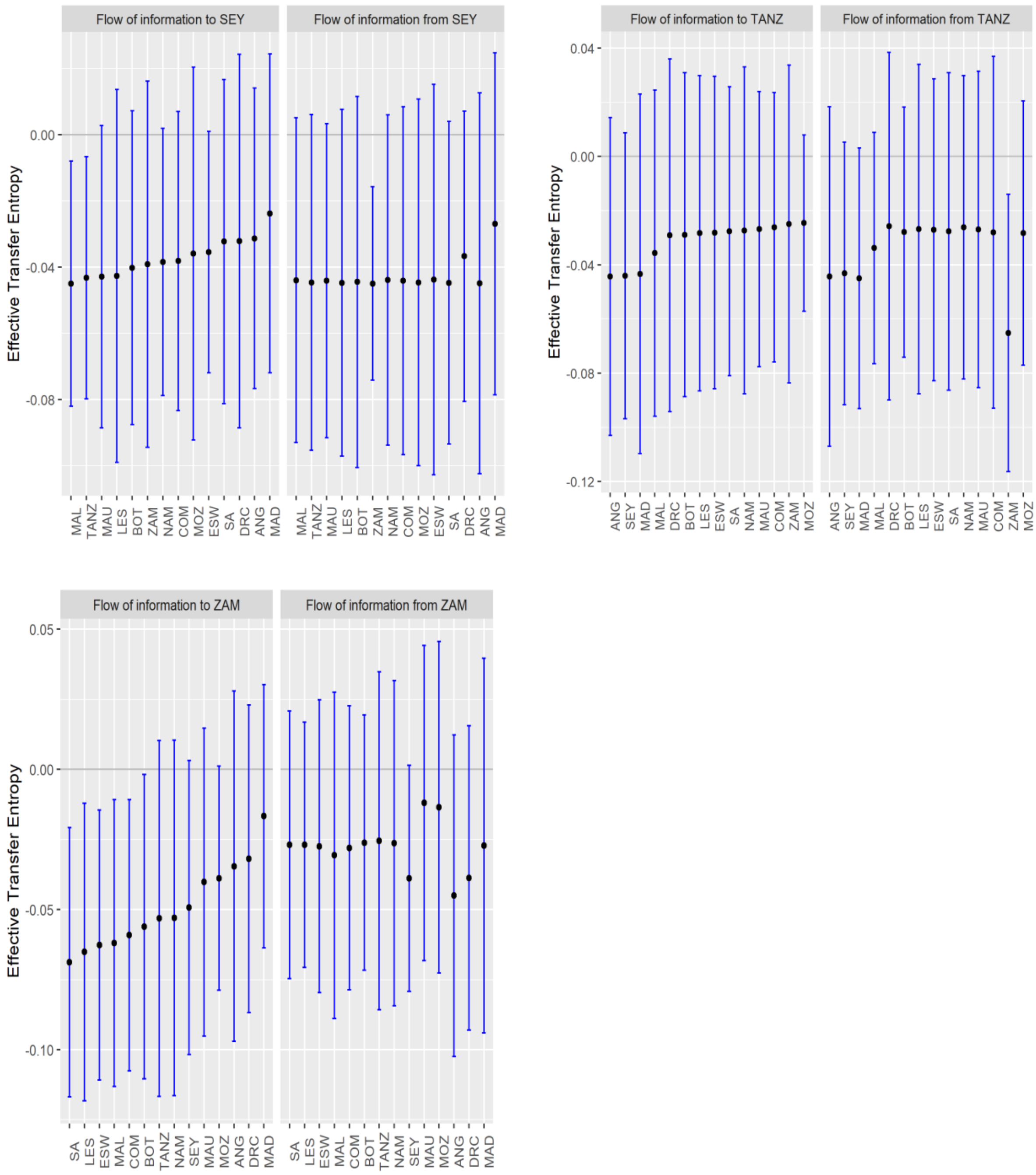
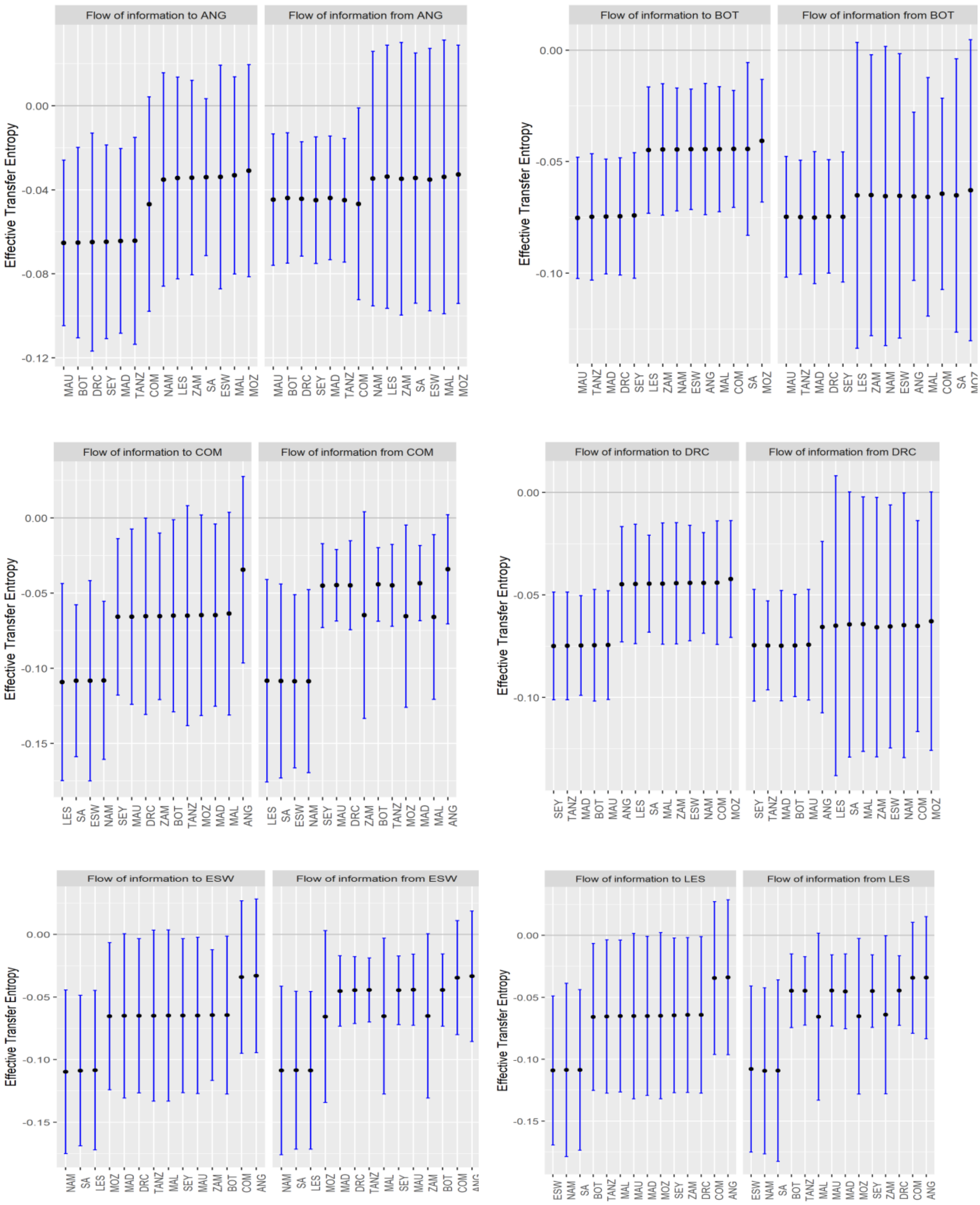


Figure 6.4 Information Flow between exchange rates at low-frequency series

6.4.4 Exchange rates Information Transfer between the Residues

Figure 6.5 shows the information flow between exchange rates at SADC for the residue representing the long-term trend or fundamental feature. It can be observed from the plots that the residue contains the most negative significant information flow between exchange rates relative to the remaining frequencies. Thus, the knowledge of the exchange rates from countries increases the risk of the exchange rate of a specific country. This is also true for information flow from a specific country's exchange rate to the remaining SADC exchange rates. These observations imply that information flow between exchange rates in SADC indicates more uncertainties. Thus, the knowledge of the history of one country's exchange rate illustrates considerably more uncertainty than knowing the history of only the remaining exchange rate(s) as compared to the high-, medium-, and low-frequencies estimates. This assertion confirms the suggestion of Duma (2001) that SADC does not form an optimum currency union. The study of Redda and Muzindusti (2017) advocated that bilateral real exchange rates in the SADC region share a common stochastic trend in the long-run, without considering the directional flow of information at multi-scales between the economies. Thus, similarities in stochastic trends are not enough to prove empirically, the conditions for optimum currency area. Notwithstanding, the study of Adam et al. (2021a; 2021b) found similarities in exchange rates of SADC at diverse frequencies via the EEMD approach. Building upon these similarities, our approach (EEMD-ETE) reveals a significant negative information flow between the exchange rates of SADC for most scales.

A glance from the plots depicts that almost all the 15 SADC economies demonstrate negative information flow between exchange rates. During this period, trade and investments between union members may escalate exchange rate risk, weaken exchange rates and lead to price instability. In this regard, knowing the history of one country's exchange rate demonstrates considerably more uncertainty than knowing the history of only the remaining economies' exchange rates, especially, from the bi-directional negative information flow. The significant negative bi-directional causality between exchange rates in SADC supports the findings of Zehirun, Breitenbach and Kemegue (2015) in the long-run.


Flow of information to ANG
Flow of information from ANG
Flow of information to BOT
Flow of information from BOT
Flow of information to COM
Flow of information from COM
Flow of information to DRC
Flow of information from DRC
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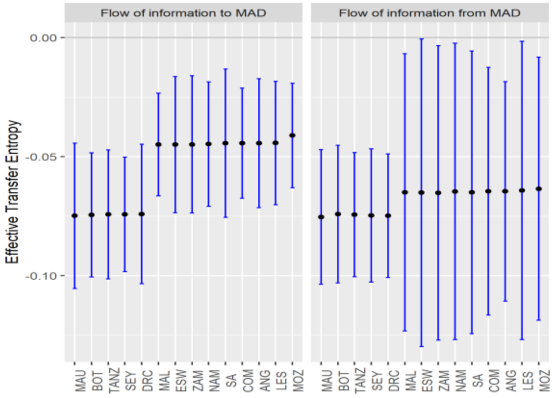
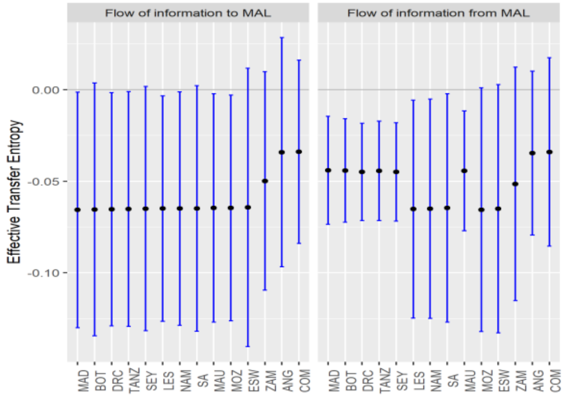
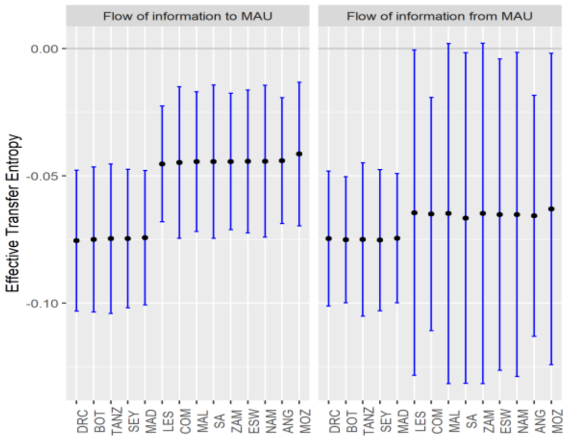
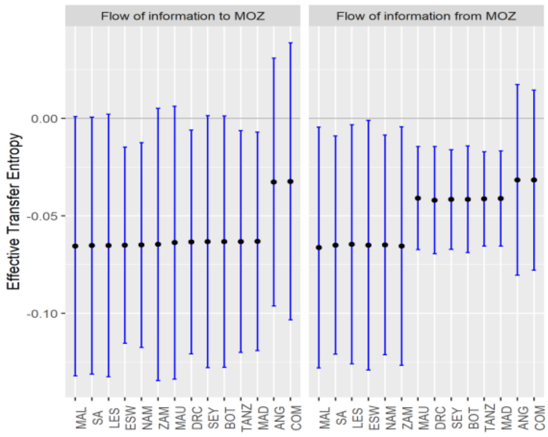
Flow of information from ESW
Flow of information to LES
Flow of information from LES
Effective Transfer Entropy


Flow of information to MAD
Flow of information from MAD
Effective Transfer Entropy


Flow of information to MAL
Flow of information from MAL
Effective Transfer Entropy


Flow of information to MAU
Flow of information from MAU
Effective Transfer Entropy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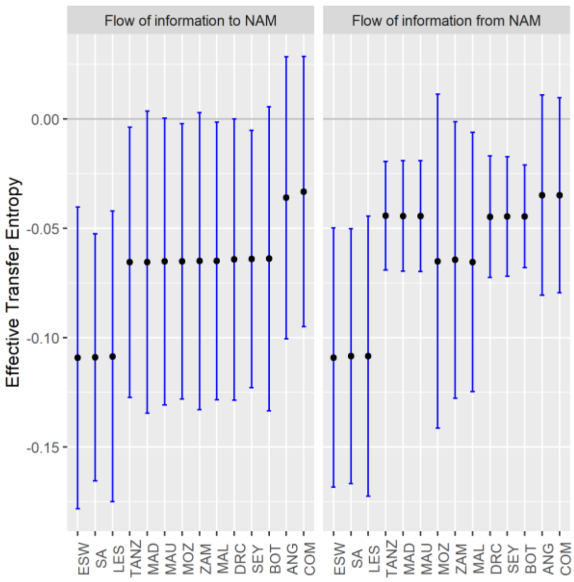
Effective Transfer Entropy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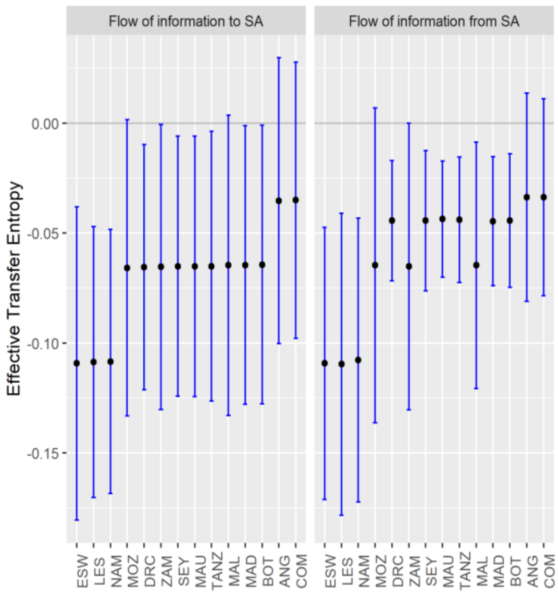
Flow of information to SA
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Effective Transfer Entropy

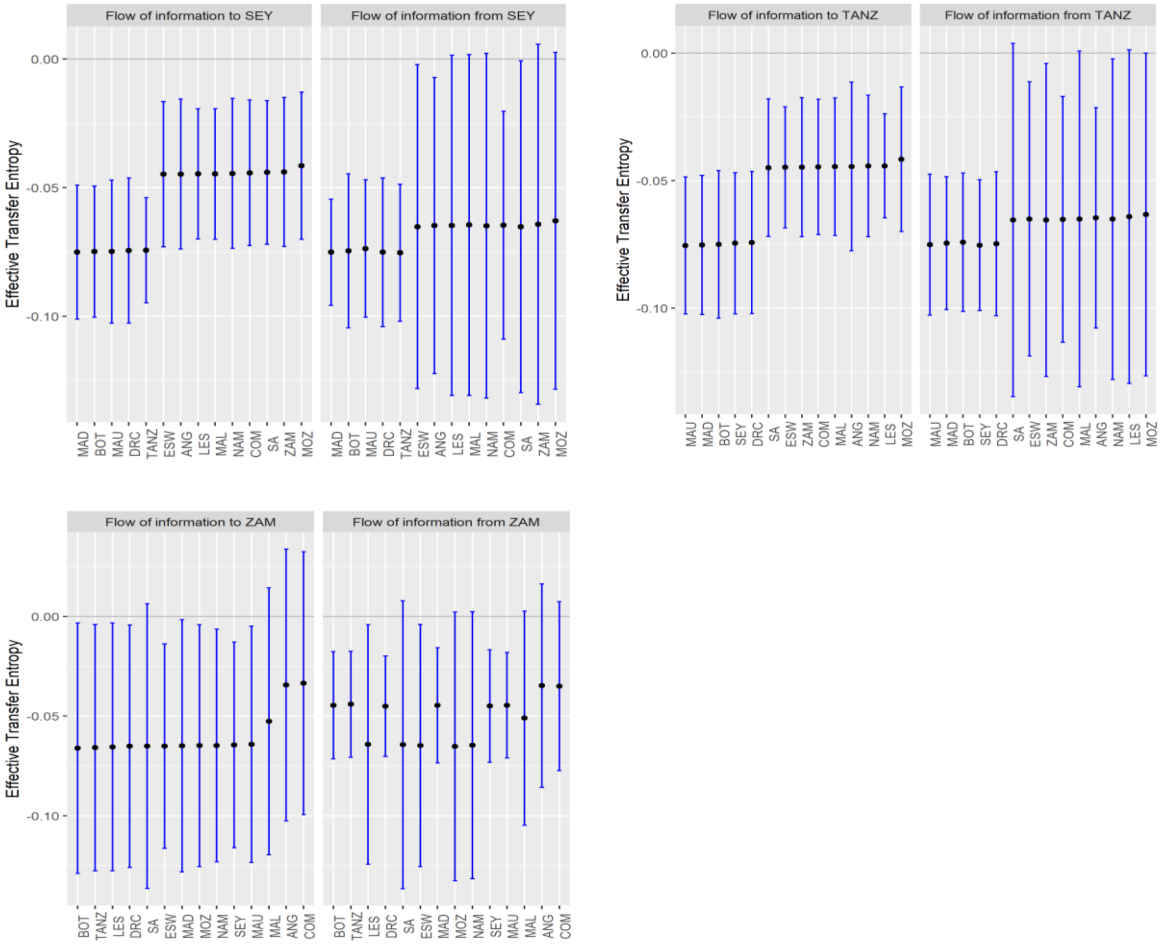
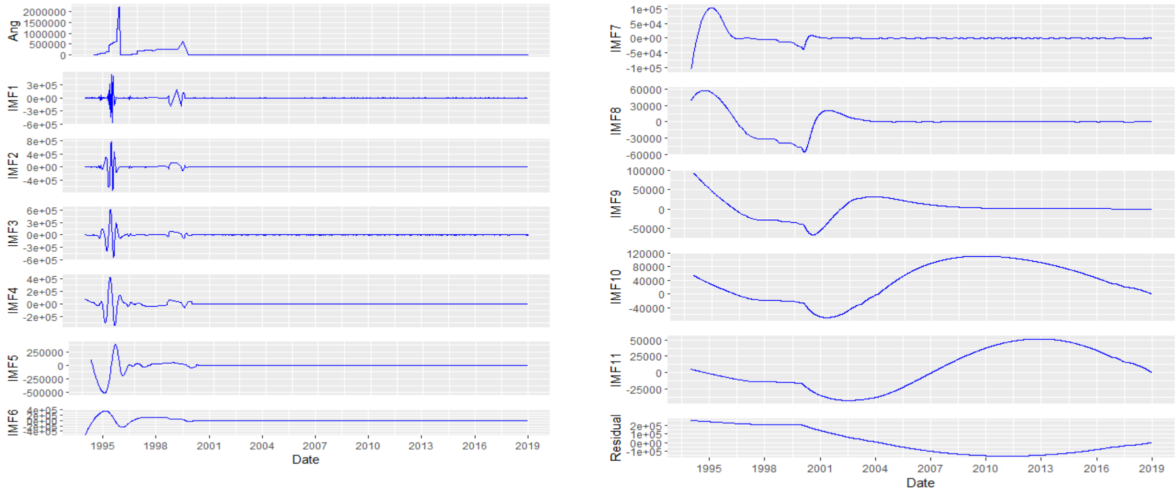


Figure 6.5 Information Flow between exchange rates at the residue

6.5 Conclusion

The EEMD-ETE was utilised in this study to quantify the direction and strength of information transfer between exchange rates at the frequency-domain. In this regard, we investigated the multi-scale information that might be disregarded. Owing to the non-linearity of most financial time series, we adopt a log-likelihood ratio transfer entropy which quantifies information from a probability density function. We set q from the Rényi transfer entropy to 0.3 to account for extreme events specifically, low probability events. This indicates that it is tail events rather than observations in the centre that become imperative to be studied when information flow is employed.
Analysis of the study was presented for four frequency-domains, these are; high, medium, and low frequencies, in addition to the trend. We find a mixture of asymmetric and non-linear bi-directional and unidirectional causality between exchange rates in SADC for the sampled period. The study reveals a significant negative information flow in the medium and long-terms, but a more positive flow in the short-term (high frequency). However, at the residue (fundamental feature), we gauge a bi-directional significant negative information flow within all the 15 economies. This suggests a higher risk of uncertainties in exchange rates of SADC.

Our findings for low property events at multi-scales have policy implications for the direction of the future of SADC monetary union. This would require tough decisions concerning monetary and exchange rate policies. It is not surprising to see the adverse information flow between exchange rates in SADC since most SADC economies have floating exchange rates and an independent monetary policy. To have a sound system of monetary union, a period of exchange rate convergence would be essential, with all potential SADC members pursuing an agreed exchange rate policy. This would gradually minimise the adverse exchange rate fluctuations between these economies in SADC over a given time.



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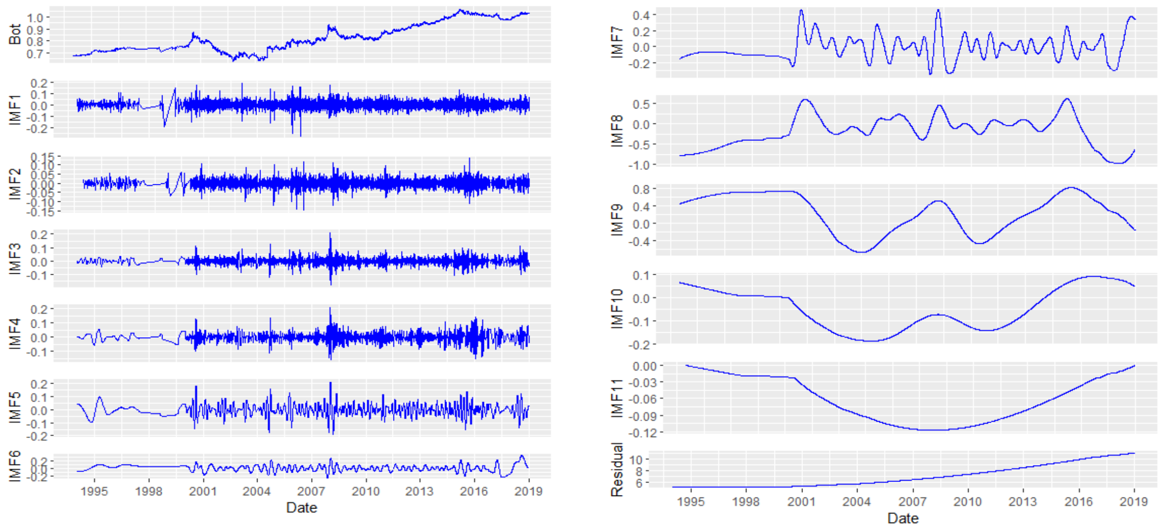

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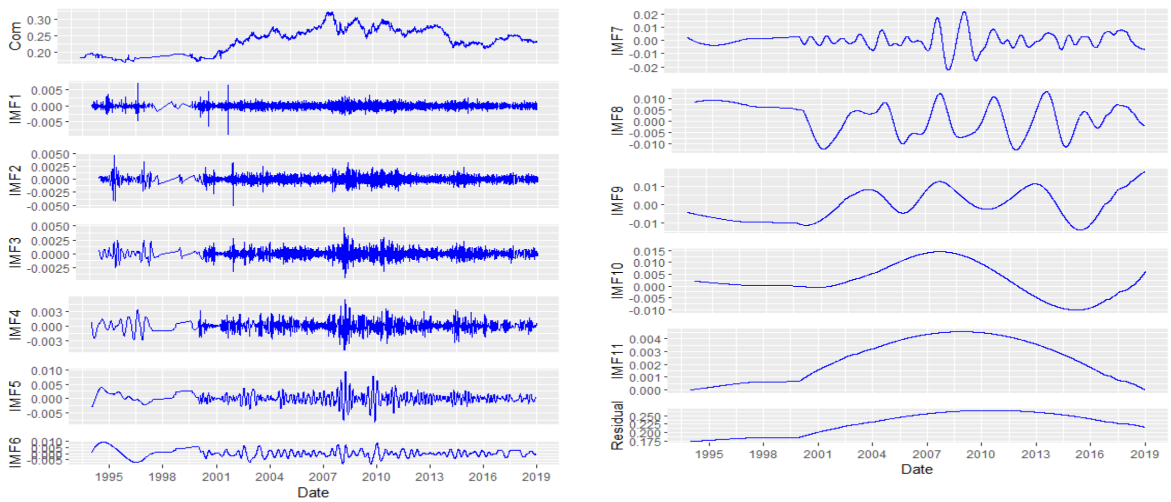
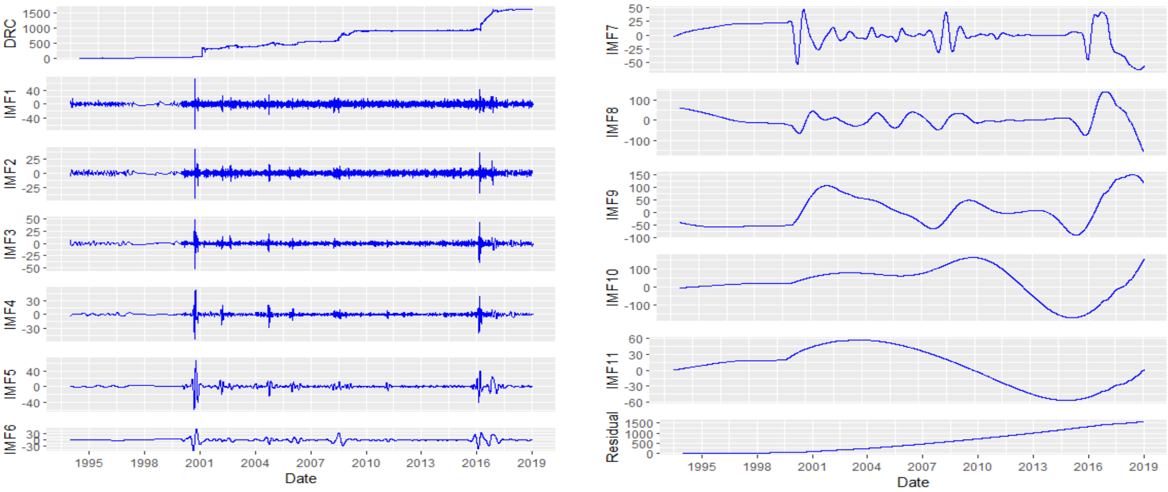


Comoros



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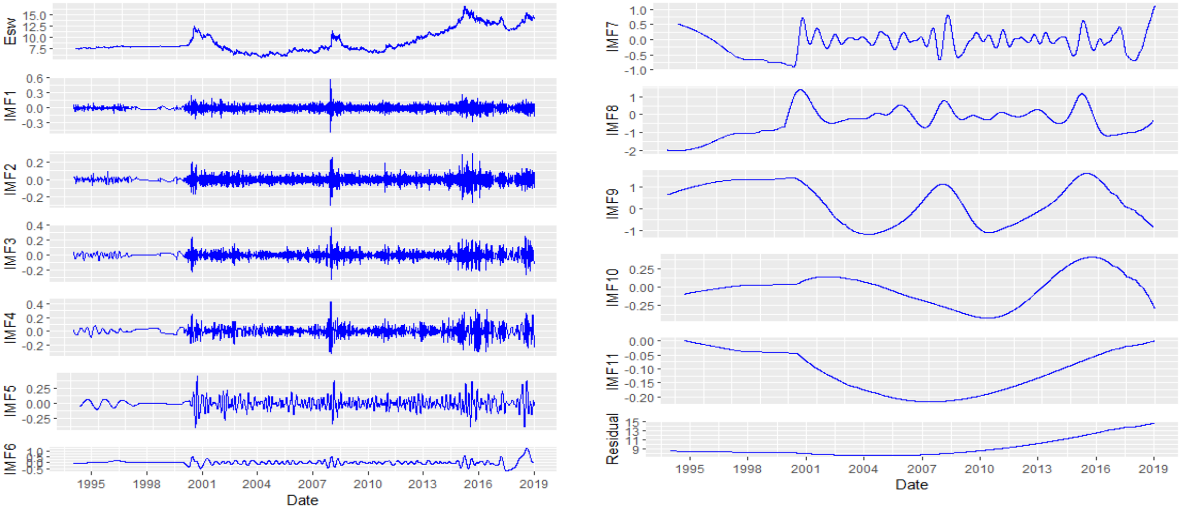
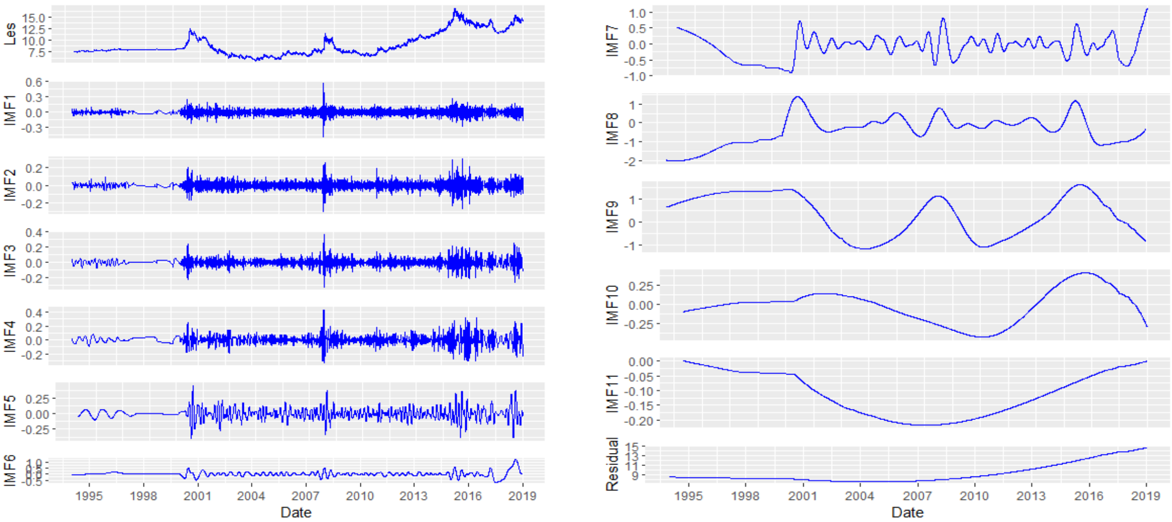


Eswatini



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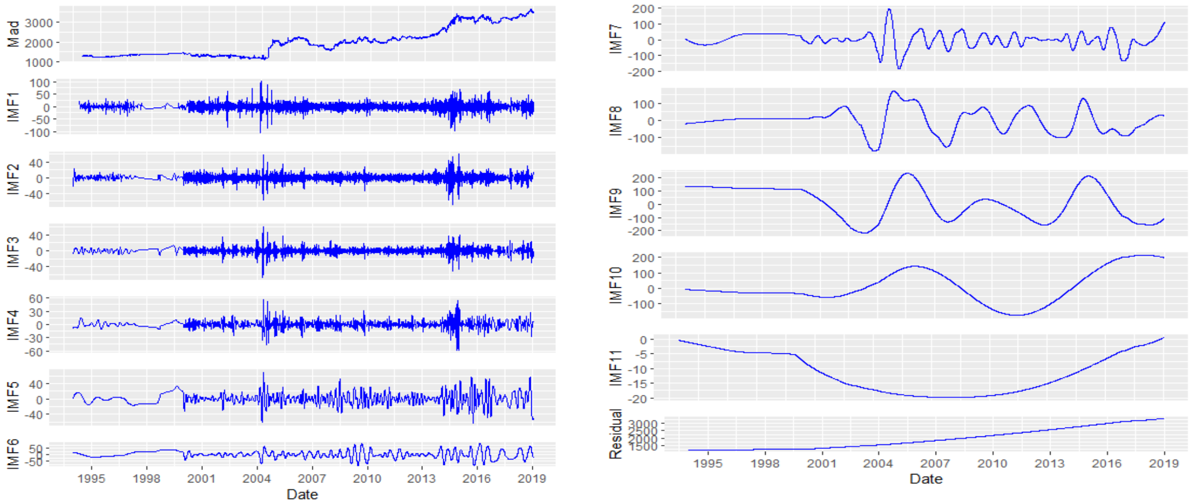

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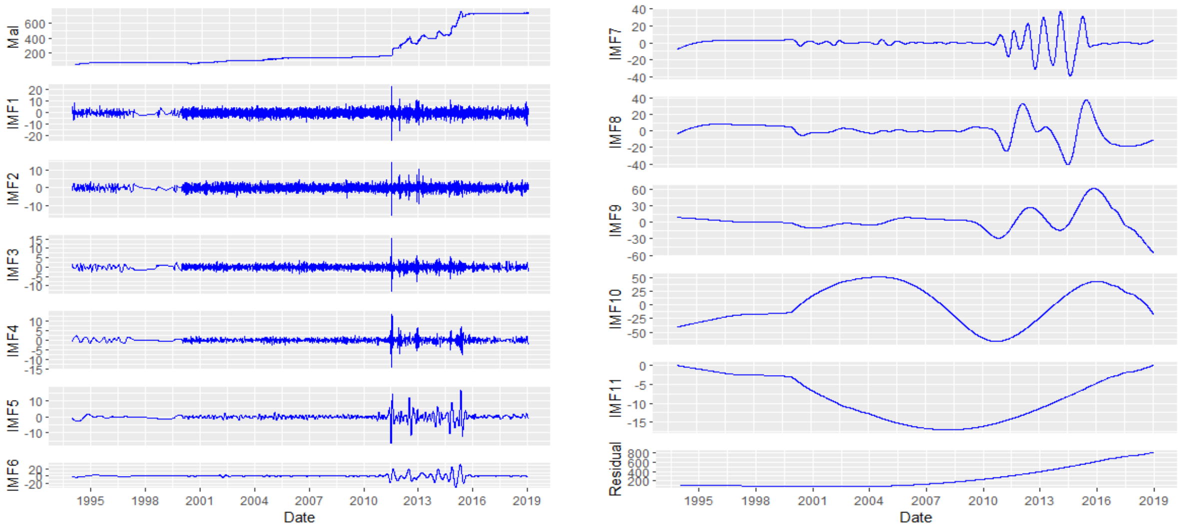

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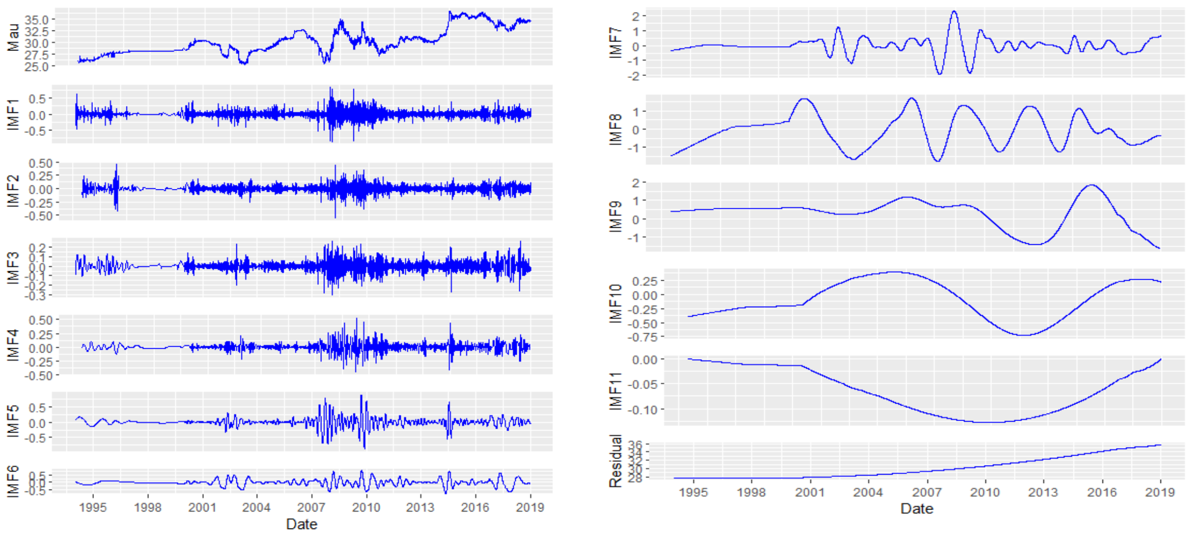

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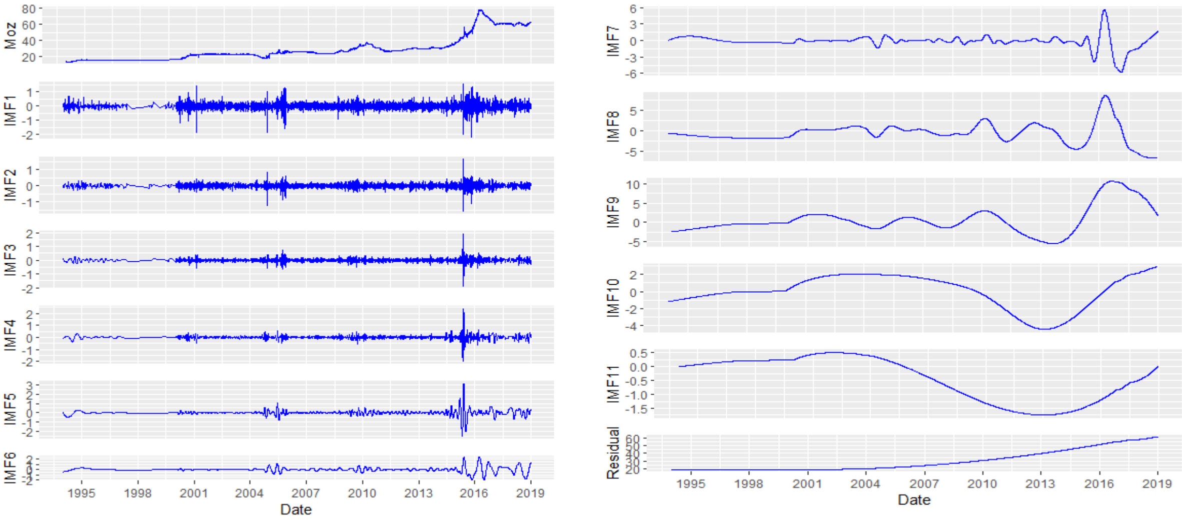

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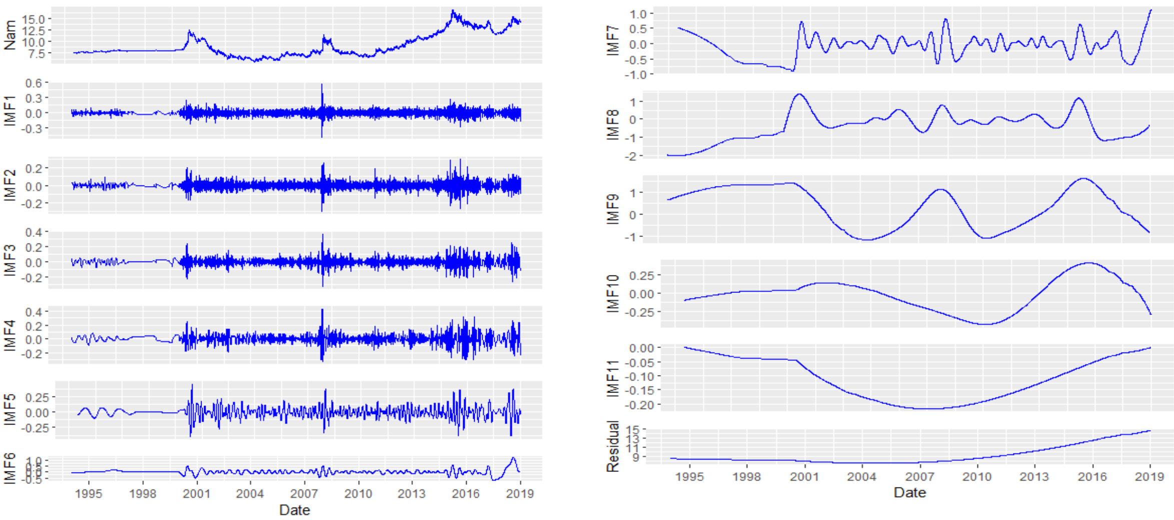

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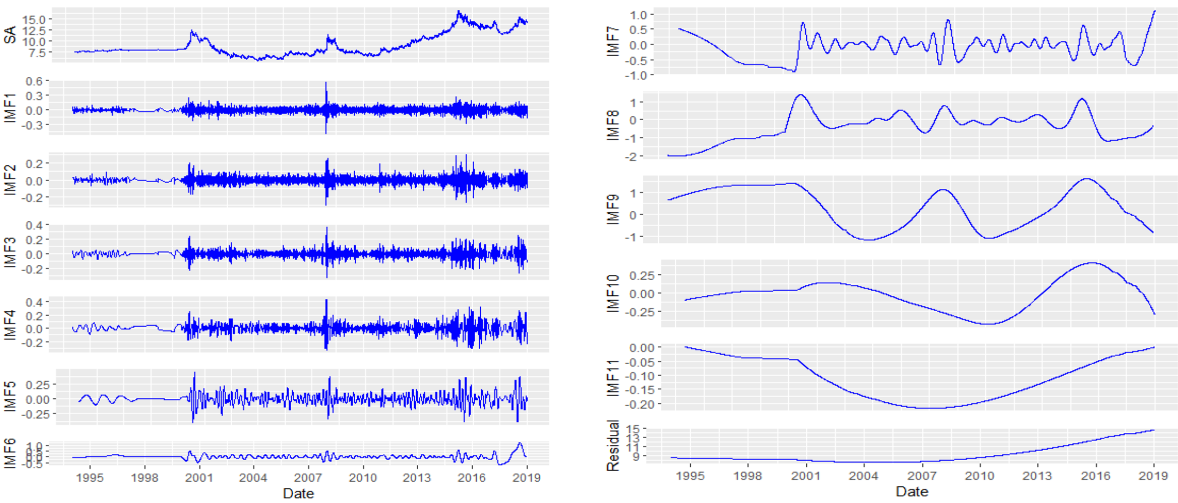
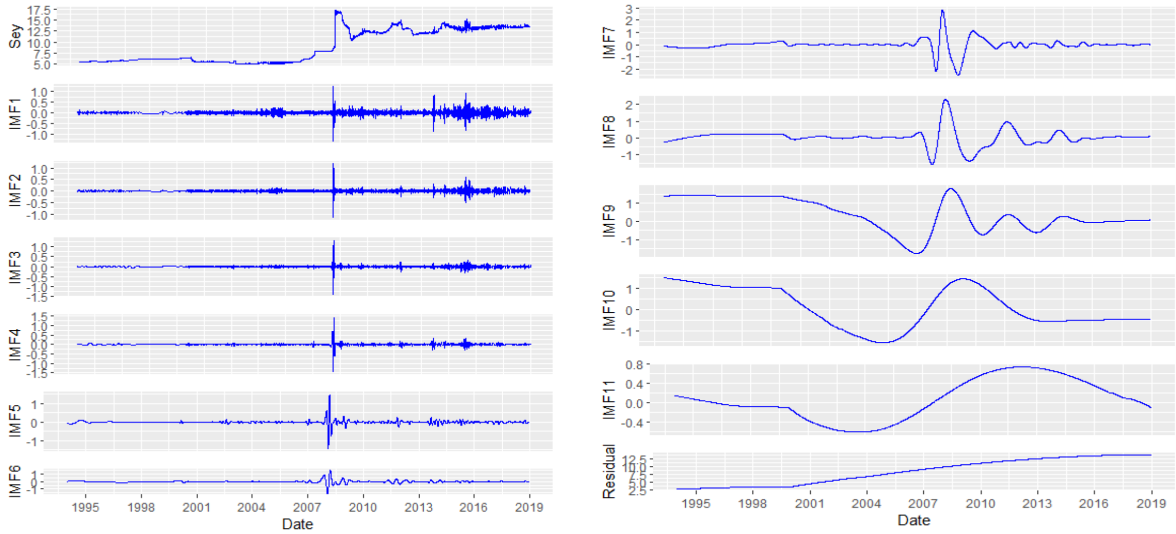
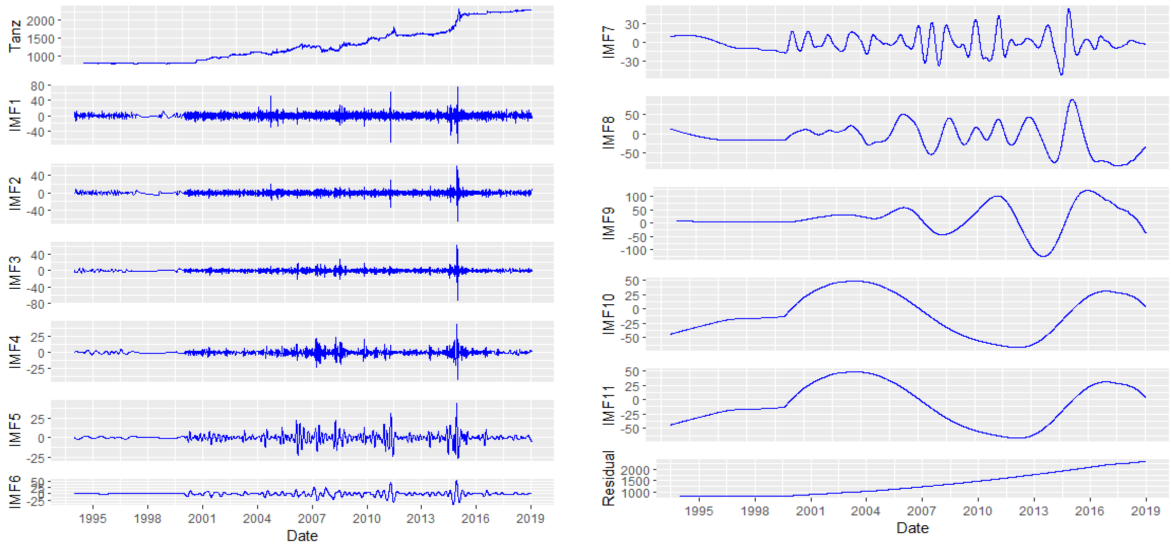


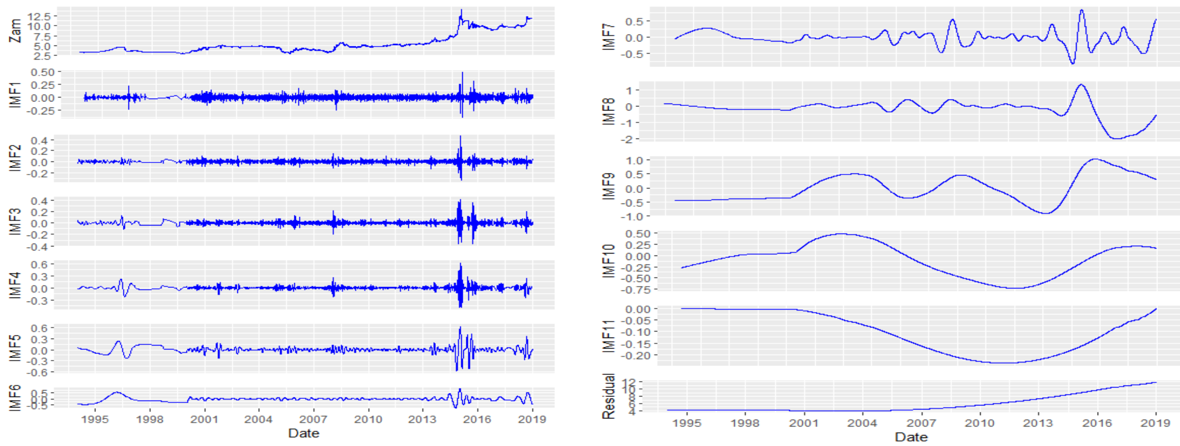
South Africa



Seychelles



Tanzania



Zambia

Figure 6.6 Plots of IMFs from EEMD Group into Various Frequencies

CHAPTER 7

General Conclusions, Contributions, Recommendations, Limitations And Further Research

7.1 General Conclusion

Following the successful implementation of European Monetary Union, the desire for economic and monetary integration of economic blocs in Africa has heightened. The success of any economic and monetary union hinges on coordination of fundamental monetary and economic variables. Therefore, any bloc that seeks to form economic and monetary integration ought to assess the extent of coordination of fundamental economic and monetary variables. Of these, the exchange rate market integration has been cited as a key indicator for stable economic integration because of its pass-through effect on other financial markets. The rising uncertainty in the global economic development makes the comovement of exchange rates important for the formation of monetary unions. Accordingly, exchange rate markets integration in various economic communities have been studied and cited as a key indicator for stable monetary union. The evidence from Rational Expectation Theory (RET) and the Efficient Market Hypothesis (EMH) shows that participants (speculators, central banks, dealers, individuals, etc.) in the exchange rate market are rational and homogeneous with different information, objective interest and investment behaviour as explained by the Heterogeneous Market Hypothesis (HMH). This makes the price and data generation of the exchange rate mixed and noisy. In addition, it suffers from one or more of the following problems: short data span, non-stationarity, non-linearity, and long memory limiting its usage in research and practice. The decision to form a monetary union is a critical one which should emanate from correct modelling. However, the intrinsic characteristics of exchange rate data hinder the use of symmetric models in analysing exchange rate data and could lead to spurious results and conclusion. In response to these, this thesis proposes novel methods based on Huang transforms, specifically empirical mode decompositions (EMDs), to analyse the similarities, interdependencies and information transfer using exchange rate markets data from SADC sub-region. The EMD decomposes a time series into a small number of independent and concretely implicational intrinsic modes based on scale separation and explains the generation of time-series data from an alternative perspective method. EMD is intuitive, direct, posteriori and adaptive. EMD performs a time-adaptive decomposition of a complex signal into elementary, almost orthogonal components that do not overlap in frequency. It thus, improves in the analysis of time series data over detrended fluctuation analysis (DFA) and wavelet transform. In the ensuing paragraphs, a chapter-by-chapter conclusions of the thesis are presented.

In chapter one, a general introduction of the study which set the tone of the thesis is discussed, background of the study, rationale, problem statement, research objectives, scope and the expected contributions of the study to literature were presented.

In the second chapter, we reviewed empirical mode decompositions and its competing models such as Fourier transform and variations of wavelet transforms. It was observed that Huang transform outperforms Fourier and Wavelet transforms because of its adaptiveness, ability to handle non-linear data, localisation of frequency and being empirical. Therefore, using Hilbert-Huang transform as a transformation technique in financial time series will improve the accuracy of the results and implied policy.

In chapter three, we reviewed and examined the stylised facts of non-stationarity and non-linearity of exchange rate data. The three approaches reviewed and used to test non-stationary were ADF, PP and KPSS. The results from these tests showed that SADC exchange rate data are non-stationary. The review and results from BDS test, NN test, Keenan and Tsay tests, TAR-LR test and Engle LM test also showed that SADC exchange rate data are non-linear. Thus, exchange rate markets in SADC are non-stationary and non-linear which requires models that delineate the influence of noise and able to deal with non-stationarity.

In the fourth chapter, we proposed a new way of analysing short and long-run comovement through the analysis of the characteristics of IMFs and residue. First, we compared the performance of EMD and EEMD to decompose SADC exchange rate markets and found EEMD to be superior. We then examined the component of the decomposed series to determine the important component that explains/defines the exchange rate trajectory in SADC. The residue of all the market explained over 80% of the variation of the original series, except for Angola. The analysis of the IMFs and residue obtained from EEMD showed that exchange rate markets in SADC are driven by economic fundamentals and 12 out of 15 countries examined showed some level of similarity in the long-term trend.

In chapter five, we proposed multifrequency network based EEMD-DCCA to study the dynamic interdependence structure of exchange rate markets in SADC. This was done by first decomposing all series into intrinsic mode functions using EEMD and reconstructing the series into three frequency modes: high, medium, and low frequencies, and residue. The DCCA method was used to analyse the cross-correlation between the various frequencies, residues and original series. These were meant to address the non-linearity and non-stationarity in observed exchange rate data. A correlation network was formed from the cross-correlation coefficients in all cases which revealed richer information than would have been obtained from the original series. We found that similarities between the nature of cross-correlation in the high-frequency series mimic the original series and the significant cross-correlation among the long-term trend of most SADC countries exchange rate markets.

In the sixth chapter, EEMD-Effective transfer entropy-based model was developed to study exchange rate market information transmission in SADC. To examine the degree of asymmetry and non-linear directional causality between exchange rates in SADC in the frequency-domain, we employed both the Ensemble Empirical Mode Decomposition (EEMD) and the Rényi effective transfer entropy techniques to investigate the multi-scale information that might be disregarded, and further quantify the directional flow of information. Analysis of the study was presented for four frequency-domains: high, medium, and low frequencies, representing short-, medium-, and long-terms respectively, in addition to the residue (fundamental feature). The study reveals a significant positive information flow in the high frequency, but negative flow in the medium and low frequencies.

7.2 Main Contributions of the Study

The main contributions of this thesis are summarised as follows:

1. The study contributes to the literature by revealing the performance of EMD in comparison with EEMD in decomposing exchange rate data for the first time and studying the underlying characteristics of exchange rates in SADC using the descriptive statistics of the IMFs and residue. The use of the descriptive statistics of IMFs to understand fundamental characteristics of financial time series is novel and provides useful importation of the data generation process as in HMH.

2. The proposal to use the residue from the decomposition to analyse similarity of financial time series structure is novel. This is an improvement in the long-run analysis in time series compared to traditional cointegration methods as it reveals comovement structure at different timescales. This proposed method could be used in analysing similarities in structure of various financial markets.

3. The EEMD-based DCCA model developed offers the opportunity to understand the extent of cross-correlation at different frequency scales. The use of this proposed approach to analyse independency of variables provides rich information for policy implementation and recommendation.

4. For the cross-correlation of financial time series to provide information on fundamental independence, the proposed analytical framework contributes to the literature on the

analysis of dependencies by introducing a new approach to the analysis of multifrequency interdependence. This allows policymakers to gain new insight into the cross-correlations.

5. The EEMD-ETE methodology developed provides perspective of information transmission and quantification to capture information spillover and interactions among different markets, which provides useful information on the spillover direction between variables. This novel method in financial markets microstructure literature.

6. Financial time series often exhibit different characteristics at different time frequencies, and the relations between different variables vary widely across time scales. The utilization of EEMD-ETE offers the opportunity to understand the extent of information transmission at different frequency scales. Thus, delineates the influence of noise from the quantification of information flow across the exchange rate markets.

7. From the financial markets' perspective, this thesis for the first time provides literature on the extent of similarity, independence and information transfer between financial markets in SADC. In terms of similarities, at least 12 countries were similar in structure excluding Angola, Comoros and Seychelles with again Angola and Comoros being obvious outliers but showing some form of orientation toward the SADC market structure. The interdependence analysis showed an increased correlation with increasing frequency of the series and the long-term trend of exchange rates of SADC countries are stronger. Comparing the original series with the levels of decomposed series, the sources of deviation of the exchange rate markets have been identified as the high-frequency component which is linked to speculation activities, short-term policies, and timing of the response to external shocks. The information transfer analysis detected a mixture of asymmetric and non-linear bi-directional and unidirectional causality between exchange rates in SADC for the sampled period. It also reveals a significant negative information flow in the medium and long-terms, but a more positive flow in the short-term (high frequency). However, at the residue (fundamental feature), a bi-directional significant negative information flow within all the 15 economies were detected. These findings has serious implication for policy toward monetary integration in the region.

7.3 Recommendations

The Huang transform approach methodology, specifically EEMD based, proposed for the analysis of similarities, independence and information transfer method can help obtain more internal characteristics and detailed information on financial data generations process, intrinsic characteristics, driving components, cross-correlation and information transfer at various timescales which will help policymakers make a more accurate analysis of exchange rate dynamics, especially for non-linear and non-stationary data. It is therefore recommended that EEMD based method is more effective than EMD and should be used in the analysis of financial time series that are susceptible to non-linearity and non-stationary to elicit the time-frequency information.

The adherence to similarity in long-term fundamentals of 12 countries of SADC give some hope that SADC can form a monetary union. Hence, it is recommended that there should be gradual formation by expanding the existing CMA to accommodate those new countries orientating toward CMA countries taking into consideration other economics agents necessary for optimum currency area such as business cycle synchronisation and macroeconomic convergence. The revelation that the high-frequency component serve as sources of deviation of the exchange rate markets in SADC requires that attention of policymaker be drawn towards activities of speculator, short-term policies, and timing of the response to external shocks.

The findings for low property events at multi-scales have policy implications for the direction of the future of SADC monetary union. This would require tough decisions concerning monetary and exchange rate policies. To have a sound system of monetary union, a period of exchange rate convergence would be essential, with all potential SADC members pursuing an agreed exchange rate policy. This would gradually minimise the adverse exchange rate fluctuations between these economies in SADC over a given time.

7.4 Limitations of the Study

The class of Empirical Mode Decomposition (EEMD), though it corrects the issue of mode-mixing, introduces the problem of exact reconstruction of signals. The ability of the added noise to affect the extrema of the original signal is paramount so that the intermittency of the components will be removed or decreased as much as possible. However, it has been observed that in the predefined constant amplitude value, the extrema are being affected (and therefore decreasing the existing mode mixing) by a random noise, which might not effectively change some extrema. This weakness of EEMD could affect the accuracy of the decomposition and ought to have compared to all variant of EMD class of decompositions. However, Wei et al. (2013) observed through empirical analysis that performance in reconstructing frequencies do not follow the extension sequence and therefore performance of these classes depends on the behaviour of the financial time series. The most profound weakness of empirical mode decomposition is the lack of theoretical foundation. Notwithstanding, the results obtained through combination of battery of methods showed some congruence which reinforced the robustness of the proposed methods. In terms of applied analysis of information transfer, the timing and speed of the information transfer is necessary for monetary policy decision. This is however not employed in the analysis.

7.5 Suggestion for Future Research

Further research could extend the proposed framework by considering an approach to extend variant of EMD with intension to compare the performance of the variant in decomposing financial time series. The emergence of improved decomposition methods such as synchrosqueezing transforms, Improved Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (I-CEEMDAN), among others to address the limitation of the usual empirical decomposition requires that future works explore the use of these approaches. The lack of theoretical foundation for EMD cannot remain forever. Future research may embark on the theoretical underpinnings to broaden the literature and application of EMD.

The timing and speed of transfer of information across financial market has serious implications for the conduct of monetary policy. Future studies should extend this study to examine the speed of information transfer from across financial markets.

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Appendix

R-Codes for Data Validation

```
tt2<-seq(0,237,by=0.05)
noise.amp <- 6.4e-07
trials <- 1000
library(hht)
ceemd.result <- CEEMD(Zam, tt2, noise.amp, trials)
PlotIMFs(ceemd.result)
ceemd.result=Sig2IMF(Zam, tt2)
ceemd.result
PlotIMFs(ceemd.result)
write.table(ceemd.result,file="ceemdtry.csv")
names(xt)
library(EMD)
library(hht)
library(Rlibeemd)
plot(Ang)
eemd.results<- eemd(Zam, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Zam, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdZam3.csv")
names(xt)
setwd("C:/Users/ano77/Desktop/project1/lsadc")
#EEMD
```

```
Angola<- eemd(Ang, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Ang, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdAng21.csv")
plot(Angola[,7:12])
Botswana<- eemd(Bot, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Bot, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdBot.csv")
plot(Botswana)

Comoros<- eemd(Com, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Com, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdCom1.csv")
plot(Comoros)

DRCongo<- eemd(DRC, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(DRC, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdDRC.csv")
plot(DRCongo)

Eswatini<- eemd(Esw, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Esw, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdEsw.csv")
plot(Eswatini)
Lesotho<- eemd(Les, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Les, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdLes.csv")
plot(Lesotho)

Madagascar<- eemd(Mad, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Mad, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdMad.csv")
plot(Madagascar)
```

```
Malawi<- eemd(Mal, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Mal, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdMal.csv")
plot(Malawi)

Mauritius<- eemd(Mau, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Mau, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdMau.csv")
plot(Mauritius)

Mozambique<- eemd(Moz, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Moz, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdMoz.csv")
plot(Mozambique)

Namibia<- eemd(Nam, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Nam, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdNam.csv")
plot(Namibia)

Seychelles<- eemd(Sey, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Sey, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdSey.csv")
plot(Seychelles)

SouthAfrica<- eemd(SA, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(SA, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdSA.csv")
plot(SouthAfrica[,7:12])

Tanzania<- eemd(Tanz, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Tanz, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdTanz.csv")
plot(Tanzania)
```

```
Zambia<- eemd(Zam, num_imfs = 10, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L)
write.table(eemd(Zam, num_imfs = 0, ensemble_size = 250L,
noise_strength = 0.2,S_number = 4L, num_siftings = 50L, rng_seed = 0L,
threads = 0L),file="eemdZam.csv")
plot(Zambia)


install.packages("xts")
install.packages("tidyverse")
library(tidyverse)
library(xts)

library(scales)
autoplot(tss[,1:11]) +
  scale_y_continuous(breaks = 0.02^(0:ceiling(log10(max(tss)))), labels =
comma)

tss <- as.xts(eemd.results)
autoplot(tss)
pt<-read.table("SADC3.txt",header=TRUE)
head(pt); tail(pt)
dim(pt)
pp <- pt[,2:16]
names(pp)
date <- pt$date
date

md <- read.table("eemdZam2.txt", header=T)
dim(md);
head(md); names(md)
mm <- md[1:12]

## Decompose with EMD
c <- emd(Zam, num_imfs = 10, num_siftings = 10)
Angola<- emd(Ang, num_imfs = 10, num_siftings = 10)
write.table(emd(Ang, num_imfs = 0,num_siftings = 10),file="emdAng.csv")
plot(Angola)
Botswana<- emd(Bot, num_imfs = 10, num_siftings = 10)
write.table(eemd(Bot, num_imfs = 0, num_siftings = 10),file="emdBot.csv")
plot(Botswana)

Comoros<- emd(Com, num_imfs = 10, num_siftings = 10)
write.table(emd(Com, num_imfs = 0, num_siftings =
10),file="emdCOM.csv")
plot(Comoros)

DRCongo<- emd(DRC, num_imfs = 10, num_siftings = 10)
write.table(emd(DRC, num_imfs = 0, num_siftings =
10),file="emdDRC.csv")
plot(DRCongo)

Eswatini<- emd(Esw, num_imfs = 10, num_siftings = 10)
```

```
write.table(emd(Esw, num_imfs = 0, num_siftings =
10),file="emdEsw.csv")
plot(Eswatini)
Lesotho<- emd(Les, num_imfs = 10, num_siftings = 10)
write.table(emd(Les, num_imfs = 0, num_siftings = 10),file="emdLes.csv")
plot(Lesotho)

Madagascar<- emd(Mad, num_imfs = 10, num_siftings = 10)
write.table(emd(Mad, num_imfs = 0, num_siftings =
10),file="emdMad.csv")
plot(Madagascar)

Malawi<- emd(Mal, num_imfs = 10, num_siftings = 10)
write.table(emd(Mal, num_imfs = 0, num_siftings = 10),file="emdMal.csv")
plot(Malawi)

Mauritius<- emd(Mau, num_imfs = 10, num_siftings = 10)
write.table(emd(Mau, num_imfs = 0, num_siftings =
10),file="emdMau.csv")
plot(Mauritius)

Mozambique<- emd(Moz, num_imfs = 10, num_siftings = 10)
write.table(emd(Moz, num_imfs = 0, num_siftings =
10),file="emdMoz.csv")
plot(Mozambique)

Namibia<- emd(Nam, num_imfs = 10, num_siftings = 10)
write.table(emd(Nam, num_imfs = 0, num_siftings =
10),file="emdNam.csv")
plot(Namibia)

Seychelles<- emd(Sey, num_imfs = 0,  num_siftings = 10)
write.table(emd(Sey, num_imfs = 0, num_siftings = 10),file="emdSey.csv")
plot(Seychelles[,7:12])

SouthAfrica<- emd(SA, num_imfs = 0, num_siftings = 10)
write.table(emd(SA, num_imfs = 0, num_siftings = 10),file="emdSA.csv")
plot(SouthAfrica[,1:7])+ plot(SouthAfrica[,7:12])

Tanzania<- emd(Tanz, num_imfs = 10, num_siftings = 10)
write.table(emd(Tanz, num_imfs = 0, num_siftings =
10),file="emdTanz.csv")
plot(Tanzania)

Zambia<- emd(Zam, num_imfs = 10, num_siftings = 10)
write.table(emd(Zam, num_imfs = 0, num_siftings =
10),file="emdZam.csv")
plot(Zambia)



EXTREME
#####################################################################
###
kk1<-read.csv("eemdAng.csv",header=TRUE)
```

```
kk2<-read.csv("emdSA.csv",header=TRUE)
attach(kk2)
names(kk2)
library(EMD)
b<-extrema(kk2[,11])
b
length(kk2[,3])/11


kk3<-read.csv("emdAng.csv",header=TRUE)
attach(kk3)
names(kk3)
kk4<-read.csv("eemdbot.csv",header=TRUE)
attach(kk4)
names(kk4)
length(minima)
a<-length(kk4[,4])
a
b<-extrema(kk4[,4])
b
a/b
a*b
var(kk1[,2])
var(kk2[,14])/var(kk2[,2])


p<-kk2[,3]+kk2[,14]
var(p)/var(kk2[,2])

library(Hmisc)

cor.test(kk2[,13], kk2[,2], method = "pearson", alternative = "greater")
cor.test(kk2[,13], kk2[,2], method = "kendall", alternative = "greater")
cor.test(xt, kk1[,1], method = "spearman", alternative = "greater")
cor.test(kk2[,13], kk1[,3])
install.packages("devtools")
install.packages("pvclust")
library(pvclust)
fit <- pvclust(kk1[,3:13], method.hclust="ward",method.dist="euclidean")
fit
plot(fit) # dendogram with p values
box()
# add rectangles around groups highly supported by the data
pvrect(fit, alpha=.95)
CEEMD(Ang, tt, noise.amp, trials, verbose = TRUE,  spectral.method =
"arctan", diff.lag = 1, tol = 5, max.sift = 10, stop.rule = "type5", boundary =
"wave", sm = "none", smlevels = c(1), spar = NULL, max.imf = 1000,
interm = NULL,  noise.type = "gaussian", noise.array = NULL)
ceemd.result <- CEEMD(Zam, tt, noise.amp, trials)
# PlotIMFs(ceemd.result, imf.list = 1:6, time.span = c(5, 10))
CEEMD(sig, tt, noise.amp, trials, verbose = TRUE,spectral.method =
"arctan", diff.lag = 1, tol = 5, max.sift = 200,stop.rule = "type5", boundary =
"wave", sm = "none",smlevels = c(1), spar = NULL, max.imf = 1000, interm
= NULL,noise.type = "gaussian", noise.array = NULL)
```

```
P <- CEEMD(Ang, tt, noise.amp, trials, verbose = TRUE, spectral.method
= "arctan", diff.lag = 1, tol = 5, max.sift = 200, stop.rule = "type5", boundary
= "wave", sm = "none", smlevels = c(1), spar = NULL, max.imf = 1000,
interm = NULL, noise.type = "gaussian", noise.array = NULL)

PLOTING CORRELATION MATRIX
################################################################
###
library(devtools)
Compute a correlation matrix
data(kk2)
corr <- round(cor(kk2[,2:5]), 1)
head(corr[, 2:13])
install.packages("ggcorrplot")
library(ggcorrplot)
# method = "square" (default)
library
ggcorrplot(corr)


PLOTING CORRELATION MATRIX
################################################################
###
kk5<-read.csv("resadc.csv",header=TRUE)
attach(kk5)
names(kk5)
install.packages("corrplot")
library(corrplot)
M<-round(cor(kk5[,2:16]), 1)
corrplot(M, method = "color")







################################################################
#####
Entropy Estimation
################################################################
#########
install.packages("RTransferEntropy")
install.packages("xts")
library(parallel)
library(RTransferEntropy)

setwd("C:/Users/ano77/Desktop/project1")
xt<-read.csv("paper2.csv",header=TRUE)
attach(xt)
names(xt)
Z<-returns(ESWT)
x=ESWLF
y=NAMLF
```

```
transfer_entropy(x, y,
                 lx = 1, ly = 1, q = 0.3,
                 entropy = c('Shannon', 'Renyi'), shuffles = 100,
                 type = c('quantiles', 'bins', 'limits'),
                 quantiles = c(5, 95), bins = NULL, limits = NULL,
                 nboot = 300, burn = 50, quiet = FALSE, seed = NULL)
#################################################################
###################
#PLOT
#################################################################
#################
df<-read.csv("df.csv",header=TRUE)

ggplot(df, aes(x = ticker, y = ete)) +
  facet_wrap(~dir) +
  geom_hline(yintercept = 0, color = "gray") +
  theme(axis.text.x = element_text(angle = 90)) +
  labs(x = NULL, y = "Renyi Effective Transfer Entropy") +
  geom_errorbar(aes(ymin = ete - qnorm(0.95) * se,
                    ymax = ete + qnorm(0.95) * se),
                width = 0.25, col = "blue") +
  geom_point()
#################################################################
#################
# Load required packages
library(RTransferEntropy)
library(xts)
library(zoo)
library(forecast)
library(e1071)
library(psych)
library(rgl)
library(ggplot2)
library(scatterplot3d)
library(hrbrthemes)
library(dplyr)
library(fBasics)
library(tseries)
library(quantmod)
library(FinTS)
library(nloptr)
library(fUnitRoots)
library(QRM)
library(data.table)
library(parallel)
cl <-makeCluster(8)

# Importing data for effective transfer entropy calculation
paper3a<- read.csv("paper3mal.csv",header=T)
attach(paper3a)
names(paper3a)

# Estimating effective transfer entropy
x=ret
y=MALT
```

```
res <- lapply(split(paper3a,paper3a$ticker),function(d) {
te <- transfer_entropy(d$ret, d$MALT,
                lx = 1, ly = 1, q = 0.3,
                 entropy = "Renyi", shuffles = 10,
                 type = c('quantiles', 'bins', 'limits'),
                 quantiles = c(5, 95), bins = NULL, limits = NULL,
                 nboot = 30, burn = 50, quiet = FALSE, seed = NULL)
data.table(ticker = d$ticker[1],dir = c("X->Y", "Y->X"),
coef(te)[1:2, 2:3])
})
df <- rbindlist(res)

write.csv(df, file = "HMPMAL.csv")
# order the ticker by the ete of X->Y
df[,ticker := factor(ticker,
levels = unique(df$ticker)[order(df[dir == "X->Y"]$ete)])]

# rename the variable (xy/yx)
df[, dir := factor(dir, levels = c("X->Y", "Y->X"),
 labels = c("Flow of information to MAL","Flow of information from MAL"))]

# Charting the effective transfer entropy between the market and the
stocks
ggplot(df, aes(x = ticker, y = ete)) +
facet_wrap(~dir) +
geom_hline(yintercept = 0, color = "gray") +
theme(axis.text.x = element_text(angle = 90)) +
labs(x = NULL, y = "Effective Transfer Entropy") +
geom_errorbar(aes(ymin = ete - qnorm(0.95) * se,
ymax = ete + qnorm(0.95) * se),
width = 0.25, col = "blue") +
geom_point()
ggsave("MAL.png",width=5,height=5)



stopCluster(cl)

##############################

write.csv(df, file = "HMDATA1.csv")

####################################

# uploading data for heat maps
dat<-read.csv("HMPTO.csv",header=T)

# Drawing heat maps of effective transfer entropy among the stocks
ggplot(dat,aes(x=dat$variable,y=dat$ticker)) +
geom_tile(aes(fill = dat$ete),colour = "grey", na.rm = TRUE) +
scale_fill_gradient2(high="blue",
midpoint=0,space="Lab",na.value="grey70",guide="colourbar",aesthetics=
"fill")+
guides(fill=guide_legend(title="ETE")) +
theme_bw() + theme_minimal() +
```

```
labs(title = "Information flow from Country  X to Country Y",x = "Country
X", y = "Country  Y") +
theme(axis.text.x = element_text(angle = 90))+
theme(panel.grid.major = element_blank(), panel.grid.minor =
element_blank())
ggsave("HMPTO.png",width=5,height=5)

##################################################################
#################
rm(list=ls())
setwd("C:/Users/ano77/Desktop/project1")
paper3<- read.csv("paper3a3.csv",header=T)
attach(paper3)
names(paper3)
library(dplyr)
ANGT <- tibble(x = c("ANG"), y = ANGT)
ANGT %>% slice(rep(1:n(), each = 1))

write.csv(ANGT, file = "Trend.csv", row.names = FALSE)

BOTT <- tibble(x = c("BOT"), y = BOTT)
BOTT %>% slice(rep(1:n(), each = 1))

COMT <- tibble(x = c("COM"), y = COMT)
COMT %>% slice(rep(1:n(), each = 1))

DRCT <- tibble(x = c("DRC"), y = DRCT)
DRCT %>% slice(rep(1:n(), each = 1))

ESWT <- tibble(x = c("ESW"), y = ESWT)
ESWT %>% slice(rep(1:n(), each = 1))

LEST <- tibble(x = c("LES"), y = LEST)
LEST %>% slice(rep(1:n(), each = 1))

MADT <- tibble(x = c("MAD"), y = MADT)
MADT %>% slice(rep(1:n(), each = 1))

MALT <- tibble(x = c("MAL"), y = MALT)
MALT %>% slice(rep(1:n(), each = 1))

MAUT <- tibble(x = c("MAU"), y = MAUT)
MAUT %>% slice(rep(1:n(), each = 1))

MOZT <- tibble(x = c("MOZ"), y = MOZT)
MOZT %>% slice(rep(1:n(), each = 1))

NAMT <- tibble(x = c("NAM"), y = NAMT)
NAMT %>% slice(rep(1:n(), each = 1))


SEYT <- tibble(x = c("SEY"), y = SEYT)
SEYT %>% slice(rep(1:n(), each = 1))

SAT <- tibble(x = c("SA"), y = SAT)
```

```
SAT %>% slice(rep(1:n(), each = 1))

TANZT <- tibble(x = c("TANZ"), y = TANZT)
TANZT %>% slice(rep(1:n(), each = 1))

ZAMT <- tibble(x = c("ZAM"), y = ZAMT)
ZAMT %>% slice(rep(1:n(), each = 1))

##append subsequent estimates ##recall to change month name every
time
write.table(BOTT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(COMT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(DRCT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(ESWT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(LEST, "Trend.csv", append = TRUE,col.names = FALSE, quote
= FALSE, sep = ',')

write.table(MADT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MALT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MAUT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MOZT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(NAMT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(SEYT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(SAT, "Trend.csv", append = TRUE,col.names = FALSE, quote
= FALSE, sep = ',')

write.table(TANZT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')
```

```
write.table(ZAMT, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

#######################HF####################################
#########
rm(list=ls())
setwd("C:/Users/ano77/Desktop/project1")
paper3<- read.csv("paper3a3.csv",header=T)
attach(paper3)
names(paper3)
library(dplyr)

ANGHF <- tibble(x = c("ANG"), y = ANGHF)
ANGHF %>% slice(rep(1:n(), each = 1))

write.csv(ANGHF, file = "Trend.csv", row.names = FALSE)

BOTHF <- tibble(x = c("BOT"), y = BOTHF)
BOTHF %>% slice(rep(1:n(), each = 1))

COMHF <- tibble(x = c("COM"), y = COMHF)
COMHF %>% slice(rep(1:n(), each = 1))

DRCHF <- tibble(x = c("DRC"), y = DRCHF)
DRCHF %>% slice(rep(1:n(), each = 1))

ESWHF <- tibble(x = c("ESW"), y = ESWHF)
ESWHF %>% slice(rep(1:n(), each = 1))

LESHF <- tibble(x = c("LES"), y = LESHF)
LESHF %>% slice(rep(1:n(), each = 1))

MADHF <- tibble(x = c("MAD"), y = MADHF)
MADHF %>% slice(rep(1:n(), each = 1))

MALHF <- tibble(x = c("MAL"), y = MALHF)
MALHF %>% slice(rep(1:n(), each = 1))

MAUHF <- tibble(x = c("MAU"), y = MAUHF)
MAUHF %>% slice(rep(1:n(), each = 1))

MOZHF <- tibble(x = c("MOZ"), y = MOZHF)
MOZHF %>% slice(rep(1:n(), each = 1))

NAMHF <- tibble(x = c("NAM"), y = NAMHF)
NAMHF %>% slice(rep(1:n(), each = 1))


SEYHF <- tibble(x = c("SEY"), y = SEYHF)
SEYHF %>% slice(rep(1:n(), each = 1))

SAHF <- tibble(x = c("SA"), y = SAHF)
SAHF %>% slice(rep(1:n(), each = 1))

TANZHF <- tibble(x = c("TANZ"), y = TANZHF)
```

```
TANZHF %>% slice(rep(1:n(), each = 1))

ZAMHF <- tibble(x = c("ZAM"), y = ZAMHF)
ZAMHF %>% slice(rep(1:n(), each = 1))

##append subsequenHF estimates ##recall to change month name every
time
write.table(BOTHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(COMHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(DRCHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(ESWHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(LESHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MADHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MALHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MAUHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MOZHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(NAMHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(SEYHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(SAHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(TANZHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(ZAMHF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')
```

```
##########################LF#################################
###
rm(list=ls())
setwd("C:/Users/ano77/Desktop/project1")
paper3<- read.csv("paper3a3.csv",header=T)
attach(paper3)
names(paper3)
library(dplyr)

ANGLF <- tibble(x = c("ANG"), y = ANGLF)
ANGLF %>% slice(rep(1:n(), each = 1))

write.csv(ANGLF, file = "Trend.csv", row.names = FALSE)

BOTLF <- tibble(x = c("BOT"), y = BOTLF)
BOTLF %>% slice(rep(1:n(), each = 1))

COMLF <- tibble(x = c("COM"), y = COMLF)
COMLF %>% slice(rep(1:n(), each = 1))

DRCLF <- tibble(x = c("DRC"), y = DRCLF)
DRCLF %>% slice(rep(1:n(), each = 1))

ESWLF <- tibble(x = c("ESW"), y = ESWLF)
ESWLF %>% slice(rep(1:n(), each = 1))

LESLF <- tibble(x = c("LES"), y = LESLF)
LESLF %>% slice(rep(1:n(), each = 1))

MADLF <- tibble(x = c("MAD"), y = MADLF)
MADLF %>% slice(rep(1:n(), each = 1))

MALLF <- tibble(x = c("MAL"), y = MALLF)
MALLF %>% slice(rep(1:n(), each = 1))

MAULF <- tibble(x = c("MAU"), y = MAULF)
MAULF %>% slice(rep(1:n(), each = 1))

MOZLF <- tibble(x = c("MOZ"), y = MOZLF)
MOZLF %>% slice(rep(1:n(), each = 1))

NAMLF <- tibble(x = c("NAM"), y = NAMLF)
NAMLF %>% slice(rep(1:n(), each = 1))


SEYLF <- tibble(x = c("SEY"), y = SEYLF)
SEYLF %>% slice(rep(1:n(), each = 1))

SALF <- tibble(x = c("SA"), y = SALF)
SALF %>% slice(rep(1:n(), each = 1))

TANZLF <- tibble(x = c("TANZ"), y = TANZLF)
TANZLF %>% slice(rep(1:n(), each = 1))

ZAMLF <- tibble(x = c("ZAM"), y = ZAMLF)
```

```
ZAMLF %>% slice(rep(1:n(), each = 1))




##append subsequenLF estimates ##recall to change month name every
time
write.table(BOTLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(COMLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(DRCLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(ESWLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(LESLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MADLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MALLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MAULF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(MOZLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(NAMLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(SEYLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(SALF, "Trend.csv", append = TRUE,col.names = FALSE, quote
= FALSE, sep = ',')

write.table(TANZLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')

write.table(ZAMLF, "Trend.csv", append = TRUE,col.names = FALSE,
quote = FALSE, sep = ',')


#########################MF##################################
###############
rm(list=ls())
```

```
setwd("C:/Users/ano77/Desktop/project1")
paper3<- read.csv("paper3a3.csv",header=T)
attach(paper3)
names(paper3)
library(dplyr)

ANGMF <- tibble(x = c("ANG"), y = ANGMF)
ANGMF %>% slice(rep(1:n(), each = 1))

write.csv(ANGMF, file = "Trend.csv", row.names = FALSE)

BOTMF <- tibble(x = c("BOT"), y = BOTMF)
BOTMF %>% slice(rep(1:n(), each = 1))

COMMF <- tibble(x = c("COM"), y = COMMF)
COMMF %>% slice(rep(1:n(), each = 1))

DRCMF <- tibble(x = c("DRC"), y = DRCMF)
DRCMF %>% slice(rep(1:n(), each = 1))

ESWMF <- tibble(x = c("ESW"), y = ESWMF)
ESWMF %>% slice(rep(1:n(), each = 1))

LESMF <- tibble(x = c("LES"), y = LESMF)
LESMF %>% slice(rep(1:n(), each = 1))

MADMF <- tibble(x = c("MAD"), y = MADMF)
MADMF %>% slice(rep(1:n(), each = 1))

MALMF <- tibble(x = c("MAL"), y = MALMF)
MALMF %>% slice(rep(1:n(), each = 1))

MAUMF <- tibble(x = c("MAU"), y = MAUMF)
MAUMF %>% slice(rep(1:n(), each = 1))

MOZMF <- tibble(x = c("MOZ"), y = MOZMF)
MOZMF %>% slice(rep(1:n(), each = 1))

NAMMF <- tibble(x = c("NAM"), y = NAMMF)
NAMMF %>% slice(rep(1:n(), each = 1))


SEYMF <- tibble(x = c("SEY"), y = SEYMF)
SEYMF %>% slice(rep(1:n(), each = 1))

SAMF <- tibble(x = c("SA"), y = SAMF)
SAMF %>% slice(rep(1:n(), each = 1))

TANZMF <- tibble(x = c("TANZ"), y = TANZMF)
TANZMF %>% slice(rep(1:n(), each = 1))

ZAMMF <- tibble(x = c("ZAM"), y = ZAMMF)
ZAMMF %>% slice(rep(1:n(), each = 1))
```

```
##append subsequenMF estimates ##recall to change month name every time
write.table(BOTMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(COMMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(DRCMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(ESWMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(LESMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(MADMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(MALMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(MAUMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(MOZMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(NAMMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(SEYMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(SAMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(TANZMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')

write.table(ZAMMF, "Trend.csv", append = TRUE,col.names = FALSE, quote = FALSE, sep = ',')
```