

Market Efficiency of African Stock Markets

by

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Declaration

I, Emmanuel Numapau Gyamfi declare that this thesis is my original work and has not been submitted for any degree at any other university or institution. The thesis does not contain other persons' writing unless specifically acknowledged and referenced accordingly.

Signed (Student): Date:

Abstract

There has been a growing interest in investment opportunities in Africa. The net foreign direct investment (FDI) to Sub-Saharan Africa has increased from \$13 billion in 2004 to about \$54 billion in 2015. Investing on the stock markets is one of such investment opportunities. Stock markets in Africa have realised growth in market capitalization, membership, value and volume traded due to an increase in investments. This level of growth in African stock markets has raised questions about their efficiency. This thesis examined the weak-form informational efficiency of African stock markets. The aim therefore of this thesis is to test the efficiency of African stock markets in the weak-form of the Efficient Market Hypothesis (EMH) for eight countries, namely, Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia. Since, the researcher will be testing the weak-form of the EMH, the data to be used is on past price information on the markets of the eight countries. Data for the eight countries were obtained from DataStream for the period between August 28, 2000 to August 28, 2015. The data is for a period of 180 months which resulted in 3915 data points.

Although there have been studies on the weak-form market efficiency of African stock markets, the efficiency conclusions on the markets have been mixed. This problem might be due to the methods used in the analyses. First, most of the methods used were linear in nature although the data generating process of stock market data is nonlinear and hence nonlinear methods maybe more appropriate in its analysis. Also these linear methods tested the efficiency of African markets in absolute form, however, an efficiency conclusion relying solely on absolute efficiency might be misleading because, stock markets become efficient with time due to improvements in the quality of information processing from reforms on the markets. The researcher solved this problem of using absolute frequency by comparing the results when the presence of long-memory in frequency and time domains of the markets were examined. The researcher used a semi-parametric estimator, the Local Whittle estimator to test for long-memory in frequency domain and the Detrended Fluctuation Analysis (DFA) to test for long-memory in time domain. The DFA method is suitable for both stationary and nonstationary time series which makes it to have more power over methods like the rescaled range analysis (R/S) in the estimation of Hurst exponent.

Second, the researcher examined whether the markets were predictable under the Adaptive Market Hypothesis (AMH). The researcher employed the Generalised Spectral (GS) test to examine the Martingale difference hypothesis (MDH) of the markets. The Generalised spectral (GS) test is a non-parametric

test designed to detect the presence of linear and nonlinear dependencies in a stationary time series. The GS test considers dependence at all lags.

Third, because of the nonlinear nature in the data-generating process on the markets, the stationarity of the market returns under a nonlinear Exponential Smooth Threshold Autoregressive (ESTAR) model was examined. A nonlinear ADF unit root test against ESTAR and a modified Wald-type test against ESTAR in the analysis were employed. Fourth, the self-exciting threshold Autoregressive (SETAR) method was employed to model the returns when non-linear patterns were observed as a result of nonlinear data generating process on the markets.

The literature on market efficiency of African stock markets has shown that variations exist in the study characteristics. There are variations in the method of analysis, type of test, type of data employed, time period chosen and the scope of analysis for the studies. The researcher therefore quantitatively reviewed previous studies by means of meta-analysis to identify which study characteristics affects efficiency conclusions of African markets using the mixed effects model.

The findings showed the presence of long-memory in the returns of the stock markets when the whole sample was used. This made the markets weak-form inefficient, however, when the researcher tested for the persistence of long-memory through time, there were periods the markets were efficient in the weak-form. The memory effect was low in the South African market but high in the Mauritian market.

Furthermore, it was observed that, the returns for Egypt, which were highly predictable when the whole data was analysed became not highly predictable when the rolling window approach of the GS test was used. Egypt had one of the lowest percentages of the windows that had a p-value less than 0.05 after South Africa.

The results obtained from using the non-linear unit root tests on the logarithmic price series of the markets under study showed that, the markets were non-stationary and hence weak-form efficient under an ESTAR framework but for Botswana. Thus the markets were weak-form efficient when analysed using a non-linear method. This observation means that Africa's foreign direct investment would have been increased over the years if the appropriate methods are used. This is because, over the years, studies on the weak-form efficiency African stock markets have ended with mixed conclusions with most of the markets being concluded to be weak-form inefficient as a result of the use of linear methods in the analysis. This finding, to us, has had an effect on investors commitments to Africa because the right methodology was not employed.

The findings from modelling the returns under the non-linear SETAR model showed that, the SETAR model performs better than the standard AR(1) and AR(2) model for all the markets under study after the non-linear patterns were identified in the returns series. The SETAR (2,2,2) model is a threshold model, therefore, investors are able to move freely in search of higher opportunities between the low and high regimes. Investors main aim is to make profits, hence, the threshold model of SETAR gives them the freedom to move to a regime where the rate of returns is increasing unlike the standard AR(1) and AR(2) linear models where there are no switching of regimes.

Finally, none of the study characteristics in the market efficiency studies was found to be significant in efficiency conclusions of African stock markets but the indicator for publication bias was significant. This means that there has been a change in attitude in recent years towards studies on informational market efficiency whose results do not support the Efficient Market Hypothesis (EMH), unlike the earlier years when the EMH was formulated and acclaimed to be one of the best propositions in economics.

It was therefore concluded that when time-varying methods are used in analysing weak-form efficiency, the dynamics of the markets become known to investors for proper decision-making. Also, nonlinear methods should be used in order to reflect the nonlinear nature of data capturing on the stock markets to prevent arbitrageurs from making abnormal returns as a result of indecisive conclusions due to the use of linear methods.

Keywords: Market efficiency, DFA, Hurst exponent, Non - linear models, Meta analysis

Jel Classification: C12, C13, C24, C83, D53, G10, G14

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Dedication

To my wife Emily and children, Ryan and Kayla.

Summary

This thesis is based on the following six original articles published in accredited journals. The articles make contributions to the extant literature on the efficiency of African stock markets.

1. **Gyamfi, E. N**, Kyei, K.A & Gill, R (2016a), Long-Memory in Asset Returns and Volatility: Evidence from West Africa, *Investment Management and Financial Innovations*, 13(2), 26 - 30.
2. **Gyamfi, E. N**, Kyei, K.A & Gill, R (2016b), Long-Memory Persistence in African Stock Markets, *EuroEconomica*, 35(1), 83 - 91.
3. **Gyamfi, E. N** & Kyei, K.A (2016c), Modeling Stock Market Returns under Self-exciting Threshold Autoregressive Model: Evidence from West Africa, *International Journal of Economics and Financial Issues* 6 (3), 1194-1199.
4. **Gyamfi, E. N**, Kyei, K.A & Gill, R (2016d), African Stock Markets and Return Predictability, *Journal of Economics and Behavioral Studies* 8 (5), 91-99.
5. **Gyamfi, E. N**, Kyei, K.A & Gill, R (2016e), Stationarity of African Stock Markets under an ESTAR framework, *EuroEconomica* 35 (2), 93-101.
6. **Gyamfi, E. N**, Kyei, K.A & Gill, R (2017), Market Efficiency of African Stock Markets - A Meta analysis, *The Journal of Developing Areas*, 51 (4), 69-80.

The thesis proceeds as follows:

A general introduction and a literature review are presented in Chapters 1 and 2 respectively. In Chapter 3, an extension was done on the study by *Gyamfi et al.* (2016a) on testing whether long-memory was present in West African markets using the Local Whittle estimator to evaluate the Hurst exponent. The extension is done for the eight African markets under study.

A discussion on whether African stock markets are becoming efficient with time using a detrended fluctuation analysis (DFA) method in a rolling window approach is presented in Chapter 4.

From the rolling window approach in Chapter 4, it was observed that there were periods markets became efficient. This led us to test the markets under the Adaptive Markets Hypothesis (AMH). The Generalised Spectral Test was employed in the analysis in Chapter 5.

In Chapter 6, the non-linearity of the markets were analysed. This was because, financial data are believed to follow non-linear pattern, hence, the stationarity of the returns of the markets were analysed under an Exponential Smooth Threshold Autoregressive (ESTAR) model.

Following the non-linear patterns of the returns of African stock markets, the researcher then modelled

the returns using a threshold model-Self Exciting Threshold Autoregressive (SETAR). The article by Gyamfi & Kyei (2016c) was extended for West African markets to cover the eight markets under this study and compared the findings of the SETAR model with that of two linear models; AR(1) and AR(2). The results and conclusions of these studies are reported in Chapter 7.

In Chapter 8, the researcher quantitatively reviewed previous studies on efficiency of African stock markets through meta-analysis. The researcher wanted to know which study characteristics are statistically significant in concluding a market to be efficient. This was necessary because of the mixed conclusions on efficiency of African markets that have been reported in the literature.

In Chapter 9, the general conclusions of the study are drawn and recommendations for industry and further studies are given.

1. General Introduction

Introduction

Market efficiency is important in the area of finance. When a market is efficient, it leads to economic development. Market efficiency can be defined in three ways - informational, operational and allocative, Bauer (2004).

1.0.1 Informational Market Efficiency

A market is efficient informationally when current prices reflect all the available information on the market. This means, it is difficult to make profit by analysing the information about a particular market.

1.0.2 Operational Market Efficiency

Operational efficiency is concerned with transaction costs on the market. It talks about how liquid the market is; that is, how easily it is for assets on the market to be bought and sold without their value being lost.

1.0.3 Allocative Market Efficiency

When investors provide funds for projects with the highest present value, the market is said to be 'allocatively efficient'. Allocative efficiency depends on the degree of informational and operational efficiency of the markets.

1.1 Background

Market efficiency studies started with the work of Bachelier (1900) and Cowles (1934). Bachelier (1900) assumed that market prices were a time series which would have patterns. Bachelier (1900) observed that prices were random and hence stated that "past, present and even discounted future events are reflected in market price, but which often show no apparent relation to price changes". Cowles (1934) also found that it was difficult to outguess the market. After the work of Cowles (1934), researchers

like Working (1934), Kendall (1953), Osborne (1959) and Roberts (1959) found that it was difficult to find predictable patterns in commodity prices.

Furthermore, Samuelson (1965) tried to prove why properly anticipated prices fluctuate randomly. Fama (1965) reviewed and added some new tests to the previous technical analysis and also discussed the value of the useful life of new information coming into the market.

Fama (1970a) stated that in an efficient market, prices always fully reflected all available information. A fully efficient market is characterised by zero transaction costs since all relevant information is costless and available to all market participants who agree on the implications of current information for the current price and the distributions for future prices. Therefore, in an efficient market, the current price of a security fully reflects all available information. These conditions ensure that investors possessing available information cannot earn above competitive returns.

1.1.1 Types of Informational Efficiency

Fama (1970b) classified the Efficient Market Hypothesis (EMH) into weak, semi-strong and strong forms.

Weak-form Efficiency

The prices on the market reflect information on past prices. The information set includes only the history of prices or returns themselves. Thus a market is said to be weak - form efficient if a trader cannot make abnormal returns by only analysing past prices. A technique of analysing past prices is known as 'technical analysis'.

Semi-strong form Efficiency

The information set includes all information known to all market participants, that is, publicly available information such as interest rates, exchange rates, annual earnings, dividends announcements, among others. In this case, investors cannot earn abnormal profits by analysing macroeconomic and financial data or any other public information about the company. This technique is known as 'fundamental analysis'.

Strong-Form Efficiency

Here, the information set becomes extensive enough to include all private information known to any market participant, therefore even those with privileged or inside information cannot use it to make

abnormal profits. This means that private or inside information is difficult to access for making abnormal returns because it is highly competitive to have such information. Thus, there is perfect incorporation of all private information in market prices.

1.2 Problem Statement

There have been studies on market efficiency of African stock markets. The problem has been with the methods used. First, most of the methods used were linear in nature. Also these linear methods tested the efficiency of African markets in absolute form. Methods such as serial correlations, multiple variance ratio (MVR) tests, Runs test, ARIMA, linear unit root tests, event studies and residual analysis have been utilised in the market efficiency literature of African stock markets. These approaches have resulted in some problems which are discussed below:

Stock markets as argued by Cajueiro and Tabak (2004b) move towards efficiency with time, therefore, testing market efficiency in absolute form might not let us observe the dynamics of the markets through time. Thus, the researcher solved this problem by testing the weak-form efficiency of the markets using a time-varying method, the detrended fluctuation analysis, to test for the presence of long-memory. Second, market efficiency is a continuous process and does not occur instantaneously as the EMH makes us believe, (Dyckman and Morse, 1986) and *Lee et al.*, (2001). The researcher therefore used the Generalised spectral (GS) test to test for the Martingale difference hypothesis (MDH) in a time-varying window approach. Testing the MDH which talks about return predictability means testing the Adaptive Market Hypothesis (AMH) of Lo (2004, 2005).

Dyckman and Morse (1986) and *Lee et al.* (2001) also posit that market friction and transaction costs make financial data non-linear, however, researchers who studied the African stock markets relied on linear methods in analysing efficiency. The assertion of nonlinearity in the data-generating process makes testing the efficiency of African stock markets, using linear methods, to result in misleading conclusions, therefore, in this thesis, two non-linear unit root tests under an exponential smooth transition autoregressive (ESTAR) models were employed to examine the stationarity of African stock markets.

Third, because of the nonlinearity in the data-generating process, the researcher compared the modelling performance of the returns of the markets between a non-linear Self-Exciting threshold autoregressive (SETAR) model and standard linear AR(1) and AR(2) models.

Fourth, because of the mixed conclusions that have characterised the efficiency of African stock markets,

(Afego 2015), the researcher examined, through a meta-analysis if a study characteristic is significant in efficiency conclusions on African stock markets.

1.3 Justification of the Study

There is a need for investigating market efficiency. This is because international and domestic investors assess how efficient a market is, before investing. When a market is not efficient, it can be inferred that a sizeable amount of the stock prices listed on that market are either undervalued or overvalued. An inefficient market is not attractive for companies to list on it. This is because arbitrageurs can make abnormal profit by studying past prices of the stocks listed, therefore, there is the need to research into African stock markets using appropriate methodologies so that concrete conclusions can be made about whether they are weak-form efficient or not. Globally, there has been numerous studies on market efficiency. A review of the literature shows that most of the studies are biased towards developed markets. Studies on market efficiency on African markets have been few with mixed conclusions.

First, the concept of evolving efficiency on African stock markets has not been well researched. Therefore this study is interested in knowing if African markets are becoming efficient with time by testing for long-memory in a rolling-window approach.

Second, African markets have been tested on the weak - form by linear methods (Afego 2015). Researchers such as Dyckman and Morse (1986) and *Lee et al.* (2001) however posit that price adjustment to new information is a continuous process and does not occur instantaneously as the Efficient Market Hypothesis makes us believe. In view of this, the researcher tested the markets under study to ascertain whether they are weak - form efficient by employing non-linear unit root tests under an ESTAR framework. The researcher also compared the modelling performance of linear and non-linear models to find out which of the models fit the data well. The study will find out what study characteristics bring about mixed efficiency conclusions on African markets. A meta-analysis will be performed to see which of the study characteristics have a significant effect in efficiency conclusions.

1.4 Objectives of the Study

1.4.1 General Objective

This thesis examined efficiency of African stock markets. It contributes to the existing body of knowledge in the area of informational efficiency of African stock markets.

1.4.2 Specific Objectives

The general objective has been narrowed and divided into the following specific objectives:

1. to examine whether African markets are weak - form efficient by investigating long-memory using frequency and time domain methods
2. to test whether Adaptive Market Hypothesis (AMH) or Efficient Market Hypothesis (EMH) should be the focus in terms of market efficiency of African stock markets
3. to test if African markets are weak - form efficient using non-linear models
4. to compare modelling performance of African stock markets between linear and non-linear models
5. to identify which factor(s) is/are significant in efficiency conclusions of African stock markets

1.5 Significance of the Study

The study will be of enormous benefit to the extant literature. It will fill the gap in informational market efficiency of African stock markets due to the new approaches employed. The meta - analysis will let us know the factors that cause efficiency conclusions on African markets and which of the factors have a significant effect in efficiency conclusions. The findings from this study will keep investors well - informed about the dynamics of the markets to be studied. Market efficiency brings about economic development, therefore policy makers in various African countries will know from this research whether their markets are becoming efficient with time. This will let them know whether or not new regulations and institutional reforms should be considered.

1.6 Data

For Chapters 3 to 7, the researcher studied stock indices of eight African countries based on the availability of data from 28/8/2000 to 28/8/2015. Daily closing prices were used so as to improve the accuracy of the serial dependence estimate, (Bollerslev and Wright 2000a). Data were obtained for each country from DataStream denominated in their respective local currency units. Each of the eight countries - Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa, and Tunisia had a stock index representing it. Table 1.1 displays the countries and their corresponding stock index.

Country	Representative Index
Botswana	S& P Botswana BMI
Egypt	EGX 30
Kenya	NSE 20
Mauritius	SE Semdex
Morocco	Morocco All Share (MASI)
Nigeria	Nigeria All Share
South Africa	FTSE/JSE All Share
Tunisia	Tunindex

Table 1.1: Countries with corresponding stock index

For Chapter 8, a search of empirical studies on the literature of informational market efficiency of African stock markets constituted the dataset. The R and MATLAB softwares were used in this thesis.

1.7 Limitations of the Study

1. This study did not include all African countries. The choice of these eight countries was because of data availability for the 15 year period chosen.
2. The meta-analysis did not include unpublished works such as theses and working papers on informational market efficiency of African stock markets.

1.8 Conclusion

In this chapter, the types of market efficiency were reviewed. The study which concentrated on the informational market efficiency described the three types of informational market efficiency as explained by Fama (1970). The problem statement was clearly stated; the justification and significance of the study were presented. The general and specific objectives were explained. The choice of data, the data period chosen and the sources of data were described. Finally, the limitations of the study were presented.

2. Literature Review

Introduction

In this chapter, the theoretical and empirical literature on informational market efficiency were reviewed. The martingale and random walk theories were also discussed. Empirical studies of the Efficient Market Hypothesis (EMH) on African stock markets were also reviewed.

2.1 Theoretical Literature Review

2.1.1 Martingale

The Martingale Model is the oldest theory about asset pricing. It dates back to Cardano's manuscripts (1565). Cardano's modern formulation was by Bachelier (1900) and according to this model, any attempt to predict the future prices of an asset will not have a statistically significant explanatory power.

Let P_t represent an asset's price at date t and, a set of information available at date t , and let Ω_t consist of all the past prices of the asset, i .e., $\Omega_t = \{P_t, P_{t-1}, P_{t-2}, \dots\}$.

The Martingale hypothesis asserts that tomorrow's price is expected to be equal to today's price, given the asset's price history. That is, if P_t is considered as a stochastic variable then, P_t is said to be a martingale when it satisfies the following condition:

$$E[P_{t+1}|\Omega_t] = P_t \quad (2.1)$$

In some applications, Ω_t is assumed to contain additional information. For example, the prices of other assets or companies' earnings data.

From equation (2.1)

$$E[P_{t+1} - P_t|\Omega_t] = 0 \quad (2.2)$$

The fair game (2.2) which Bachelier defined as the mathematical expectation of the speculator is zero, that is, the expected return is zero given the asset's price history. The following assumptions imply that the asset price evolves according to (2.1) or (2.2): investors believe that holding the asset is just like playing a fair game, and they have access to the information contained in the set Ω_t .

The next property of the Martingale hypothesis is that non - overlapping price changes are uncorrelated

at all leads and lags, so all linear forecasting rules for future price changes based on historical prices alone have no predictive power. Depending on the Martingale concept, in an efficient market, the current prices reflect all historical prices, and it should not be possible to make profit by expectation of future price changes from price history. The market hence is efficient when price changes are random and unpredictable, however, in finance, there is a trade-off between risk and return, and the Martingale hypothesis does not involve risk considerations in any way. Some economic models like the Capital Asset Pricing Model (CAPM) determine the equilibrium return of the asset according to the risk of the asset, so there is a trade-off between risk and expected return. The Martingale hypothesis however puts a restriction on expected return, and does not take risk into consideration, which means the Martingale property is not a sufficient condition for rationally determined asset prices.

Nevertheless, the Martingale assumption has become a powerful tool in modern theories of asset prices (Campbell, 1997). Theoretically, once asset returns are adjusted properly for risk, then the Martingale property does hold. For instance, an asset's risk may imply that it must offer some level of positive return to an investor. As a result, in an efficient market, the asset's price change is expected to be positive but the actual return is still unforecastable. This leads to Random Walk model of the asset price where one can show that if returns are properly adjusted for risk (given the equilibrium model) then the Martingale property holds for the adjusted returns.

2.1.2 Random Walks

A model that is associated with the Martingale process is the 'Random Walk Model'. It is used in testing whether returns can be forecasted. A Random Walk model without a drift is represented by:

$$P_{t+1} = P_t + \varepsilon_{t+1} \quad (2.3)$$

A Random Walk Model with a drift is represented by:

$$P_{t+1} = \mu + P_t + \varepsilon_{t+1} \quad (2.4)$$

This model in eqn 2.4 shows that the asset price at time $t + 1$ is given by the price at the immediately previous moment, a term of expected change μ which is the mean of P_t for all t known as the 'drift'. The 'drift' reflects how prices change on average to provide the expected rate of returns from holding the asset over time, plus an unpredictable error component. The error term ε_t is an independent and identically distributed (iid) variable with a mean of 0 and variance σ_ε^2 . The Random Walk Model can

be obtained through the Martingale process by restrictions on the error term ε_t . The behaviour of error term ε_t is extremely important, and restrictions on the behaviour of this term produce three versions of the Random Walk model, as stated by *Campbell et al.* (1997).

Random Walk I- IID Increments

The stronger version of the Random Walk Model is the one in which increments at price P_t given by error term, ε_t belongs to the same distribution (identically distributed) and are independent. In addition, the original distribution can be used, which in the most common cases, is the same as assuming that term ε_t belongs to a normal distribution with zero mean and constant variance σ^2 . Random Walk I, also known as RWI, is even more restrictive than the Martingale Model, since in the latter model the increments are nonlinearly uncorrelated and any nonlinear combination of the increments should also be uncorrelated.

Random Walk II - Independent Increments

The RWI model is extremely restrictive, therefore, it should not be used in real financial series because it rules out the possibility of structural changes in the data - generating process, such as parameter changes, of which the most relevant are the changes in volatility. A more appropriate version, known as Random Walk II (RW2), only requires that the increments should be independent, but not necessarily originate from the same distribution. This maintains the characteristic of linear unpredictability and allows for changes in unconditional volatility.

Random Walk III - Uncorrelated Increments

The most general form of the Random Walk Model requires only that ε_t be uncorrelated over time. This is referred to as RW3 and is the least restrictive form of Random Walk and it is more likely to prove consistent with the behaviour observed in real financial series. RW3 is usually the most widely tested form of Random Walk.

2.2 Empirical Literature Review

The researcher reviewed the Efficient Market Hypothesis literature on African markets as in Afego (2015).

2.2.1 Weak - Form Efficiency Studies on African Stock Markets

Weak - form efficiency studies on African markets was started by Jammie & Hawkins (1974). They studied the South African stock exchange using the method of serial correlations. Their conclusion was that the market was weak-form inefficient. Affleck-Graves & Money (1975) also studied the South African market using serial correlations and concluded it has a weak-form efficiency; a result opposite to the study of Jammie & Hawkins (1974).

Other studies using serial correlations were done by Dickinson & Muragu (1994), Osei (1998), Olowe (1999) and Bundoo (2000) who concluded weak-form efficient, weak-form inefficient, weak-form efficient and weak-form inefficient on Kenya, Ghana, Nigeria and Mauritius markets, respectively. The studies continued with other researchers such as Magnusson & Wydick (2000) who studied the South African, Nigerian, Ghanaian and Zimbabwean markets using partial autocorrelations and white noise. They concluded that only the South African market was weak-form efficient. Smith et al. (2002) used multiple variance ratio (MVR) tests on South Africa, Egypt and Nigeria. Again, only the South African market was found to be weak-form efficient.

Furthermore, researchers like Appiah-Kusi & Menyah (2003), Simons & Laryea (2005) and Jefferis & Smith (2005) used EGARCH-M, ARIMA and MVR tests and time varying GARCH, respectively, to study on different African markets. Appiah-Kusi & Menyah concluded that only Mauritius and Kenya were weak-form efficient. South Africa was the only market seen to be weak-form efficient by Simons & Laryea (2005). The markets of Nigeria and South Africa were concluded weak-form efficient by Jefferis & Smith (2005).

Smith (2008) studied on the South African, Zimbabwean, Ghanaian and Nigerian markets using MVR tests and concluded that none of these markets were weak-form efficient, a conclusion opposite to that of Jefferis & Smith (2005). Other studies by Mollah (2007), Mollah & Vitali (2011) and *Ntim et al.* (2011) gave mixed conclusions about the markets they studied.

2.2.2 Semi - Strong Form Efficiency Studies on African Stock Markets

The studies of Bhana (1991), Osei (2002), Adelegan (2003), Adelegan (2009) and Afego (2011) using event - studies method on the markets in South Africa, Ghana and Nigeria showed that none of these markets was semi-strong form efficient.

There are a few studies on calendar effects. The works of Coutts and Sheikh (2002), Chukwuogur (2007), and *Mlambo et al.*(2009) found no evidence of seasonality for any of the markets studied, however Onyuma (2009) found the presence of seasonality in their study on Kenya. Alagidede (2008b) found mixed results for the markets studied using GARCH-type models.

2.2.3 Conclusion

A review of the weak and semi-strong forms of efficiency on African stock markets has shown that the conclusions are mixed; an example of this is the study by Jefferis and Smith (2005). This study tested for weak-form efficiency in South Africa, Zimbabwe, Nigeria, Morocco, Kenya, Egypt and Mauritius with the GARCH model with time varying parameters. They found the South African stock market to be efficient over the study period. Nigeria, Egypt and Morocco only became efficient in the latter part of the period, while Zimbabwe and Kenya failed the efficiency test over the whole period. This conclusion contradicts a study by Smith (2008) who studied 11 African markets-South Africa, Zimbabwe, Ghana, Nigeria, Egypt, Tunisia, Botswana, Kenya, Morocco, Mauritius and Cote d'Ivoire. Smith (2008) analysed the data using Wright's joint variance ratio test, proposed by Wright (2000a), and Chow-Denning multiple variance ratio test, proposed by Chow and Denning (1993). Smith (2008) observed that none of the markets studied was weak - form efficient.

This observation makes this study relevant to the literature on EMH on African markets in the sense that, it will be finding the factors that cause efficiency conclusions on these markets by the method of meta - analysis.

Furthermore, a review of the weak-form efficiency literature shows that most of the methods used in testing weak - form efficiency on African markets are linear. However, researchers such as Dyckman and Morse (1986) and *Lee et al.* (2001) posit that price adjustment to new information is a continuous process and does not occur instantaneously as the Efficient Market Hypothesis makes us to believe. The researcher therefore revisited the weak-form efficiency testing by employing non-linear unit root tests in

an ESTAR framework to validate or disprove the assertion by Dyckman and Morse (1986) and *Lee et al.* (2001). Also the researcher used methods that are of high power than previously used methods to test if African markets are becoming weak-form efficient with time.

3. Long-Memory in Frequency Domain

Abstract

This chapter examined long-memory in price returns and volatilities of stock markets in eight African countries. The researcher employed the Hurst exponent as the measure of long-memory on a 15 year sample.

The Hurst exponent was evaluated by a semi-parametric method, the Local Whittle method.

The researcher found strong evidence of long memory in both returns and volatility for all the countries as the Hurst exponent $H > 0.5$. This meant that none of the markets was weak-form efficient. However, the decay exponent of the autocovariance function showed that the autocorrelations on the South African stock market seemed to be going to zero at the fastest rate when compared to the rest of the markets.

Keywords: Long memory, Hurst exponent, Local Whittle, Market efficiency

3.1 Introduction

The Efficient market hypothesis (EMH) states that prices fully and instantaneously reflect all the available information on the market. According to the weak form efficiency of the EMH, one cannot make abnormal returns by analysing past price information in predicting future prices of a market, Fama(1970). In other words, there should be no long memory in asset returns and volatilities. When there is long memory in returns and volatilities, it means a market responds to new information gradually but not as quickly as the EMH makes us believe (*Mukherjee et al.*, 2011), a violation of the weak form market efficiency. Henry (2002) states that evidence of persistence in equity returns means that stock returns will be predictable, while *Barkoulas et al.* (2000) suggests that if a market is predictable then speculators can exploit for profit. The work of Los (2003) showed that the GARCH processes which are common models are inefficient models because they exhibit incorrect empirical long-term dependence.

Baillie (1996) observed long-memory behaviour in financial time series. Researchers such as Chan and Hammed (2006) explain the presence of long-memory in emerging markets. They believe that long-memory in emerging markets is caused by lack of flow of firm-specific information to investors. There are however other explanations of the presence of long memory in emerging markets by Kim and Wu (2008), Rajan and Zingales (2003), Harvey (1994), Tolvi (2003) and Kim and Shamsuddin (2008). It is to be noted that there have been numerous works on long memory in returns and volatilities of

stock markets (*Barkoulas et al.*, 1997; Crato and Ray, 2000; Panas, 2001; *Chen et al.*, 2006; Elder and Jin, 2007; Lien and Yang, 2010). Although, many authors also believe that long memory phenomenon is an illusion. Aydogan and Booth (1988) re-examined earlier papers and concluded that evidence of long memory processes in American stock returns was spurious. They concluded that it arose from the existence of pre-asymptotic behaviour in statistical estimates. Cheung and Lai (1995) could find little evidence for long memory processes in a variety of international stock returns. *Chow et al.* (1995) found no compelling evidence to support long memory in the equity returns they examined. Barkoulas and Baum (1996) failed to find any significant evidence of long memory in the American stock market. Grau-Carles (2005) failed to find evidence of long memory processes from the log return series from either the S&P 500 or the Dow Jones Industrial Average using a wide variety of statistical techniques. *Zhuang et al.* (2000) could uncover little evidence of long memory processes in UK stock returns.

McMillan and Thupayagale (2008) examined long memory in equity returns and volatility for the South African stock market using the ARFIMA-FIGARCH model in order to assess the efficiency of the market. The results show that volatility exhibits a predictable component in both sample periods, while returns in both sample periods do not. Also *Morris et al.* (2009) extended the work of Jefferies and Thupayagale (2008) by testing for efficiency of the South African stock market with Wavelet and Markov Switching Regime analyses. They observed that the Wavelet analysis indicated that most of the individual share prices and the share index time series were mean reverting over the long run and follow a long memory process, giving evidence against weak-form efficient market hypothesis (EMH). The Markov model established the presence of patterns in the historic time series, which gives support against the weak-form EMH.

The aim of this chapter is to examine the presence of long memory in returns and volatilities of the stock markets in Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia. The researcher employed the Hurst exponent which is evaluated by the Local Whittle method (Robinson 1995).

The chapter is organised as follows:

In section 3.2, the methodology used is provided. In section 3.3, summary statistics and empirical analysis are provided. Section 3.4 discusses and gives the concluding remarks.

3.2 The Local Whittle Estimator

The Local Whittle estimator proposed by Robinson (1995) is used for analysing long-memory in the frequency domain. Since the interest is to know the degree of long memory in a given market, the researcher used the Local Whittle estimator which is a semi-parametric estimator to provide the Hurst exponent. The estimator requires specifying the parametric form of the spectral density when the frequency λ degenerates to zero.

$$f(\lambda) \sim G(H)|\lambda|^{1-2H} \quad \text{as } \lambda \rightarrow 0 \quad (3.1)$$

where G is a strictly positive constant. The computation involves an additional parameter m known as the bandwidth parameter which is an integer less than $N/2$, where N is the size of the time series Y_t where $t = 1, \dots, N$ and as $N \rightarrow \infty$

$$\frac{1}{m} + \frac{m}{N} \rightarrow 0 \quad (3.2)$$

This means that as N gets large, m gets large as well, although at a slower rate. For a spectral density of the form of equation 3.1, the Whittle approximation of the Gaussian likelihood function is obtained by minimizing

$$Q(G, H) = \frac{1}{m} \sum_{j=1}^m \left(\frac{I(\lambda_j)}{G\lambda_j^{1-2H}} + \log(G\lambda_j^{1-2H}) \right) \quad (3.3)$$

where $\lambda_j = \frac{2\pi j}{N}$ and $I(\lambda_j) = \frac{1}{2\pi N} \left| \sum_{t=1}^N Y_t e^{i\lambda_j t} \right|^2$ is the periodogram of the time series. Therefore this estimator sums the frequencies up to $2\pi m/N$. When G above is replaced by its estimate \hat{G} , we get

$$\hat{G} = \frac{1}{m} \sum_{j=1}^m \frac{I(\lambda_j)}{\lambda_j^{1-2H}} \quad (3.4)$$

$R(H)$ may be defined as

$$\begin{aligned} R(H) &= Q(\hat{G}, H) - 1 \\ &= \log \left(\frac{1}{m} \sum_{j=1}^m \frac{I(\lambda_j)}{\lambda_j^{1-2H}} - \frac{2H-1}{m} \sum_{j=1}^m \log(\lambda_j) \right) \end{aligned}$$

Under finiteness of the fourth moment and other assumptions, Robinson (1995) showed that

$$\hat{H} = \operatorname{argmin} R(H) \quad (3.5)$$

converges in probability to actual value H , i.e.,

$$m^{1/2} (\hat{H} - H) \rightarrow_d \text{Normal}(0, 1/4) \quad (3.6)$$

Hence, choosing m is important. As m gets larger, \hat{H} converges to H faster. On the other hand, m should be small if the series presents short memory. This study, makes use of a limiting value of $m = (N/2) - 1$ to ensure \hat{H} converges to H faster.

The Hurst exponent H which measures the size and direction of persistence in a time series is a bounded real number, $H \in [0, 1]$. The following are to be noted about H :

1. If $H = 0.5$, it means that all autocorrelations tend rapidly to zero and the time series is a random walk. Hence it is concluded that no long memory in time series, thus the market is said to be weak-form efficient.
2. If $H > 0.5$, there is a stronger memory effect which means persistence in the time series. This means that an increase (decrease) of asset price is likely to follow another increase (decrease).
3. If $H < 0.5$, it suggests anti-persistence or mean reversion which means an increase (decrease) of asset price is likely to follow a decrease (increase).

The researcher adopted Perron and Qu (2010) convention of accounting for zero returns by eliminating returns with absolute magnitudes below 1.0×10^{-6} .

The researcher followed especially the work of *Anderson et al.* (2001) and researchers such as Taylor (1986), Crato and Lima (1994), Starica and Granger (2005) and *Bentes et al.* (2008) who used the squared of log returns as the best approximation for volatility.

In using the Hurst exponent in determining the degree of long memory. Let Y_t be the price of an index at time t and r_t is the logarithmic return denoted by:

$$r_t = \log \left(\frac{Y_t}{Y_{t-1}} \right) \quad (3.7)$$

Volatility is therefore denoted by:

$$v_t = r_t^2 \quad (3.8)$$

Evaluation of Hurst Exponent

The Hurst exponent was estimated for each of the series (returns and volatilities) for the eight indices. Each series was split into smaller values each having n observations.

3.3 Results

Market	No of observations	Mean	Std deviation	Skewness	Kurtosis	Jarque-Bera	ADF
Botswana	3915	0.0006	0.0078	5.8172	124.9661	2572200	-14.823
Egypt	3915	0.0006	0.0169	-0.4309	10.1907	17084	-14.491
Kenya	3915	0.0002	0.0093	0.3047	31.6902	164070	-13.932
Mauritius	3915	0.0004	0.0067	0.2802	22.7257	84396	-13.543
Morocco	3915	0.0003	0.0077	-0.4954	7.1333	7712	-14.117
Nigeria	3915	0.0003	0.0133	-0.6872	368.4107	22164000	-14.862
South Africa	3915	0.0005	0.0121	-0.1164	3.6536	2190.4	-15.743
Tunisia	3915	0.0003	0.0053	-0.4453	11.2584	20833	-14.5

Table 3.1: Summary Statistics for returns

Market	Hurst Exponent (H)	Decay Exponent
Botswana	0.8892	-0.223
Egypt	0.7788	-0.443
Kenya	0.8165	-0.366
Mauritius	0.8735	-0.252
Morocco	0.8618	-0.276
Nigeria	0.8325	-0.335
South Africa	0.8953	-0.209
Tunisia	0.8875	-0.2249

Table 3.2: Hurst Estimate and Decay Exponent for returns

Market	Hurst Exponent (H)	Decay Exponent
Botswana	0.9995	-0.0010
Egypt	0.9984	-0.0031
Kenya	0.9995	-0.0009
Mauritius	0.9999	-0.0001
Morocco	0.9999	-0.0001
Nigeria	0.9822	-0.0354
South Africa	0.9998	-0.0004
Tunisia	0.9999	-4.02E-5

Table 3.3: Hurst Estimate and Decay Exponent for volatility

3.4 Discussion and Conclusions

Discussion

The results from Table 3.1 showed the summary statistics of the return series. It was observed that the return series were not normally distributed. The null hypothesis of normality by the use of the Jarque-Bera statistic at the 1% level of significance was failed to be accepted. The kurtosis coefficients were very high ($k > 3$) and the series were skewed negatively. The result from the Augmented Dickey-Fuller tests showed that the return series were stationary. These results showed that the series were not weak-form efficient, a violation of the Efficient Market Hypothesis (EMH).

Table 3.2 and Table 3.3 presented the main results of this study. The values for the Hurst exponent were stronger in Table 3.3 (volatility) than in Table 3.2 (returns). The Hurst exponent for the returns and volatility series for all the indices were greater than 0.5. It was concluded that there was the presence of long-memory in the markets. Thus memory effect was stronger and the markets exhibited mean reversion, implying an increase (decrease) of asset price is likely to follow another increase (decrease). Therefore, the markets in Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia were less liquid and information does not flow well in setting current prices on the market. This makes them inefficient in the weak-form.

Conclusions

In this chapter, the presence of long-memory in eight African stock markets were examined . A 15 year (2000-2015) data was used in estimating the Hurst exponent of the returns and volatility series by using the Local Whittle estimator proposed by Robinson (1995).

The results showed evidence of strong memory effect in both returns and volatility as the Hurst exponent for the markets are greater than 0.5.

These results provided evidence that none of the markets studied were weak-form efficient. This means that there are opportunities for abnormal returns to be made when one analyses past prices of these markets.

4. Long-Memory in Time Domain

Abstract

Emerging stock markets which are not sophisticated and under-researched are said to become efficient with time. This study examined this assertion by analysing long - memory persistence in 8 African stock markets covering the period from August 28, 2000 to August 28, 2015.

The Hurst exponent was used as an efficiency measure which was evaluated by the Detrended Fluctuation Analysis (DFA).

The findings showed strong evidence of long-memory persistence in the markets which violated the weak - form Efficient Market Hypothesis (EMH).

Keywords: Long - memory, Hurst exponent, DFA, Market efficiency

4.1 Introduction

Long - memory or long - range dependence or non - Gaussianity which is a stylized fact of financial data implies that observations which are apart in time are highly correlated. When there is persistence of long - memory on a stock market, it means a violation of the Efficient Market Hypothesis (EMH). The EMH states that information on a market is incorporated correctly and instantaneously in setting prices on that market so if long - memory persists, abnormal returns can be made by analysing information on the market.

This chapter examined the degree of long-memory persistence over time with the use of the Hurst exponent which will be evaluated by the Detrended Fluctuation Analysis (DFA). When the efficiency measure, the Hurst exponent, shows persistence or anti-persistence, then the market is inefficient. On the other hand, when there is no persistence nor anti-persistence, then abnormal returns cannot be made by analysing information on that market which goes to affirm the EMH.

It is of belief that as stock markets develop, they become efficient with an increase in the number of research analysts and arbitrageurs, hence, anything that constitutes abnormal profit-making should be identified and discarded immediately. The reason why arbitrageurs are seen on efficient markets is because, Grossman and Stigitz (1980) argue that, to bring about efficiency, there should be some level of inefficiency in order to give an incentive to arbitrageurs to find and trade the assets which are not

correctly priced.

The aim of this chapter is to examine the degree of long-memory on the returns of African stock markets over time. African stock markets which are emerging have seen little attention when it comes to long-memory persistence, *Barkoulas et al.* (2000). Therefore, this work will build on the works on emerging markets in the literature by Tolvi (2003), Cajueiro and Tabak (2004a, 2004b), Jefferis and Smith (2005), McMillan and Thupayagale (2008), Jefferies and Thupayagale (2008), and *Morris et al.* (2009).

The rest of the paper will be sectioned as follows:

Section 4.2 presents the literature review. Section 4.3 describes the Detrended Fluctuation Analysis (DFA) method. Section 4.4 presents the research findings with appropriate tables and graphs. Section 4.4 discusses the research findings and concludes the chapter.

4.2 Literature Review

There has been considerable amount of research on long-memory persistence in financial time series. A time series that shows presence of long - memory implies it is serially dependent, which is a violation of the EMH in that the market is said not to correctly and instantaneously reflect all the available information on that market.

The literature makes mention of the works of Mandelbrot (1971) who argues that the presence of serial dependence in a financial time series shows that new market information is not incorporated in setting current prices. This assertion was affirmed by Mukherjee et al. (2011) who found that Indian stock returns were autocorrelated. He therefore concluded that markets response to new information is a gradual process and not instantaneous as the EMH suggests. Also, Los (2003) showed that the use of (G)ARCH processes cannot correctly model long-term dependence. Barkoulas and Baum (1996) extended the argument by Los (2003) stating that since techniques such as linear modelling and forecasting assume EMH-consistent Gaussian distributions, the presence of long - memory in financial time series would make conclusions based on these techniques invalid. The works of Costa and Vasconcelos (2003) show that the Brazilian stock market displayed a memory effect which lasted for up to six months. *Barkoulas et al.* (2000) and Panas (2001) both found significant and robust evidence of long memory processes in the Greek stock market. Rege and Martin (2007) discovered pronounced long memory effects in the Portuguese stock market.

The work of Sadique and Silvapulle (2001) found evidence of long - memory persistence in emerging

markets of Korea, Malaysia, Singapore and New Zealand but not in developed markets such as USA, UK and Japan. These findings suggest that emerging markets exhibit long - memory while developed markets do not. The reasons behind this assertion were given much emphasis in the paper by Chan and Hameed (2006) who found out that long - memory persists in emerging markets because flow of information of a firm to potential investors is poorer than the flow of information in developed markets. Also, Tolvi (2003) observed that serial dependence in smaller markets make them less efficient than developed markets; a conclusion which was affirmed by Kim and Shamsuddin (2008).

But Cajueiro and Tabak (2004b) however make a conclusion that emerging markets move towards efficiency with time. An investigation of this is one of the aims of this chapter; to see if African stock markets are becoming efficient with time. Cajueiro and Tabak (2004b) give reasons such as increase in foreign capital inflow and increase in trading volumes as the drivers of emerging markets towards efficiency.

The Hurst exponent developed by Hurst (1951) to compute the optimum size of a dam on River Nile will be used to measure the degree of long - memory in the stock markets. The exponent will be evaluated by the use of a technique for detecting long range autocorrelations in time series, the Detrended Fluctuation Analysis (DFA) developed by Peng *et al.* (1994).

4.3 Long-Memory

Long - memory is defined based on frequency or time domains. It is commonly defined with respect to its autocorrelation function (ACF). Ding *et al.* (1993) describe a series to have long - memory if its ACF declines slowly and has an infinite spectrum at zero frequency.

Given a time series Y_t , the frequency domain definition of long - memory is when a spectral density $f(\lambda)$ of low frequencies obeys

$$f(\lambda) \sim C_f |\lambda|^{-2d} \quad \text{as } \lambda \rightarrow 0 \quad (4.1)$$

where λ is the frequency, C_f is a strictly positive constant and d is the fractional degree of integration. In the time domain, long - memory is said to be present in a series Y_t if there exists a real number H and a finite constant C such that the ACF, $\rho(k)$ has the following rate of decay:

$$\rho(k) \approx C k^{2H-2} \quad \text{as } k \rightarrow \infty \quad (4.2)$$

where $H \in (0, 1)$, is called the Hurst exponent, which measures the degree of long - memory of the time series.

This chapter seeks to measure the degree of long - memory in the time domain hence the Hurst exponent will be evaluated by the Detrended Fluctuation Analysis (DFA) is thus used.

4.4 Detrended Fluctuation Analysis

The Detrended Fluctuation Analysis (DFA) is an important tool for detecting long - range autocorrelations in time series with non - stationarities which affect experimental data.

The researcher followed *Peng et al.* (1994) who proposed the DFA.

Suppose $Y(t)$ be an integrated financial time series of logarithmic returns, i.e.

$$Y(t) = \ln(P_t) - \ln(P_{t-1}) \quad (4.3)$$

where P_t and P_{t-1} are the daily closing prices of the index on two consecutive trading days and $t = 1, \dots, N$. In this method, the integrated time series is divided into blocks of the same length n .

The ordinary least squares method is used to estimate the trend in each block. In each block, the ordinary least square line is expressed as $Y_n(t)$. The trend of the series is removed by subtracting $Y_n(t)$ from the integrated series $Y(t)$ in each block.

This procedure is applied to each block and the fluctuated magnitude is defined as

$$\sigma_{DFA} = \sqrt{\frac{1}{N} \sum_{t=1}^N (Y(t) - Y_n(t))^2} \quad (4.4)$$

This step is repeated for every step n and to estimate Hurst exponent, the following scaling relationship is defined

$$\sigma_{DFA} \propto n^H. \quad (4.5)$$

This equation (4.5) can be written as

$$\log(\sigma_{DFA}) \propto H \log(n). \quad (4.6)$$

This linear relationship between σ_{DFA} and n on a log - log plot supports the presence of power law (fractal) scaling which indicates there is self - similarity in the series. This means the fluctuation over small time scales are related to fluctuations over larger time scales. The following are to be noted about H :

If $H = 0.5$, it means that all autocorrelations tend rapidly to zero and the time series is a random walk.

The market is thus weak- form efficient.

If $H > 0.5$, the memory effect is stronger which means persistence. Thus, an increase (decrease) of an asset price is likely to follow another increase (decrease).

If $H < 0.5$, it suggests anti-persistence or mean reversion in the time series.

4.5 Results

Year	Botswana	Egypt	Kenya	Mauritius	Morocco	Nigeria	South Africa	Tunisia
2000	0.4829	0.4655	0.5969	0.5604	0.5712	0.6150	0.4623	0.4117
2001	0.6704	0.6072	0.5525	0.6916	0.5318	0.4972	0.5785	0.4754
2002	0.5145	0.5278	0.6944	0.6153	0.6257	0.5808	0.4473	0.6726
2003	0.6128	0.6347	0.5305	0.7955	0.6395	0.4203	0.4885	0.5879
2004	0.5980	0.6521	0.7889	0.7569	0.6156	0.4173	0.6455	0.7385
2005	0.5756	0.5498	0.6049	0.5666	0.5595	0.6753	0.4793	0.6483
2006	0.5551	0.4929	0.5714	0.6390	0.6551	0.6494	0.5342	0.7036
2007	0.6007	0.5887	0.4632	0.6283	0.4282	0.6159	0.4662	0.5907
2008	0.8045	0.7127	0.6228	0.6512	0.4905	0.6949	0.4717	0.6044
2009	0.5171	0.4395	0.6631	0.5784	0.5324	0.5287	0.4139	0.7519
2010	0.6283	0.5735	0.5683	0.6235	0.4549	0.5037	0.5298	0.4925
2011	0.6017	0.5437	0.5883	0.5252	0.5609	0.5315	0.3857	0.5728
2012	0.7107	0.4417	0.5492	0.6727	0.5618	0.6803	0.5091	0.5222
2013	0.6504	0.6663	0.5128	0.6644	0.4692	0.6398	0.4858	0.5753
2014	0.7952	0.4375	0.5673	0.6032	0.4916	0.5406	0.4254	0.6527
2015	0.6435	0.4476	0.5763	0.6233	0.4917	0.5507	0.4752	0.6321
All years	0.6532	0.6007	0.6290	0.6667	0.5879	0.6081	0.5279	0.6450

Table 4.1: Hurst exponent for returns

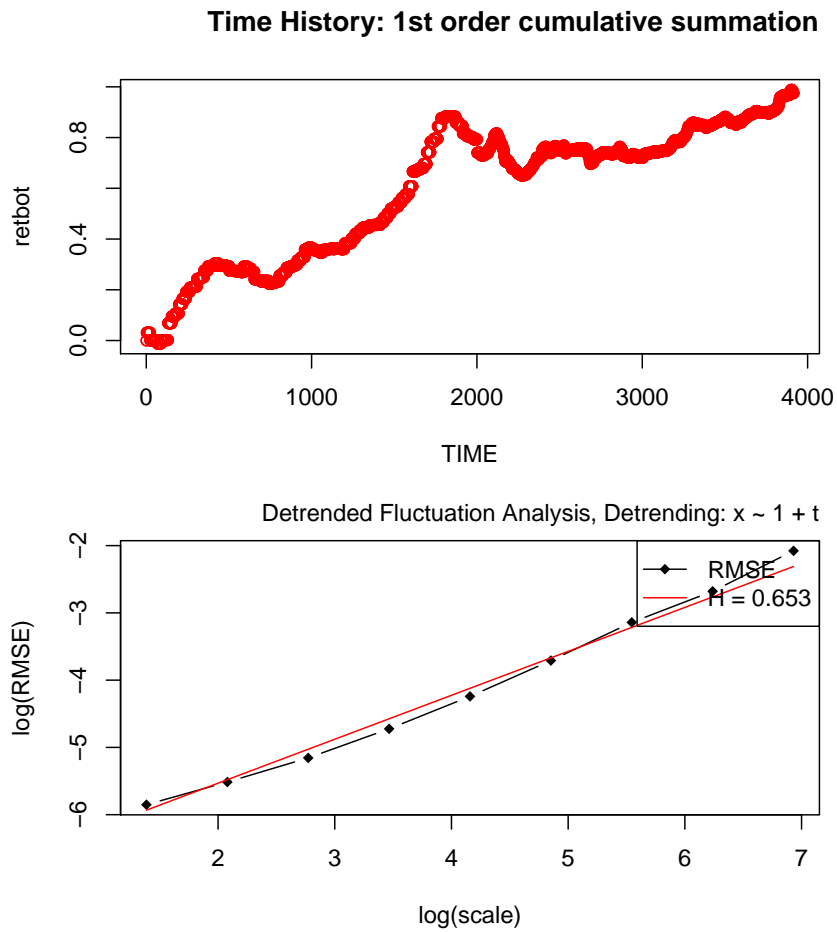


Figure 4.1: DFA plot of Botswana

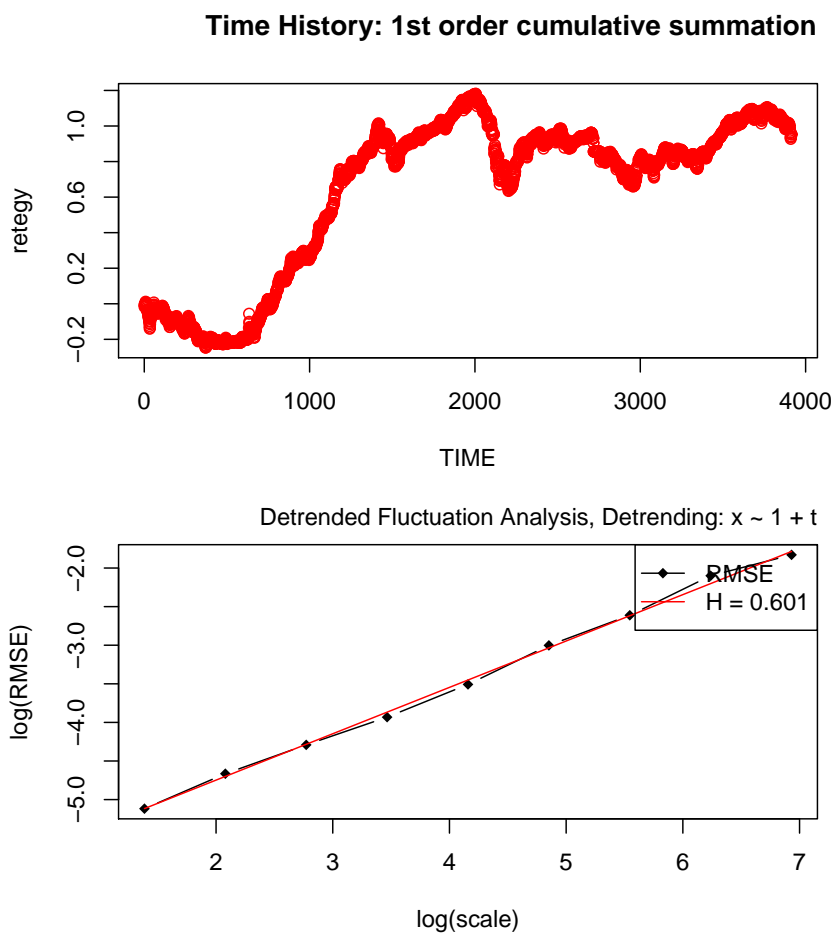


Figure 4.2: DFA plot of Egypt

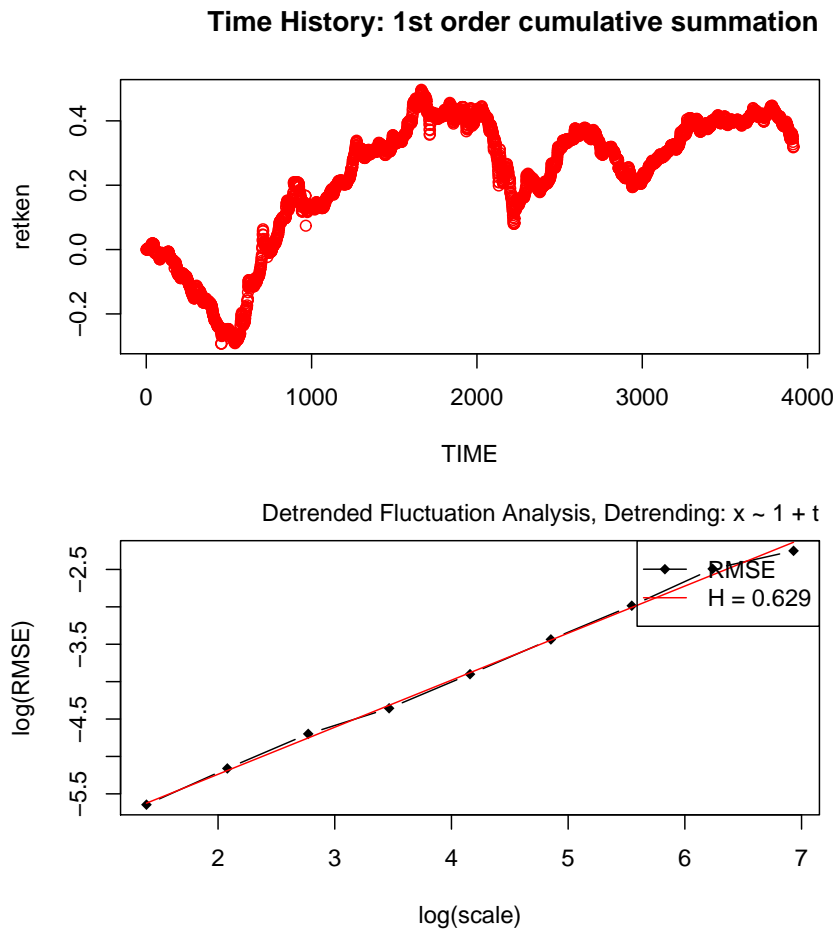


Figure 4.3: DFA plot of Kenya

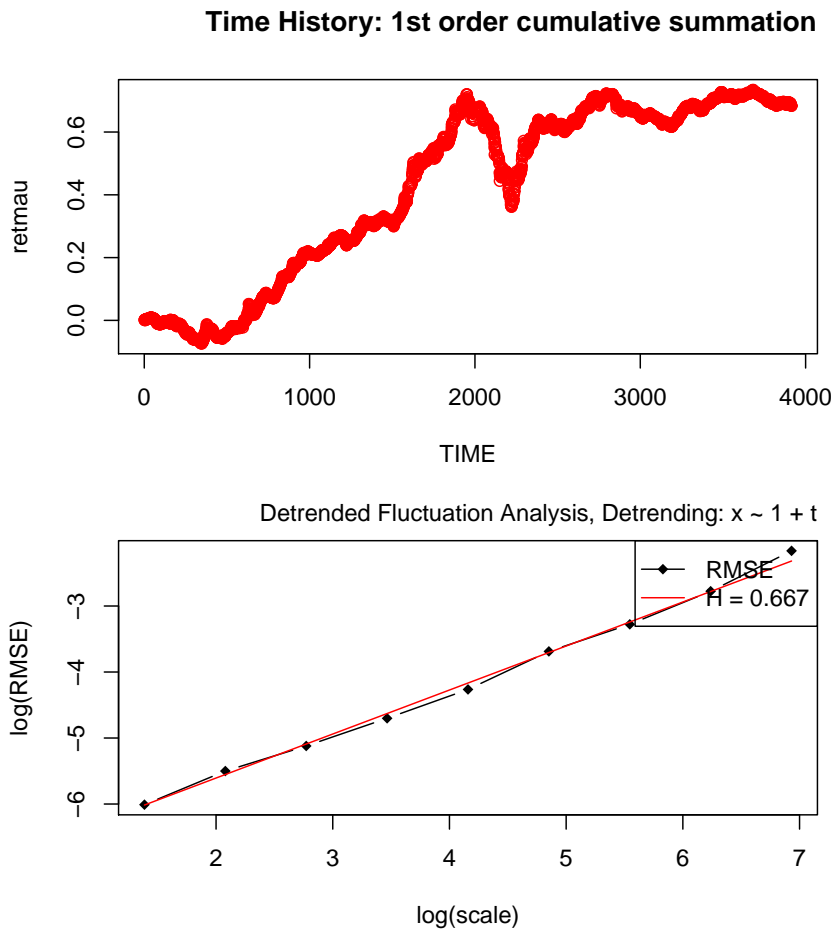


Figure 4.4: DFA plot of Mauritius

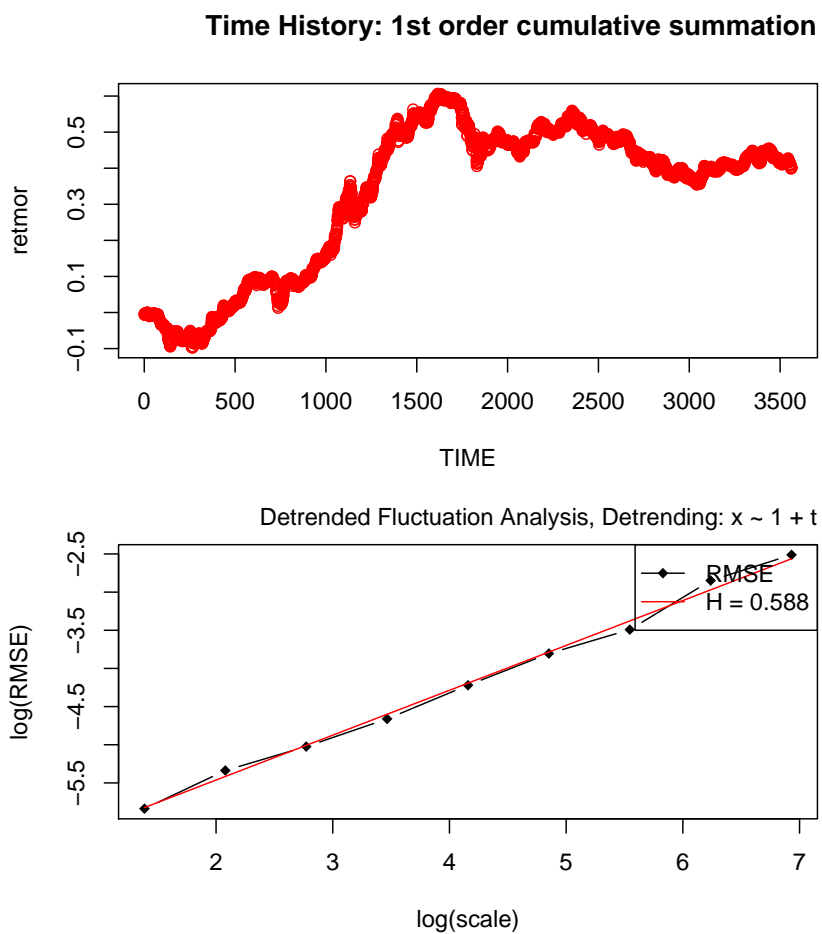


Figure 4.5: DFA plot of Morocco

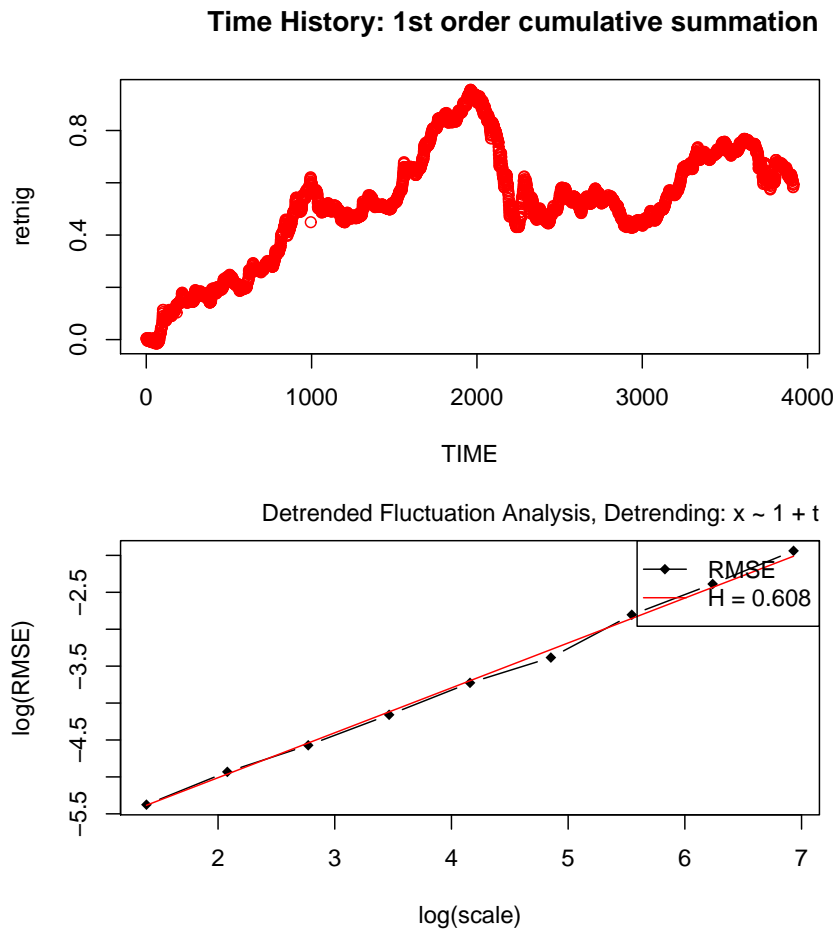


Figure 4.6: DFA plot of Nigeria

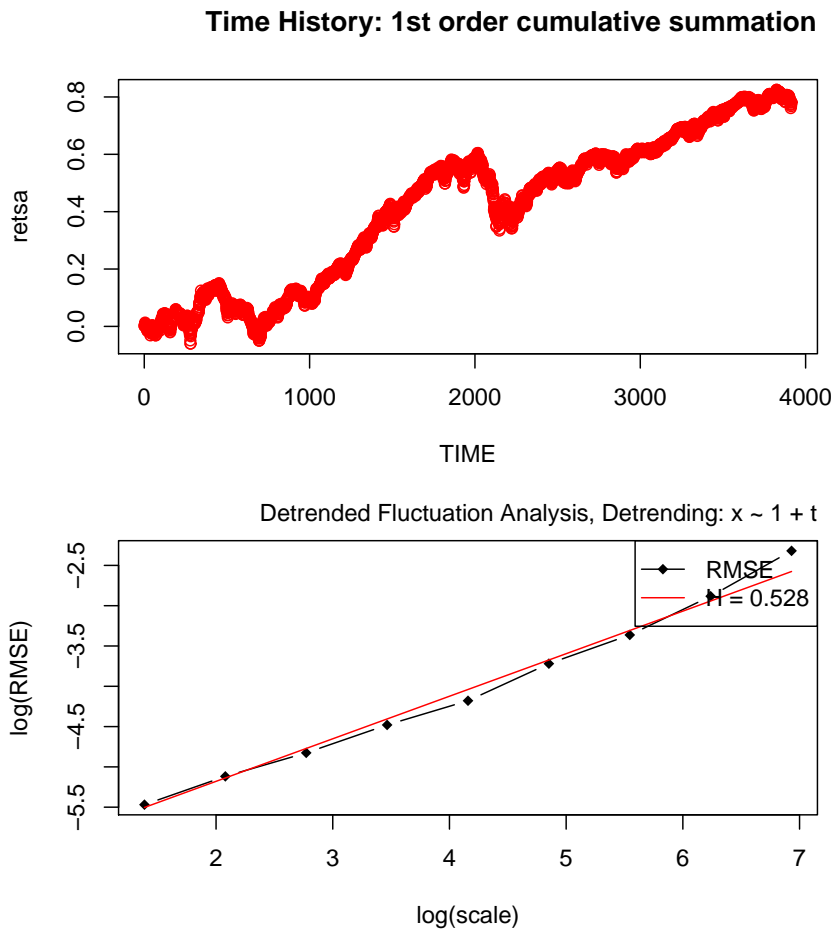


Figure 4.7: DFA plot of South Africa

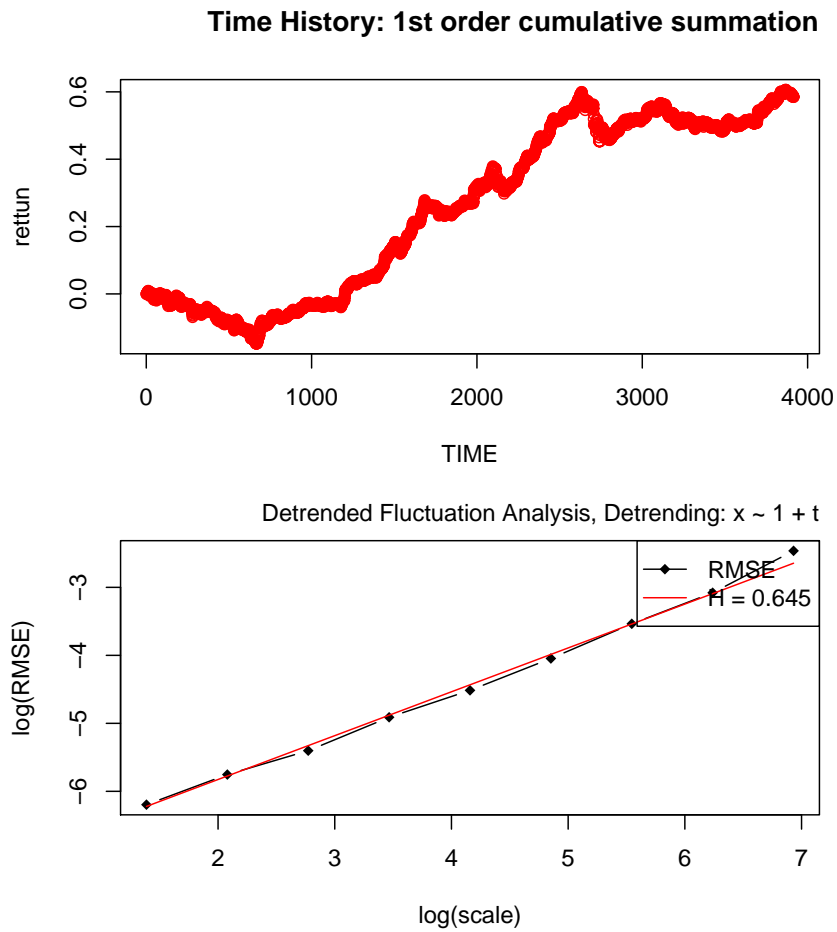


Figure 4.8: DFA plot of Tunisia

4.6 Discussion and Conclusions

Referring to Table 3.1, the summary statistics of the return series for the markets showed that the series were non - Gaussian. The kurtosis coefficients were large and mostly skewed negatively. The Jarque - Bera test statistic null hypothesis was rejected at the 1% level of significance. The Augmented Dickey - Fuller (ADF) test in Table 3.1 showed that the return series were stationary. This is because the test statistics are greater than the critical values of the ADF at all significance levels. Hence, the null of unit root of the ADF test is rejected.

From Table 4.1, there were periods that the Hurst exponent, $H < 0.5$ for some markets. This meant that the memory effect during that period was less and possibly moving towards efficiency. However, a close look at the DFA plots of $\log(\text{RMSE})$ versus $\log(\text{scale})$ in Figures 4.1 to 4.8 of the return series for the 15 year period showed a different feature. The DFA plots indicated the degree of long - memory persistence by estimating the Hurst exponent. The Hurst exponents for the markets showed persistence of long - memory in the return series because all had $H > 0.5$.

The plot of the return series in Figures 4.1 to 4.8 showed an upward trend which was in contrast to Cajueiro and Tabak (2004b) who found a downward slope as evidence of emerging markets becoming efficient with time. This new findings meant that the markets were not becoming efficient with time. In ranking the eight markets studied using the all years Hurst exponent in Table 4.1, it was observed that the market with a smaller Hurst exponent and hence less memory effect was South Africa, Morocco, Egypt, Nigeria, Kenya, Tunisia, Botswana and finally Mauritius.

Conclusions

In this chapter, the degree of long - memory persistence in return series over time in eight African stock markets were examined. The Hurst exponent was used as a measure of long - memory persistence which was evaluated by the Detrended Fluctuation Analysis (DFA).

This work built on the works on emerging markets in the literature by Tolvi (2003), Cajueiro and Tabak (2004a, 2004b), Jefferis and Smith (2005), McMillan and Thupayagale (2008), Smith (2008), Jefferies and Thupayagale (2008), and *Morris et al.* (2009).

It was observed that there was strong evidence of long - memory persistence in the markets studied which is contrary to the works of Cajueiro and Tabak (2004b), Jefferies and Smith (2005) but agrees with the work of Smith (2008) who concluded that none of the 11 African markets studied was weak - form efficient. These findings give strong evidence against the weak - form Efficient Market Hypothesis (EMH) in that long - memory persistence in these markets means existence of arbitrage opportunities.

5. Adaptive Market Hypothesis

Abstract

This chapter re-examines and analyses the return predictability of eight African stock markets using a non-parametric Generalised Spectral test in a rolling window approach.

The results support the Adaptive Market Hypothesis (AMH) in the sense that indices whose returns were observed to be predictable by analysing them in absolute form and therefore weak - form inefficient showed trends of unpredictability in a rolling window.

Keywords: Generalised Spectral Test, Martingale, Relative efficiency, Rolling window, African stock markets

5.1 Introduction

Analyses of studies on stock return predictability on the African continent have mainly relied on using the whole sample of a particular index which seeks to find absolute efficiency. In finding absolute efficiency, the conclusion of a particular market is either weak - form efficient or weak - form inefficient when the analysis is done using past price information. This has resulted in mixed conclusions about the weak-form Efficient Market Hypothesis (EMH) on stock markets in Africa. The problem with absolute efficiency approach is that stock markets become efficient with time because of improvements in the quality of information processing due to reforms on the markets; Hall and Urga (2002) hence drawing conclusions over a whole sample do not take into account time frames for lower or higher efficiency, Afego (2015). The relative efficiency reasoning is consistent with the Adaptive Markets Hypothesis (AMH) of Lo (2004, 2005) who talks about market efficiency not being an all-or-none condition but a feature that varies continuously with time.

In the literature, relative market efficiency is achieved using a rolling window or time-varying approach. For example, Cajueiro and Tabak (2004) estimate Hurst exponents in each window to test for long-term linear dependence and use the median as a statistical measure when ranking the markets. Lim (2007) and Lim and Brooks (2010) used the rolling bivariate correlation test statistic that focuses on nonlinear dependence, arguing that a more appropriate indicator for relative efficiency would be the percentage of time windows in which the market exhibited significant nonlinear dependence. On the African continent, studies on

relative efficiency of stock markets have been few with only those of Jefferis and Smith (2005) and Smith and Dyakova (2014).

This chapter therefore contributes to the extant literature on relative efficiency of African stock markets. The researcher extended the work of Smith and Dyakova (2014) by analysing returns of indices from eight African stock markets using the Generalised spectral test of Escanciano and Velasco (2006). The Generalised spectral test is a non-parametric test which does not depend on distributional assumptions. It is able to detect a wide range of linear and non-linear dependencies in conditional mean, hence has more power than other competing tests, *Charles et al.* (2010).

The rest of the chapter is as follows:

Section 5.2 briefly describes the Generalised spectral test of Escanciano and Velasco (2006). Section 5.3 presents summary statistics of the data as well as the empirical results. Section 5.4 discusses the results and concludes the chapter.

Closing prices from these indices were transformed into returns which were calculated by

$Y_t = \ln \left(\frac{P_t}{P_{t-1}} \right)$ where P_t and P_{t-1} are the daily closing prices of the index on two consecutive trading days.

5.2 The Generalised Spectral Test

The Generalised spectral (GS) test of Escanciano and Velasco (2006) is a non-parametric test designed to detect the presence of linear and nonlinear dependencies in a stationary time series. The GS test considers dependence at all lags; it is robust to conditional heteroskedasticity and it is consistent against a class of uncorrelated non-martingale sequences. Monte-Carlo tests done by researchers such as *Charles et al.* (2010) to study comparison between small sample properties of other tests for martingale difference hypothesis (MDH) conclude that the GS test performs better under nonlinear dependence and has more empirical power than other tests.

The GS test proceeds as followed in Todea and Lazar (2012):

Let $\left\{ Y_t \right\}_{t=1}^n$ be a stationary return time series. The null hypothesis of a martingale difference sequence of the return series is tested against the alternative hypothesis using a pairwise approach. Thus: $H_0 : m_j(y) = 0; j \geq 1$ almost surely where $m_j(y) = E[Y_t - \mu | Y_{t-j} = y]$ and μ is the mean against $H_1 : P(m_j(Y_{t-j}) \neq 0) > 0$ for some $j \geq 1$.

Let $\gamma_j(x) = E[(Y_t - \mu)e^{ixT_{t-j}}]$ be a nonlinear measure of dependence where $x \in \mathbb{R}$. The exponential

weighting function is used to measure the conditional mean dependence in a nonlinear time series. The null hypothesis above is therefore consistent with $\gamma_j(x) = 0$ for all $j \geq 1$ almost everywhere.

Escanciano and Velasco (2006) used the generalised spectral distribution function:

$$H(\lambda, x) = \gamma_0(x)\lambda + 2 \sum_{j=1}^{\infty} \gamma_j(x) [\sin(j\pi\lambda)/j\pi] \quad (5.1)$$

where $\lambda \in [0, 1]$. The sample estimate of H becomes

$$\hat{H}(\lambda, x) = \gamma_0(x)\lambda + 2 \sum_{j=1}^{\infty} (1 - j/n)^{1/2} \hat{\gamma}_j(x) \frac{\sin(j\pi\lambda)}{j\pi} \quad (5.2)$$

where $(1 - j/n)^{1/2}$ is a sample finite correction factor, $\hat{\gamma}_j(x) = (n - j)^{-1} \sum_{t=1+j}^n (Y_t - \bar{Y}_{n-j}) e^{ixY_{t-j}}$

and $\bar{Y}_{n-j} = (n - j)^{-1} \sum_{t=1+j}^n Y_t$. The generalised spectral distribution function under the null of Martingale difference hypothesis (MDH) therefore becomes $H(\lambda, x) = \gamma_0(x)\lambda$. The test is based on the difference between $\hat{H}(\lambda, x)$ and $\hat{H}_0(\lambda, x) = \hat{\gamma}_0(x)\lambda$ as follows:

$$S_n(\lambda, x) = \left(\frac{n}{2}\right)^{1/2} [\hat{H}(\lambda, x) - \hat{H}_0(\lambda, x)] = \sum_{j=1}^{\infty} (1 - j/n)^{1/2} \hat{\gamma}_j(x) \frac{\sqrt{2} \sin(j\pi\lambda)}{j\pi} \quad (5.3)$$

The researcher used the Cramer-von Mises norm in equation (5.4) below to evaluate the distance of $S_n(\lambda, x)$ to zero for all possible values of λ and x

$$D_n^2 = \int_R \int_0^1 |S_n(\lambda, x)|^2 W(dx) d\lambda = \sum_{j=1}^{n-1} (n - j) \frac{1}{(j\pi)^2} \int_R |\hat{\gamma}_j(x)|^2 W(dx) \quad (5.4)$$

where the weighting function $W(\cdot)$ satisfies some mild conditions. If the standard normal cumulative distribution functions is settled as the weighting function, the following statistics results:

$$D_n^2 = \sum_{j=1}^{n-1} \frac{(n - j)}{(j\pi)^2} \sum_{t=j+1}^n \sum_{s=j+1}^n (Y_t - Y_{n-j}^-)(Y_s - Y_{n-j}^-) \exp[-0.5(Y_{t-j} - Y_{s-j})^2] \quad (5.5)$$

The null hypothesis of a martingale difference hypothesis is rejected when values of D_n^2 are large.

Since the asymptotic distribution of the test depends on the data-generating process in a complicated way, the authors therefore propose implementing the test using a wild bootstrap procedure. The validity of the bootstrap procedure was proved, allowing for approximation of the critical values. The p-values for the test statistic D_n^2 are obtained by the following steps as outlined in Escanciano and Velasco (2006):

1. test if $\left\{ Y_t \right\}_{t=1}^n$ is stationary

2. compute the statistics D_n^2 for the stationary return time series, $\left\{ Y_t \right\}_{t=1}^n$
3. simulate a sequence $\left\{ w_t \right\}_{t=1}^n$ of independent random variables with zero mean, unit variance and bounded support independent of the observed sequence
4. compute $\hat{\gamma}_0^*(x) = (n-j)^{-1} \sum_{t=1+j}^n (Y_t - Y_{n-j}^-) \hat{\Phi}_{t-j}(x) w_t$ then S_n^* and D_n^{*2}
 where $\hat{\Phi}_{t-j}(x) = e^{ixY_{t-j}} - (n-j) \sum_{t=j+1}^n e^{ixY_{t-j}}$
5. repeat steps 2 and 3 many times to obtain a bootstrap distribution of the test statistic.

The p-value of the test statistic is thus estimated as the proportion of D_n^{*2} greater than D_n^2

A p-value is first computed for the first 500 observations, the first observation is then dropped and the sample rolled one point forward to re-estimate the next p-value.

In this study, a window length of 500 observations lead to 3416 rolling windows for each of the return series. Also, a statistical indicator of relative efficiency proposed by Lim (2007) which determines the percentage of time windows for which a p-value is less than 0.05 was used.

5.3 Results

First, preliminary analysis were done to to determine the time series properties of the return of the indices under study and then applied the Generalised spectral test to compute the p-value on the full sample. The sample was divided into two and computed p-values for the period from 2000 to 2007 and from 2008 to 2015 before finally the p-value was computed for each rolling window in order to observe the time variation in predictability for the indices.

Preliminary Analysis

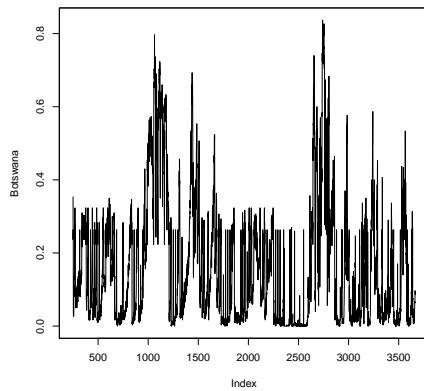
Referring to Table 3.1, it was observed that the return series were non - Gaussian. The kurtosis coefficients are large and mostly skewed negatively. The Jarque - Bera test statistic null hypothesis is rejected at the 1% level of significance. The Augmented Dickey - Fuller test showed that the return series were stationary.

P - values for full sample and sub - sample Analysis

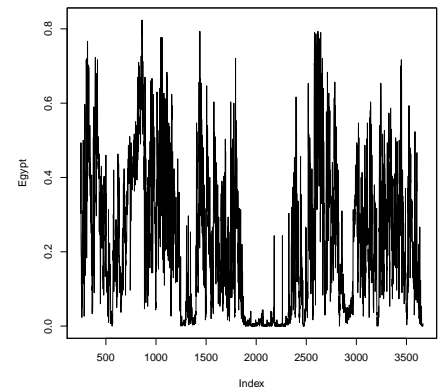
Market	Full sample	2000-2007	2008-2015
Botswana	0.3167	0.3800	0.0433
Egypt	0.0000	0.0000	0.0000
Kenya	0.0000	0.0067	0.0300
Mauritius	0.0000	0.0000	0.0300
Morocco	0.0000	0.0000	0.1000
Nigeria	0.9100	0.3133	0.0000
South Africa	0.5100	0.3933	0.4533
Tunisia	0.0000	0.0033	0.0200

Table 5.1: The p-values for Generalised Spectral test

Plots of P - values for full sample

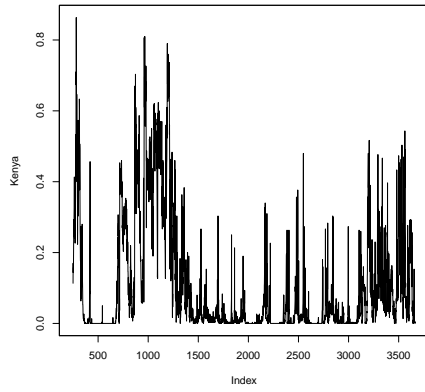


(a) Botswana

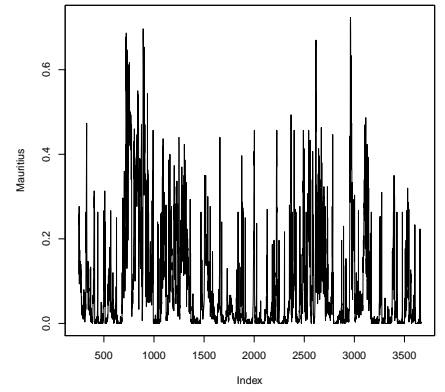


(b) Egypt

Figure 5.1: P - value plots of Botswana and Egypt

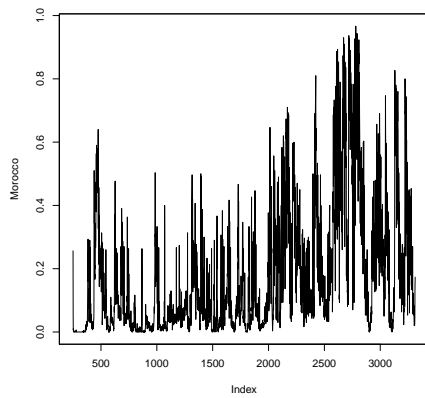


(a) Kenya

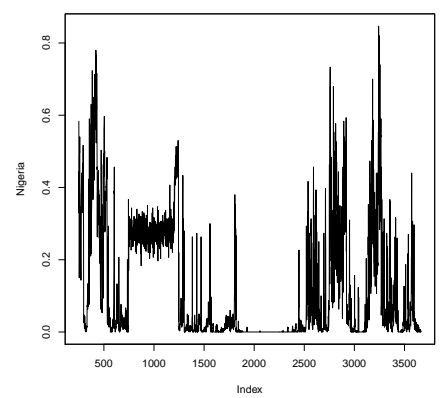


(b) Mauritius

Figure 5.2: P - value plots of Kenya and Mauritius

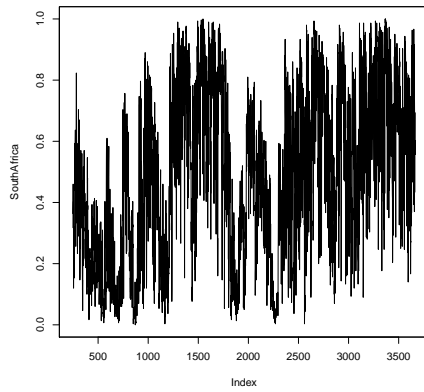


(a) Morocco

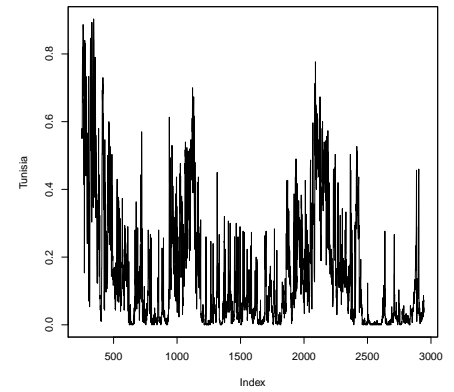


(b) Nigeria

Figure 5.3: P - value plots of Morocco and Nigeria



(a) South Africa



(b) Tunisia

Figure 5.4: P - value plots of South Africa and Tunisia

Percentage of windows for which p - value < 0.05

Market	Full sample	2000-2007	2008-2015
Botswana	36.27	18.61	53.92
Egypt	24.18	9.19	39.16
Kenya	61.21	38.70	83.72
Mauritius	59.66	38.70	80.62
Morocco	32.99	51.63	14.36
Nigeria	58.25	36.59	79.91
South Africa	3.48	4.80	2.17
Tunisia	40.32	39.91	40.73

Table 5.2: Percentage of p-values < 0.05

5.4 Discussion and Conclusions

The plots of the p - values from the rolling window approach in Figures 5.1 - 5.4 were numerically discussed.

Botswana

It was observed in Table 5.1 that the p - values using the Generalised Spectral Test (GST) in absolute form using the full sample and the sample for the period between 2000 and 2007 were not predictable (that is, $0.3167 > 0.05$). The sample became predictable for the period between 2008 and 2015 ($0.0433 < 0.05$). The rolling window results in Table 5.2 established that about 36.27% of the windows for the full sample had p - values less than 0.05. This number decreased to 18.61% for the sample between 2000 and 2007 but increases to 53.92% for the sample between 2008 and 2015. This observation shown a contradiction of the position by Cajueiro and Tabak (2004b) that markets become efficient with time.

Egypt

The p - values as reported in Table 5.1 shown that the index was predictable. The full sample and the two sub - samples had p - values of 0.0000 which is less than 0.05. This made the index weak - form inefficient, a conclusion which contradicted that of Appiah - Kusi and Menyah (2003) but conformed to the results of Simons and Laryea (2005).

The results for the rolling windows shown that only 24.18%, 9.19% and 39.16% of the windows for the full sample and the sub - samples were predictable. It will be statistically incorrect therefore to label the whole sample as rejecting the null hypothesis of return predictability as reported in Table 5.1. This observation conformed to the work of Smith and Dyakova (2014) who concluded that the Egyptian market was least predictable in a rolling window.

Kenya

The index was predictable and hence weak - form inefficient as evidenced by results in Table 5.1. More than half of the windows for the full sample have p -values less than 0.05 and it was more predictable for the period between 2008 and 2015 as 83.72% of the windows had p - values less than 0.05. This predictability conclusion contradicted the results by Dickinson and Muragu (1994) who used serial correlations and the runs test to analyse the Kenyan market for the period between 1979 to 1989. The return predictability conclusion however was in conformity with the results of Jefferis and Smith (2005) and that of Smith and Dyakova (2014).

Mauritius

The index was predictable as reported in Table 5.2 with respect to the full sample and the two sub-samples. This conformed to the results in the literature on Mauritius by Bundoo (2000) and Simons & Laryea (2005). The full sample had almost 60% of the windows being predictable. This number went up to 80.62% of the windows being predictable for the sub - sample between 2008 and 2015 in Table 5.2. This finding contradicted the assertion that markets, and for that matter, indices become efficient with time.

Morocco

The full sample had p - values of 0.00 in Table 5.1, hence, predictable but the index was not predictable for the period between 2008 and 2015. About 15% of the windows for the sample between 2008 and 2015 are predictable. This means that the index was becoming efficient with time as the number of windows that are predictable reduced from 51.63% for the 2000 to 2007 period to about 15% for the period between 2008 and 2015 as shown in Table 5.2

Nigeria

The p - value from Table 5.1 shown the index was not predictable for the full sample and the period between 2000 and 2007 but became predictable for the period between 2008 and 2015. This showed the index was not becoming efficient with time. In absolute form, the index would have been concluded as weak - form efficient. This conclusion conformed to Olowe (1999) and Jefferis and Smith (2005) but contradicted that of Magnusson & Wydick (2002), *Smith et al.* (2002), Smith (2008) and Mollah & Vitalli (2011). Results from the rolling window in Table 5.2 showed that almost 58.25% of the windows were predictable for the full sample and about 79.91% of windows predictable for the period between 2008 and 2015. These results showed that the Nigerian market was becoming predictable with time which contradicted the results obtained by Cajueiro and Tabak (2004b) but conformed to the results obtained by Smith and Dyakova (2014).

South Africa

South Africa was the most weak - form efficient index of all the indices under study as based on results from Table 5.1. It was not predictable as the p - values for the full sample and that of the sub - samples were greater than 0.05. This result was in conformity with the results obtained by Affleck-Graves & Money (1975), Magnusson & Wydick (2002), *Smith et al.* (2002), Simons & Laryea (2005), Jefferis & Smith (2005), McMillan & Thupayagale (2008) and Mollah & Vitalli (2011) but not in conformity with the results of Jammie & Hawkins (1974) and that of Smith (2008). The results from the rolling window approach in Table 5.2 showed less than 4% of the windows had p - values less than 0.05 in the full

sample and the sub - samples. This made the South African market the least predictable, a conclusion that resonated with that of Smith & Dyakova (2014).

Tunisia

The results from Table 5.1 brought about a conclusion that the index was predictable but the rolling window results in Table 5.2 showed that less than 50% of the windows of the full sample and the sub - samples had p - values less than 0.05. Therefore, it cannot be wholly concluded that the index was predictable as there were about 60% of periods of unpredictability. The index was thus the least predictable as concluded by Smith and Dyakova (2014).

Conclusions

This study re-examined and extended the work of Smith and Dyakova (2014) by analysing the return predictability of eight African stock markets. The data was analysed using the non-parametric Generalised Spectral Test of Escanciano and Velasco (2006) in a rolling window approach.

The results favour the Adaptive Market Hypothesis (AMH) of Lo (2004, 2005) in that each of the return series showed trends in the time variation of return predictability. For example, Egypt, one of the oldest markets in Africa was found to be highly predictable when the analysis was done in absolute form because its p-values were 0.0000 for the full sample and the sub - samples, thus violating the assertion that markets become efficient with time as posited by Cajueiro and Tabak (2004b). In the rolling window approach, the results for Egypt however showed that it was not highly predictable as observed earlier in the full sample results since it had one of the lowest percentages of the windows that had p-values less than 0.05 after South Africa.

6. Stationarity under Exponential Smooth Threshold Autoregressive (ESTAR)

Abstract

This chapter examined the stationarity of eight African stock markets. The results from the analyses of the logarithmic daily closing prices of eight indices from August, 28 2000 to August, 28 2015 using the non-linear ADF unit root test and the modified Wald type test under an ESTAR framework were compared.

The results showed that both non-linear unit root tests failed to reject the null of unit root in all the markets but for Botswana. It was concluded that the stock markets in Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia are non-stationary and hence weak-form efficient.

Keywords: Stationarity, African Stock Markets, ESTAR, Weak-form efficiency

6.1 Introduction

The stationarity or otherwise of a stock market has implications on the conclusions that are made about such market(s) with respect to its informational efficiency. The Efficient Market Hypothesis (EMH) as postulated by Fama (1970) states that, current prices reflect all the available information on the market. Thus information is quickly and instantaneously reflected in setting current security prices. This assertion implies a linear relationship between information and pricing, however, this linear assertion has been criticised by researchers such as Dyckman and Morse (1986) and *Lee et al.* (2001). They posit that the data-generating process of a financial data is non-linear and not linear as the EMH stipulates. According to McMillan (2003), Hasanov and Omay (2008), the non-linearity of a financial data arises because of issues such as market friction, transaction costs, noise traders, the existence of bid-ask spreads and short sales. The issues raised by these researchers make the EMH assertion of linearity not totally valid.

Notwithstanding these issues raised, there have been studies of the EMH on African stock markets especially the weak-form type of EMH. The weak-form EMH states that prices on the market reflect information on past prices. Studies of the weak-form EMH on African stock markets have ended with

mixed conclusions. Afego (2015) reviews the literature on the weak-form EMH on African stock markets. An example of a mixed conclusion about the markets was when Jefferis and Smith (2005) tested for weak-form efficiency in South Africa, Zimbabwe, Nigeria, Morocco, Kenya, Egypt and Mauritius by a GARCH model with time varying parameters. They found the JSE to be efficient over the study period between 1990 and 2001. Nigeria, Egypt and Morocco only became efficient in the latter part of the period, while Zimbabwe and Kenya failed the efficiency test over the whole period. Smith (2008) however studied the weak-form EMH on 11 African markets, namely, South Africa, Zimbabwe, Ghana, Nigeria, Egypt, Tunisia, Botswana, Kenya, Morocco, Mauritius and Cote d'Ivoire. Smith (2008) analysed the data using Wright's joint variance ratio test, proposed by Wright (2000a), and Chow-Denning multiple variance ratio test, proposed by Chow and Denning (1993), observed that none of the markets studied was weak form efficient. There are so many questions that come to mind as to why this contradiction. Could it be because of the data period chosen or the method of analysis employed?

This chapter seeks to use a non-linear method to analyse the stationarity or otherwise of eight African stock markets since according to Afego (2015), the methods that have been used in the literature in analysing data on African stock markets have been linear. Non-linear time series models such as the smooth transition autoregressive (STAR) models which have been used by researchers for some time now in analysing financial data, (Terasvirta, 1994) were considered. First, a test for linearity in the series were done by applying the Brock, Dechert and Scheinkman (1987) test for detecting serial dependence in time series. After the evidence of nonlinearity, tests for stationarity were done by employing two non-linear unit root tests under an exponential STAR (ESTAR) framework; the ADF type unit root against ESTAR developed by *Kapetanios et al.* (2003) and the modified Wald type non-linear unit root test by Kruse (2011).

The rest of the chapter is organised as follows:

Section 6.2 to section 6.4 describes the methods used in analysing the data. The results and findings are presented in section 6.5. Section 6.6 discusses the findings and concludes the chapter.

6.2 The BDS Linearity Test

The BDS test was developed by W.A. Brock, W. Dechert and J. Scheinkman in 1987 (hereafter BDS (1987)). The test is used for detecting serial dependence in time series, hence, it can be used to detect nonlinearity. The null of independent and identically distributed (I.I.D.) hypothesis is tested against an unspecified alternative. The researcher followed the procedure below in computing the BDS test: Let

y_i be a time series with N observations, which should be the first difference of the natural logarithms of the raw data. Thus

$$\{y_i\} = [y_1, y_2, y_3, \dots, y_N] \quad (6.1)$$

The researcher selected a value of m called the 'embedding dimension' and embed the time series into m -dimensional vectors by taking each m successive points in the series. This converted the series of scalars into a series of vectors with overlapping entries:

$$\begin{aligned} y_1^m &= (y_1, y_2, \dots, y_m) \\ y_2^m &= (y_2, y_3, \dots, y_{m+1}) \\ &\vdots \\ y_{N-m}^m &= (y_{N-m}, y_{N-m+1}, \dots, y_N) \end{aligned}$$

The correlation integral was computed. The correlation integral, which is a measure of the spatial correlation among the points by adding the number of pairs of points (i, j) where $1 \leq i \leq N$ and $1 \leq j \leq N$ in the m -dimensional space which are "close" in the sense that the points are within a radius of tolerance ε of each other.

$$C_{\varepsilon, m} = \frac{1}{N_m(N_m - 1)} \sum_{i \neq j} I_{i, j; \varepsilon} \quad (6.2)$$

where

$$I_{i, j; \varepsilon} = \begin{cases} 1, & \text{if } \|y_i^m - y_j^m\| \leq \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

BDS (1987) proved that if the time series is an IID

$$C_{\varepsilon, m} \approx [C_{\varepsilon, 1}]^m \quad (6.3)$$

According to Lin (1997), if the ratio $\frac{N}{m}$ is greater than 200, the values of $\frac{\varepsilon}{\sigma}$ range from 0.5 to 2 and the values of m are between 2 and 5 (Brock et al. (1988)). The quantity $[C_{\varepsilon, m} - (C_{\varepsilon, 1})^m]$ has an asymptotic normal distribution with zero mean and a variance $V_{\varepsilon, m}$ defined as:

$$V_{\varepsilon, m} = 4 \left[K^m + 2 \sum_{j=1}^{m-1} K^{m-j} C_{\varepsilon}^{2j} + (m-1)^2 C_{\varepsilon}^{2m} - m^2 K C_{\varepsilon}^{2m-2} \right] \quad (6.4)$$

where

$$K = K_{\varepsilon} = \frac{6}{N_m(N_m - 1)(N_m - 2)} \sum_{i < j < N} h_{i, j, N; \varepsilon}; h_{i, j, N; \varepsilon} = \frac{[I_{i, j; \varepsilon} I_{j, N; \varepsilon} + I_{i, N; \varepsilon} I_{N, j; \varepsilon} + I_{j, i; \varepsilon} I_{i, N; \varepsilon}]}{3}$$

The BDS test statistic is thus stated as:

$$BDS_{\varepsilon,m} = \frac{\sqrt{N}[C_{\varepsilon,m} - (C_{\varepsilon,1})^m]}{\sqrt{V_{\varepsilon,m}}}. \quad (6.5)$$

The null hypothesis of independent and identically distributed (IID) was rejected at the 5% significance level if $|BDS_{\varepsilon,m}| > 1.96$.

6.3 Nonlinear ADF Unit Root Test against ESTAR

The Augmented Dickey-Fuller (ADF) test which is to detect nonstationarity in a time series was extended by Kapetanios, Shin and Snell (2003) (hereafter, the KSS). The null of a unit root as in the ADF test is kept by KSS but the alternative hypothesis is that of a nonlinear but globally stationary process. The researcher explains the KSS test as follows: Let

$$y_t = y_{t-1} + \phi y_{t-1}(1 - \exp(-\gamma(y_{t-1} - c)^2)) + \varepsilon_t \quad (6.6)$$

be a univariate exponential smooth transition autoregressive model of order 1, ESTAR (1) where $c = 0$ is the location parameter. Equation (6.6) was transformed by KSS (2003) by making $c = 0$. Hence the researcher obtained:

$$\Delta y_t = \phi y_{t-1}(1 - \exp((-\gamma y_{t-1})^2)) + \varepsilon_t \quad (6.7)$$

where y_t is the demeaned and or detrended series of interest and $1 - \exp((-\gamma y_{t-1})^2)$ is the exponential transition function in the KSS test for nonlinear adjustment. The hypotheses of the KSS test is that $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$. Since γ cannot be identified directly under H_0 , the null hypothesis becomes impossible to test. KSS (2003) therefore reparametrized equation (6.7) using a first order Taylor series approximation and the following auxiliary regression was obtained:

$$\Delta y_t = \beta_1 y_{t-1}^3 + \mu_t \quad (6.8)$$

with μ_t being a noise term depending on ε_t , ϕ and remainder of the Taylor expansion. Equation (6.8) looks like the Dickey-Fuller test regression without a deterministic term. The cubic term y_{t-1}^3 approximates the ESTAR nonlinearity. The researcher modified equation (6.8) so that it can take care of the presence of a possible serial correlation in the error terms. Thus

$$\Delta y_t = \beta_1 y_{t-1}^3 + \mu_t + \sum_{k=1}^q \rho \Delta y_{t-k} + \varepsilon_t \quad (6.9)$$

where q is the number of augmentations that can be specified using any standard lag length selection criteria, hence, the null hypothesis becomes $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 < 0$.

KSS (2003) provided critical values for the test statistic which had no asymptotic standard normal distribution. The critical values on page 364 of their article are reproduced in the Table 6.1 below:

Significance level	Raw Data	Demeaned Data	Detrended Data
1%	-2.82	-3.48	-3.93
5%	-2.22	-2.93	-3.40
10%	-1.92	-2.66	-3.13

Table 6.1: Critical Values of the KSS test

6.4 Modified Wald-type Test against ESTAR

The argument by Kruse (2011) was that the location parameter c in the exponential transition function must be nonzero hence the nonlinear model in equation (6.6) was considered as:

$$\Delta y_t = y_{t-1} + \phi y_{t-1}(1 - \exp(-\gamma(y_{t-1} - c)^2)) + \varepsilon_t. \quad (6.10)$$

A first-order Taylor approximation to $G(y_{t-1}; \gamma, c) = (1 - \exp(-\gamma(y_{t-1} - c)^2))$ around $\gamma = 0$ was applied and the test regression was proceeded as follows:

$$\Delta y_t = \beta_1 y_{t-1}^3 + \beta_2 y_{t-1}^2 + \beta_3 y_{t-1} + \mu_t. \quad (6.11)$$

Following *Kapetanios et al.* (2003) and to improve the power of the test statistic, $\beta_3 = 0$ was imposed and obtained an estimated model as:

$$\Delta y_t = \beta_1 y_{t-1}^3 + \beta_2 y_{t-1}^2 + \mu_t \quad (6.12)$$

where $\beta_1 = \gamma\phi$ and $\beta_2 = -2c\gamma\phi$. The interest was in testing the null of a unit root; $H_0 : \gamma = 0$ against the alternative of a globally stationary ESTAR process $H_1 : \gamma > 0$.

From the test regression in equation (6.12), the pair of hypothesis is equivalent to $H_0 : \beta_1 = \beta_2 = 0$ against $H_1 : \beta_1 < 0, \beta_2 \neq 0$. It is to be noted that under H_1 , β_1 is one-sided and β_2 is two-sided so that c is allowed to take on real values. This testing problem where one parameter is one-sided under H_1 and the other parameter is two-sided is non-standard therefore employing a standard Wald test would be

inappropriate. Following the methods of Abadir and Distaso (2007), Kruse (2011) derived a modified Wald test for the null and alternative hypothesis which is simply formulated by:

$$\tau = t_{\beta_{1/2}=0}^2 + 1(\hat{\beta}_1 < 0)t_{\beta_1=0}^2 \quad (6.13)$$

where two-summands in the test statistic τ can be interpreted as follows:

The first term is a squared t-statistic for the hypothesis $\beta_{1/2} = 0$. Thus $\beta_{1/2}$ is orthogonal to β_1 while the second term is a squared t-statistic for the hypothesis $\beta_1 = 0$. The test statistic τ has a non-standard asymptotic distribution and the asymptotic critical values are derived under the standard assumptions for the error term which are shown in Table 6.2.

Significance level	Raw Data	Demeaned Data	Detrended Data
1%	13.5	13.75	17.10
5%	9.53	10.17	12.82
10%	7.85	8.60	11.10

Table 6.2: Critical Values of the Kruse test

6.5 Results

The linear unit root test results in Table 6.3 showed that the Augmented Dickey Fuller ADF and the KwiatkowskiPhillipsSchmidtShin (KPSS) tests failed to reject the assertion that the series were of unit roots in all cases. This meant that the logarithmic daily closing prices of the indices under study were non-stationary and therefore not mean reverting.

An examination on whether the logarithmic daily closing prices of the indices were characterized by a linear or nonlinear trend were done. The Brock, Dechert and Scheinkman (BDS) (1996) test for detecting serial dependence in time series was employed. The results of the BDS test of embedding dimension ($m = 2$) and metric bound (ε) equal 0.5 times the standard deviation of the logarithmic price series as shown in Table 6.4 indicate that all the test statistics are greater than the critical values. Also, the small p-values show that a non-linear relationship exists, therefore, the conclusion on the BDS test is that the null of an independent and identically distribution is rejected. This shows that the eight indices follow a non-linear pattern. This non-linear pattern in the data therefore allowed us to carry out a non-linear test on the data using the non-linear ADF test of *Kapetanios et al.* (2003) and the modified

Index	ADF	KPSS
S& P Botswana BMI	-1.55	22.51
EGX 30	-0.82	17.78
NSE 20	-1.23	15.42
SE Semdex	-0.74	23.80
Morocco All Share (MASI)	-0.51	16.61
Nigeria All Share	-1.67	12.25
FTSE/JSE All Share	-1.98	24.37
Tunindex	-1.84	25.17

Table 6.3: Results of linear Unit Root tests

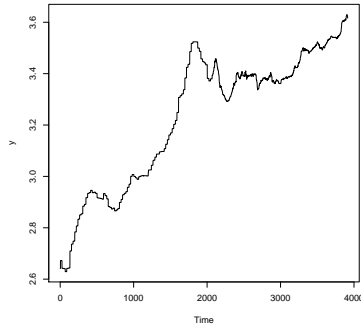
Index	ε/σ	Embedding dimension (m)	Test statistic	P-value
S& P Botswana BMI	0.5	2	192.74	0.00
EGX 30	0.5	2	178.25	0.00
NSE 20	0.5	2	150.04	0.00
SE Semdex	0.5	2	182.84	0.00
Morocco All Share (MASI)	0.5	2	176.96	0.00
Nigeria All Share	0.5	2	182.21	0.00
FTSE/JSE All Share	0.5	2	238.74	0.00
Tunindex	0.5	2	247.50	0.00

Table 6.4: BDS test results of logarithmic daily closing prices

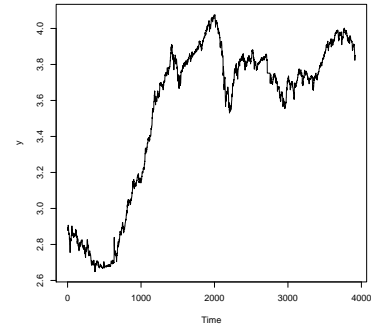
Wald type test of Kruse (2011). Figures 6.1-6.4 however showed that the plots of the logarithmic daily closing prices was characterized by a trend, hence the non-linear analysis will only be on the demeaned and detrended data.

The results obtained by analysing the data using the non-linear *Kapetanios et al.* (2003) and the non-linear Kruse (2011) tests are as shown in Tables 6.5 and 6.6 respectively. The null of unit root was failed to be rejected in all the markets studied for the demeaned and detrended data but for Botswana.

The *Kapetanios et al.* (2003) test rejects the null of unit root for Botswana when the data is demeaned at the 5% and 10% significance levels while the Kruse (2011) test rejects the null of unit root for Botswana when the data is demeaned at the 1%, 5% and 10% significance levels.

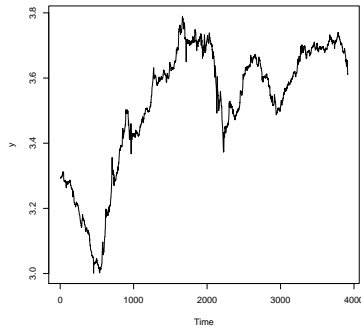


(a) Botswana

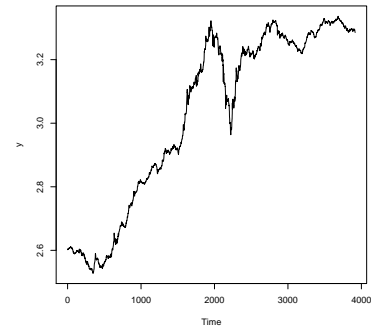


(b) Egypt

Figure 6.1: Plots of log series for Botswana and Egypt



(a) Kenya

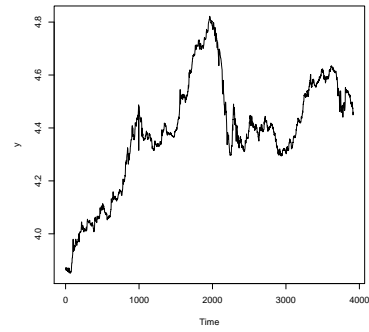


(b) Mauritius

Figure 6.2: Plots of log series for Kenya and Mauritius

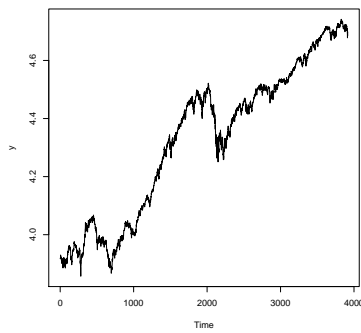


(a) Morocco

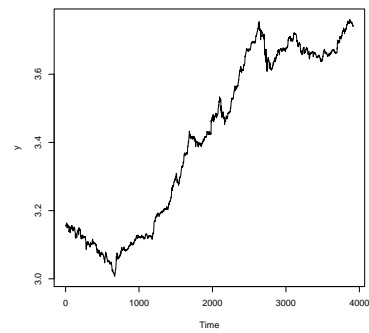


(b) Nigeria

Figure 6.3: Plots of log series for Morocco and Nigeria



(a) South Africa



(b) Tunisia

Figure 6.4: Plots of log series for South Africa and Tunisia

Index	Demeaned	Detrended
S& P Botswana BMI	-3.15	-1.28
EGX 30	-0.94	-1.15
NSE 20	-0.77	-1.18
SE Semdex	-1.08	-1.03
Morocco All Share (MASI)	-1.27	-1.16
Nigeria All Share	-1.92	-0.95
FTSE/JSE All Share	-1.03	-2.06
Tunindex	-0.31	-2.01

Table 6.5: Results of Kapetanios et al. (2003) test

Index	Demeaned	Detrended
S& P Botswana BMI	18.59	5.95
EGX 30	1.04	1.43
NSE 20	1.02	1.57
SE Semdex	1.55	1.81
Morocco All Share (MASI)	1.97	1.36
Nigeria All Share	4.31	1.66
FTSE/JSE All Share	2.47	7.19
Tunindex	1.24	4.17

Table 6.6: Results of Kruse test

6.6 Discussion and Conclusions

The non-stationarity conclusion and the subsequent implication of weak-form efficiency of seven out of the eight African markets studied contradict most studies in the weak-form efficiency literature of the EMH. In most of the studies summarized by Afego (2015), with the exception of the South African market which has shown consistency in being weak-form efficient, the rest of the markets have been weak-form inefficient.

The reason behind the results of this study which conclude a weak-form efficiency for the markets known in literature to be mostly weak-form inefficient, might be due to the assertion by Cajueiro and Tabak (2004b). They are of the view that markets become efficient through time.

The markets studied have existed for quite some time. As a result, factors given by Cajueiro and Tabak (2004b) to bring about efficiency such as increases in foreign capital inflow and in trading volumes, technological advancement in a market's operations might have affected the conclusions.

On the other hand, Botswana was proven to be weak-form inefficient. The reason for this might be due to the fact that even though the market had existed since 1989, technological advancement has been slow. For example, an automated trading system which deals with clearing and settlement was only introduced in 2012.

Conclusions

The stationarity of eight indices on eight African stock markets were examined. The BDS test of serial dependence was used to check whether the data-generating process was characterized by a linear or non-linear pattern. The BDS test results showed non-linearly behaviour in the data generating process hence the researcher employed two non-linear unit root tests under the ESTAR framework namely the non-linear ADF unit root of *Kapetanios et al.* (2003) and an extension of the *Kapetanios et al.* (2003) test known as the 'modified wald type test' of Kruse (2011).

The findings from the two non-linear tests showed that the markets were non-stationary but for Botswana, thus it was concluded that the stock markets in Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia are non-stationary, therefore, weak-form efficient while Botswana is not weak-form efficient. This observation means it is difficult for arbitrageurs to make abnormal profits by analysing the price history on these seven African stock markets.

7. Modelling Under Self Exciting Threshold Autoregressive (SETAR) Model

Abstract

The chapter examined whether non-linear patterns were present in the returns of eight indices on the stock markets in Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia between the period of 2011 and 2015.

The results of applying four linearity tests on the returns concluded that the null of linearity was rejected on all four tests for the eight indices. The indices were modelled under the non-linear self-exciting threshold autoregressive (SETAR) model. The modelling performance of the non-linear SETAR model with that of the standard AR(1) and AR(2) were compared by analysing AIC values of the respective models. The results showed that the SETAR model fitted the data well, hence, modelling stock market returns from these eight countries using linear models might lead to spurious conclusions.

Keywords: Threshold models, Linearity tests, Self-Exciting Threshold AutoRegressive (SETAR) model

7.1 Introduction

The Efficient Market Hypothesis (EMH) posits that information on a market is correctly and instantaneously incorporated in setting current asset prices. This assertion means there is a linear relationship between information flow to market participants and how prices are set on the market, thus modelling and forecasting of the returns are done using linear models. However, some researchers have raised objection to the EMH. Researchers, such as, Hinich and Paterson (1985), Fama and French (1988), Lo and McKinlay (1988), Hsieh (1991), Cochrane (1998), Terasvirta and Asbrink (1998) and Garcia and Gencay (2000) have questioned whether it is appropriate for linear models to be used in analysing complex models that had come about as a result of how prices are determined and the market negotiation process. These researchers believe that market participants do not have an even trading field. It is believed that information flow

on the market is not simultaneously relayed to all agents on the market, therefore, a non-linear model is appropriate for capturing the dynamics on the market. Nonlinearity on the market might be due to agents having different objectives and targets for trading on the market. Also agents vary in their negotiation times and how agents view risk and the need to diversify their portfolio. These sources of nonlinearity on the market has increased the interest in analysing stock returns with non-linear methods.

Although it is reported in the extant literature of the non-easiness of non-linear models because they can sometimes create spurious fits, (Granger and Terasvirta, 1993), a discrete transition regime switching model, the self-exciting threshold autoregressive (SETAR) model was employed, because of its variety and flexibility. The SETAR model is robust to heteroscedasticity in the data.

The aim of this chapter was to determine if there exists non-linear patterns on the returns of eight indices on the stock markets in Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia. First, four linearity tests were applied on the returns of the series - Keenan (1985) test, Tsay (1986) test, BDS (1987) test and the Delay Vector Variance (DVV) test developed by *Gautama et al.* (2004). If nonlinearity exists, the returns of the series were modelled with the SETAR model. The results of the SETAR model were compared with the results of the standard AR(1) and AR(2) models to see which model fitted the data well by analysing the AIC values.

The chapter is organised as follows:

Sections 7.2 to 7.6 describe the methods employed. Section 7.7 presents the empirical results while section 7.8 discusses the findings observed and concludes the chapter.

The daily closing prices were transformed into returns which were calculated as:

$$Y_t = \ln \left(\frac{P_t}{P_{t-1}} \right)$$
 where P_t and P_{t-1} are the daily closing prices of the index on two consecutive trading days.

Methodology

The Keenan test, the Tsay test, the BDS test and the Delay Vector Variance test were employed to confirm the existence or otherwise of non-linear patterns in the data. These tests were chosen based on its power. If nonlinearity was present, the returns were modelled using the non-linear SETAR.

Linearity Tests

First, the return series were tested if it exhibited non-linear patterns before proceeding with modelling using non-linear models. Four linearity tests, namely, Keenan (1985) test and Tsay (1986) test, BDS (1987) test and the Delay Vector Variance DVV (2004) test were presented.

7.2 Keenan Test

The Keenan test (1985) as in Cryer and Chan (2008) was presented. The Keenan (1985) test is used to test for nonlinearity against the null hypothesis that a time series follows some AR process. The Keenan test is based on a second-order Volterra type expansion similar to the Taylor expansion. The Volterra expansion is used for non-linear modelling and it is able to capture memory effects.

The Keenan test can be written as:

$$y_t = \mu + \sum_{i=-\infty}^{\infty} \theta_i \varepsilon_{t-i} + \sum_{i,j=-\infty}^{\infty} \theta_{ij} \varepsilon_{t-i} \varepsilon_{t-j} + \sum_{i,j,k=-\infty}^{\infty} \theta_{ijk} \varepsilon_{t-i} \varepsilon_{t-j} \varepsilon_{t-k} + \dots \quad (7.1)$$

Here, $\{\varepsilon_t, -\infty < t < \infty\}$ are a sequence of independent and identically distributed random variables with mean zero and $\sigma_\varepsilon^2 < \infty$ while $y_1, y_2, y_3, \dots, y_n$ are the observations. The process $\{y_t\}$ is linear if the double sum of the right hand side of equation (7.1) disappears. Thus, testing the nonlinearity of a series $\{y_t\}$ consists practically in testing whether the double sum is zero or not.

Alternatively, as proven by Cryer and Chan (2008), Keenan test can also be heuristically derived as follows:

$$y_t = \theta_0 + \phi_1 Y_{t-1} + \dots + \phi_m Y_{t-m} + \exp\left\{\eta \left(\sum_{j=1}^m \phi_j Y_{t-j}\right)^2\right\} + \varepsilon_t \quad (7.2)$$

where $\{\varepsilon_t\}$ are independent and normally distributed with zero mean and finite variance. If the regression coefficient $\eta = 0$ then the exponential term becomes 1. Equation 7.2 becomes an AR model with order m . However, if the regression coefficient η is different from zero, then equation 7.2 is nonlinear. Using the expansion $\exp(x) \approx 1 + x$, which holds for x of small magnitude, it was seen that for small η , Y_t follows approximately a quadratic AR model:

$$Y_t = \theta_0 + \phi_1 Y_{t-1} + \dots + \phi_m Y_{t-m} + \exp\left\{\eta \left(\sum_{j=1}^m \phi_j Y_{t-j}\right)^2\right\} + \varepsilon_t \quad (7.3)$$

The test statistic $\hat{F} = \frac{\eta^2(n - 2m - 2)}{RSS - \eta^2}$ is approximately distributed as an F-distribution with degrees of freedom 1 and $n - 2m - 2$. The null of linearity is rejected if the p-value is less than 0.05.

7.3 Tsay Test

Tsay (1986) extended the Keenan test due to its limitations. As shown in Keenan (1985), although the Keenan test is robust in detecting nonlinearity in the form of the square of the approximating linear conditional mean function, the strength of the test is sometimes low. This limitation brought about Tsay (1986) test. The Tsay test was presented as in Cryer and Chan (2008).

Tsay replaced the term $\eta(\sum_{j=1}^m \phi_j Y_{t-j})^2$ of equation 7.2 by

$$\begin{aligned} & \exp\left(\varsigma_{1,1} Y_{t-1}^2 + \varsigma_{1,2} Y_{t-1} Y_{t-2} + \dots \varsigma_{1,m} Y_{t-1} Y_{t-m} + \right. \\ & \quad \varsigma_{2,2} Y_{t-2}^2 + \varsigma_{2,3} Y_{t-2} Y_{t-3} + \dots \varsigma_{2,m} Y_{t-2} Y_{t-m} + \dots + \\ & \quad \left. \varsigma_{m-1,m-1} Y_{t-m}^2 + \varsigma_{m-1,m} Y_{t-m+1} Y_{t-m} + \varsigma_{m,m} Y_{t-m}^2 + \varepsilon_t \right) \end{aligned} \quad (7.4)$$

From this approximation in equation 7.4, it can be observed that the nonlinear model is approximately a quadratic AR model but the coefficient of the quadratic terms are unconstrained. Therefore, the Tsay test considers the following quadratic regression model:

$$\begin{aligned} Y_t = & \theta_0 + \phi_1 Y_{t-1} + \dots \phi_m Y_{t-m} + \varsigma_{1,1} Y_{t-1}^2 + \varsigma_{1,2} Y_{t-1} Y_{t-2} + \dots \varsigma_{1,m} Y_{t-1} Y_{t-m} + \\ & \quad \varsigma_{2,2} Y_{t-2}^2 + \varsigma_{2,3} Y_{t-2} Y_{t-3} + \dots \varsigma_{2,m} Y_{t-2} Y_{t-m} + \dots + \\ & \quad \varsigma_{m-1,m-1} Y_{t-m}^2 + \varsigma_{m-1,m} Y_{t-m+1} Y_{t-m} + \varsigma_{m,m} Y_{t-m}^2 + \varepsilon_t \end{aligned} \quad (7.5)$$

and tests whether all $\frac{m(m+1)}{2}$ coefficients $\varsigma_{i,j} = 0$.

The autoregressive order m must be specified using the Akaike Information Criterion (AIC) to test the null of linearity. The null of linearity is rejected if the p-value is less than 0.05.

7.4 The BDS Linearity Test

The BDS test was developed by W.A. Brock, W. Dechert and J. Scheinkman in 1987 (hereafter BDS (1987)). The test is used for detecting serial dependence in time series, hence it can be used to detect nonlinearity. The null of independent and identically distributed (I.I.D) hypothesis is tested against an unspecified alternative. The procedure below was followed in computing the BDS test: Let y_i be a time series with N observations, which should be the first difference of the natural logarithms of the raw data. Thus

$$\{y_i\} = [y_1, y_2, y_3, \dots, y_N]. \quad (7.6)$$

A value of m called the 'embedding dimension' was selected and embed the time series into m -dimensional vectors by taking each m successive points in the series. This converts the series of scalars into a series of vectors with overlapping entries:

$$\begin{aligned} y_1^m &= (y_1, y_2, \dots, y_m) \\ y_2^m &= (y_2, y_3, \dots, y_{m+1}) \\ &\vdots \\ y_{N-m}^m &= (y_{N-m}, y_{N-m+1}, \dots, y_N) \end{aligned}$$

A correlation integral was computed. The correlation integral, which is a measure of the spatial correlation among the points by adding the number of pairs of points (i, j) where $1 \leq i \leq N$ and $1 \leq j \leq N$ in the m -dimensional space which are "close" in the sense that the points are within a radius of tolerance ε of each other.

$$C_{\varepsilon, m} = \frac{1}{N_m(N_m - 1)} \sum_{i \neq j} I_{i, j; \varepsilon} \quad (7.7)$$

where

$$I_{i, j; \varepsilon} = \begin{cases} 1, & \text{if } \|y_i^m - y_j^m\| \leq \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

BDS (1987) proved that if the time series is an IID

$$C_{\varepsilon, m} \approx [C_{\varepsilon, 1}]^m \quad (7.8)$$

According to Lin (1997), if the ratio $\frac{N}{m}$ is greater than 200, the values of $\frac{\varepsilon}{\sigma}$ range from 0.5 to 2 and the values of m are between 2 and 5 (Brock *et al.* (1988)). The quantity $[C_{\varepsilon,m} - (C_{\varepsilon,1})^m]$ has an asymptotic normal distribution with zero mean and a variance $V_{\varepsilon,m}$ defined as:

$$V_{\varepsilon,m} = 4 \left[K^m + 2 \sum_{j=1}^{m-1} K^{m-j} C_{\varepsilon}^{2j} + (m-1)^2 C_{\varepsilon}^{2m} - m^2 K C_{\varepsilon}^{2m-2} \right] \quad (7.9)$$

where

$$K = K_{\varepsilon} = \frac{6}{N_m(N_m-1)(N_m-2)} \sum_{i < j < N} h_{i,j,N;\varepsilon}; h_{i,j,N;\varepsilon} = \frac{[I_{i,j;\varepsilon} I_{j,N;\varepsilon} + I_{i,N;\varepsilon} I_{N,j;\varepsilon} + I_{j,i;\varepsilon} I_{i,N;\varepsilon}]}{3}$$

The BDS test statistic is thus stated as:

$$BDS_{\varepsilon,m} = \frac{\sqrt{N}[C_{\varepsilon,m} - (C_{\varepsilon,1})^m]}{\sqrt{V_{\varepsilon,m}}} \quad (7.10)$$

The null of independent and identically distributed (IID) was rejected at the 5% significance level if $|BDS_{\varepsilon,m}| > 1.96$.

7.5 Delay Vector Variance (DVV) Test

The delay vector variance method, hereafter, (DVV) was developed by *Gautama et al.* (2004a) for signal characterization. Characterizing signal nonlinearities have been adopted in predicting survival in heart failure cases, practical engineering situations (*Ho et al.*, 1997; Chambers and Mandic, 2001) and in economic time series (Caraianni (2011), *Addo et al.* 2013a, 2013b, 2013c). DVV is based on surrogate data and it is more suitable for signal processing application because the data-generating process can be described by a linear or non-linear equations theoretically. The combination with the concept of surrogate data gives an additional account of the nonlinear behavior of the time series. The DVV analysis calculates the target variance σ^{*2} which is an inverse measure of the predictability of a time series. The summarized algorithm of the DVV analysis is presented below:

1. For an optimal embedding dimension m which are obtained via a differential entropy based method using wavelet-based surrogates and time lag τ , generate delay vector (DV) : $y(k) = [y_{k-\tau}, \dots, y_{k-m\tau}]$ and corresponding target y_t

2. The mean μ_d and standard deviation σ_d are computed over all pairwise distances between DVs, $\|y(t) - y(j)\|$ for $t \neq j$
3. The sets Ω_k are generated such that $\Omega_k = \{y(t) \mid \|y(k) - y(t)\| \leq \varrho_d\}$. That is sets which consists of all DVs that lie closer to $x(k)$ than a certain ϱ_d taken from the interval $[\min\{0, \mu_d - n_d\sigma_d\}; \mu_d + n_d\sigma_d]$, for example, uniformly spaced, where n_d is a parameter controlling the span over which to perform the DVV analysis.
4. For every set Ω_k , the variance of the corresponding targets σ_k^2 is computed. The average over all sets Ω_k normalized by the variance of the time series, σ_k^2 yields the target variance σ^{*2} as:

$$\sigma^{*2} = \frac{\frac{1}{N} \sum_{k=1}^N \sigma_k^2(\varrho_d)}{\sigma_x^2} \quad (7.11)$$

where N denotes the total number of sets $\Omega_k(\varrho_d)$.

The DVV analysis can be represented by graphically plotting of $\sigma^{*2}(\varrho_d)$ as a function of the standardized distance (ϱ_d). The minimum target variance $\sigma_{\min}^{*2} = \min_{(\varrho_d)}[\sigma^{*2}(\varrho_d)]$. A measure of the amount of noise present corresponds to the lowest point of the curve.

For a DVV analysis where the surrogate and the original time series provide similar plots, the series is said to be linear or non-linear. Also, because of the standardization of the distance axis, the plots can be combined within a scatter diagram, where the horizontal axis corresponds to the DVV plot of the original time series and the vertical axis corresponds to the surrogate time series. If the DVV scatter diagram deviates from the bisector line, the series is said to be non-linear.

7.6 Non-linear SETAR Model

The SETAR model is the simplest form of threshold autoregressive models (TAR). Tong (1978) and Tong and Lim (1980) proposed TAR models where the regime was determined by the value of an observable variable relative to a threshold value. The SETAR model can account for conditional heteroscedasticity in the data because the error variance may be different in the regimes.

The researcher employed a SETAR model of order two for the analysis. The SETAR model is

presented in summary from Cryer and Chan (2008) as below:

$$y_t = \begin{cases} \mu_{1,0} + \rho_{1,1}y_{t-1} + \rho_{1,2}y_{t-2} + \sigma_1\varepsilon_t, & \text{if } y_{t-1} \text{ and } y_{t-2} < \theta \\ \mu_{2,0} + \rho_{2,1}y_{t-1} + \rho_{2,2}y_{t-2} + \sigma_2\varepsilon_t, & \text{if } \theta < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.12)$$

where ρ are the autoregressive parameters, σ are noise standard deviations, θ is the threshold parameter and ε is a sequence of i.i.d random variables with mean 0 and variance 1. Therefore, if the lag 1 and lag 2 values of y_t are not greater than the threshold, then the conditional distribution of y_t is similar to the first AR (2) process and we are in the lower regime, else the second AR (2) model is operational and we will be in the upper regime. This means the process switches between two linear models depending on the position of the lag 1 and lag 2 values.

7.7 Results

The results of the summary statistics as referred to in Table 3.1 indicate that the return series is non-normal with high values of kurtosis and highly skewed. The Jarque - Bera test statistic null hypothesis is rejected at the 1% level of significance. The Augmented Dickey-Fuller (ADF) test result shows that the return series for each country is stationary. The AIC was used to select autoregressive order m in testing for linearity using the Keenan, Tsay, BDS and DVV tests. The autoregressive orders m for the eight indices are given in Table 7.1:

Market	Botswana	Egypt	Kenya	Mauritius	Morocco	Nigeria	South Africa	Tunisia
Autoregressive Order (m)	23	10	34	31	1	10	7	31

Table 7.1: Autoregressive order for returns

Keenan Test Results

Market	Test Statistic	m	P-value
Botswana	22.147	23	2.6134E-6
Egypt	7.074	10	0.00785
Kenya	64.196	34	1.48E-15
Mauritius	1.079	31	0.2988
Morocco	0.288	1	0.5912
Nigeria	0.05	10	0.816
South Africa	3.5127	7	0.0609
Tunisia	5.7802	31	0.016

Table 7.2: Keenan Test Results for returns

Tsay Test Results

Market	Test Statistic	m	P-value
Botswana	0.7054	23	0.999
Egypt	2.485	10	1.05E-8
Kenya	3.835	34	1.14E-132
Mauritius	6.11	31	9.11E-231
Morocco	1.208	1	0.2717
Nigeria	33.72	10	0.000
South Africa	3.378	7	5.06E-9
Tunisia	2.94	31	7.80E-73

Table 7.3: Tsay Test Results for returns

BDS Test Results for Botswana

```

BDS Test
data:  retbot
Embedding dimension =  2 3
Epsilon for close points =  0.0017 0.0034 0.0051 0.0068
Standard Normal =
      [ 0.0017 ] [ 0.0034 ] [ 0.0051 ] [ 0.0068 ]
[ 2 ]      6.3381      0.2082      -0.7931      -0.7678
[ 3 ]      7.3133      0.3456      -1.5965      -1.4001
p-value =
      [ 0.0017 ] [ 0.0034 ] [ 0.0051 ] [ 0.0068 ]
[ 2 ]           0      0.8351      0.4277      0.4426
[ 3 ]           0      0.7297      0.1104      0.1615

```

BDS Test Results for Egypt

```

BDS Test
data:  retegy
Embedding dimension =  2 3
Epsilon for close points =  0.0037 0.0073 0.0110 0.0147
Standard Normal =
      [ 0.0037 ] [ 0.0073 ] [ 0.011 ] [ 0.0147 ]
[ 2 ]     12.8425     15.4025     17.2053     17.4617
[ 3 ]     17.9674     20.2909     21.5552     20.8346
p-value =
      [ 0.0037 ] [ 0.0073 ] [ 0.011 ] [ 0.0147 ]
[ 2 ]           0           0           0           0
[ 3 ]           0           0           0           0

```

BDS Test Results for Kenya

BDS Test
data: retken
Embedding dimension = 2 3
Epsilon for close points = 0.002 0.004 0.006 0.008
Standard Normal =
[0.002] [0.004] [0.006] [0.008]
[2] 19.0538 24.8017 26.8223 26.7647
[3] 21.6966 27.7344 29.5227 28.9204
p-value =
[0.002] [0.004] [0.006] [0.008]
[2] 0 0 0 0
[3] 0 0 0 0

BDS Test Results for Mauritius

BDS Test
data: retmau
Embedding dimension = 2 3
Epsilon for close points = 0.0015 0.0029 0.0044 0.0058
Standard Normal =
[0.0015] [0.0029] [0.0044] [0.0058]
[2] 24.5888 28.0327 27.9632 26.1034
[3] 28.5608 30.9452 30.0497 27.4644
p-value =
[0.0015] [0.0029] [0.0044] [0.0058]
[2] 0 0 0 0
[3] 0 0 0 0

BDS Test Results for Morocco

BDS Test
data: retmor
Embedding dimension = 2 3
Epsilon for close points = 0.0017 0.0033 0.0050 0.0067
Standard Normal =
[0.0017] [0.0033] [0.005] [0.0067]
[2] 17.0785 19.2109 20.4631 20.1491
[3] 21.5662 23.1873 24.1205 23.3655
p-value =
[0.0017] [0.0033] [0.005] [0.0067]
[2] 0 0 0 0
[3] 0 0 0 0

BDS Test Results for Nigeria

BDS Test
data: retnig
Embedding dimension = 2 3
Epsilon for close points = 0.0029 0.0058 0.0087 0.0116
Standard Normal =
[0.0029] [0.0058] [0.0087] [0.0116]
[2] 25.8292 30.0394 32.2699 32.7657
[3] 29.3465 32.3744 32.7138 32.5435
p-value =
[0.0029] [0.0058] [0.0087] [0.0116]
[2] 0 0 0 0
[3] 0 0 0 0

BDS Test Results for South Africa

BDS Test

data: retsa

Embedding dimension = 2 3

Epsilon for close points = 0.0026 0.0052 0.0079 0.0105

Standard Normal =

	[0.0026]	[0.0052]	[0.0079]	[0.0105]
[2]	9.7299	10.8777	11.7043	11.7907
[3]	13.9980	15.5036	16.8803	17.3351

p-value =

	[0.0026]	[0.0052]	[0.0079]	[0.0105]
[2]	0	0	0	0
[3]	0	0	0	0

BDS Test Results for Tunisia

BDS Test

data: rettun

Embedding dimension = 2 3

Epsilon for close points = 0.0012 0.0023 0.0035 0.0046

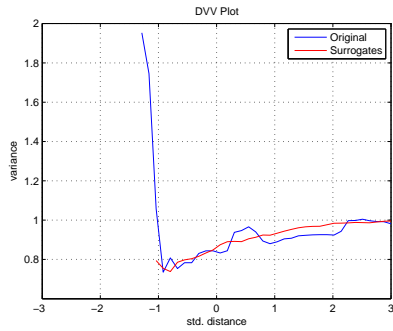
Standard Normal =

	[0.0012]	[0.0023]	[0.0035]	[0.0046]
[2]	16.4955	19.7912	22.7766	25.0425
[3]	18.8614	22.0569	24.5979	26.2061

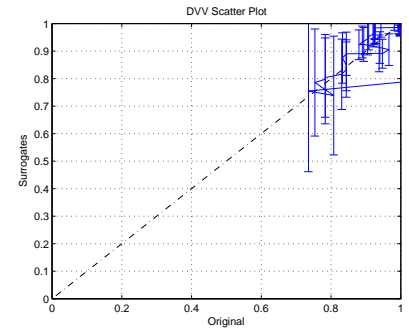
p-value =

	[0.0012]	[0.0023]	[0.0035]	[0.0046]
[2]	0	0	0	0
[3]	0	0	0	0

DVV Test Results

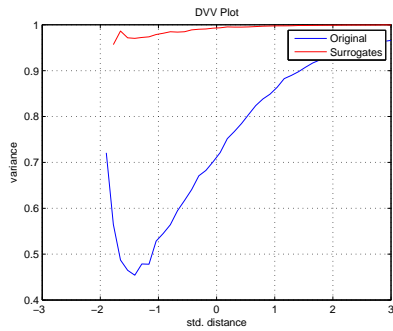


(a) DVV Plot

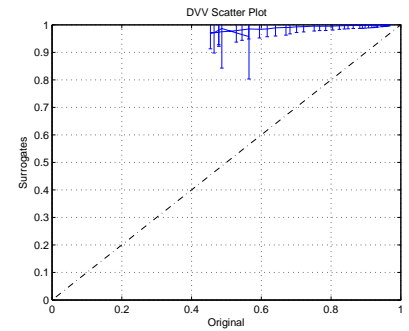


(b) DVV Scatter Plot

Figure 7.1: DVV Plot and DVV Scatter Plot of Botswana

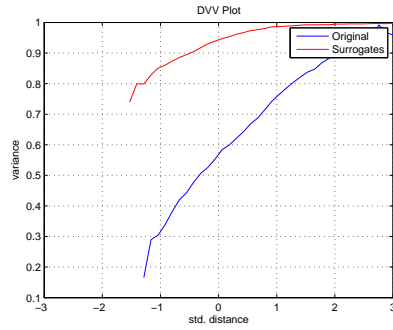


(a) DVV Plot

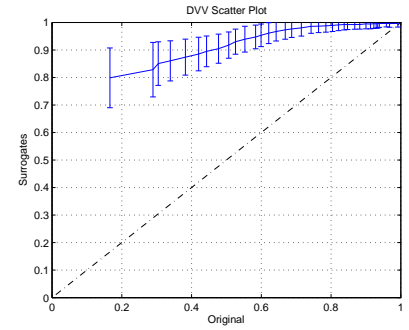


(b) DVV Scatter Plot

Figure 7.2: DVV Plot and DVV Scatter Plot of Egypt

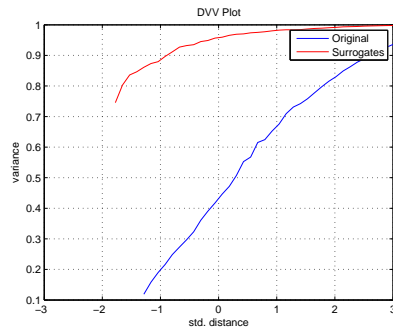


(a) DVV Plot

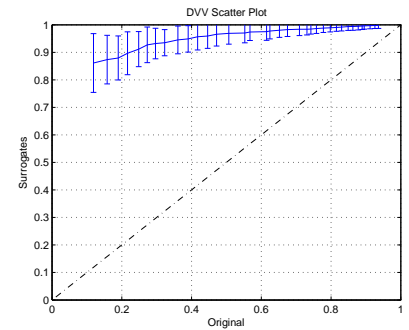


(b) DVV Scatter Plot

Figure 7.3: DVV Plot and DVV Scatter Plot for Kenya

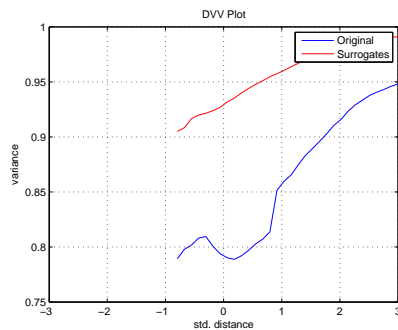


(a) DVV Plot

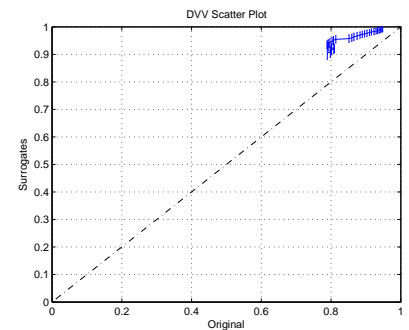


(b) DVV Scatter Plot

Figure 7.4: DVV Plot and DVV Scatter Plot of Mauritius

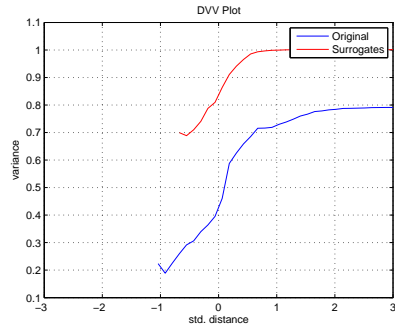


(a) DVV Plot

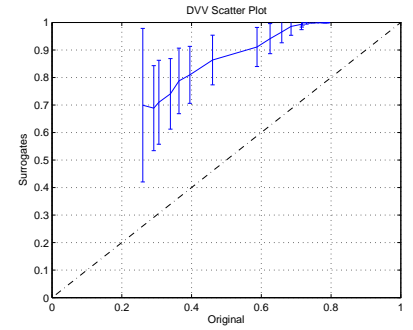


(b) DVV Scatter Plot

Figure 7.5: DVV Plot and DVV Scatter Plot of Morocco

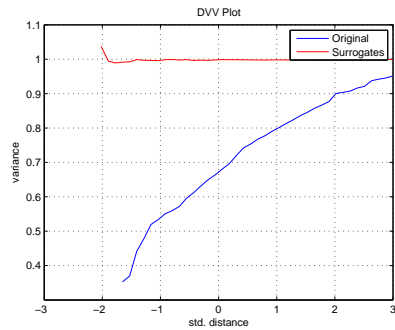


(a) DVV Plot

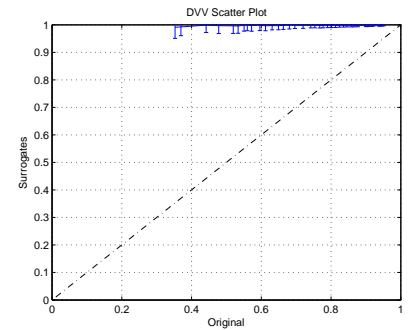


(b) DVV Scatter Plot

Figure 7.6: DVV Plot and DVV Scatter Plot of Nigeria

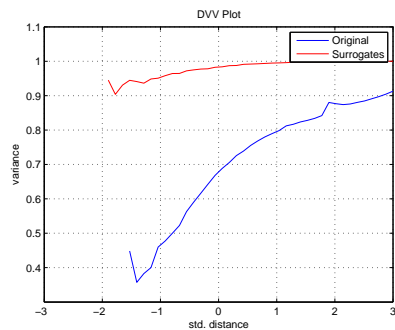


(a) DVV Plot

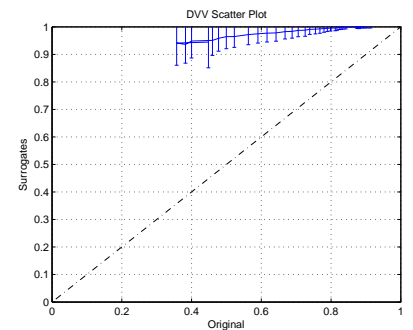


(b) DVV Scatter Plot

Figure 7.7: DVV Plot and DVV Scatter Plot of South Africa



(a) DVV Plot



(b) DVV Scatter Plot

Figure 7.8: DVV Plot and DVV Scatter Plot of Tunisia

The results of using the Keenan test as shown in Table 7.3 failed to reject the null of linearity for Mauritius, Morocco, Nigeria and South Africa, but, the Tsay test in Table 7.4 rejected the null of linearity for Egypt, Kenya, Mauritius, Nigeria, South Africa and Tunisia but not for Botswana and Morocco.

Notwithstanding these mixed conclusions about the linearity of the returns under study, the results from the BDS test showed that, the null of linearity was rejected in all the indices in almost all the epsilon for close points values. The results gave an indication of the presence of nonlinearity in the returns because the p-values were less than 0.05.

Also the DVV analysis with iAAFT surrogates performed on the returns of the eight indices via the differential entropy-based method (*Gautama et al. 2003*) showed that there was a clear deviation from the bisector line on the DVV scatter plots in Figures 7.1 to 7.8. The DVV plots in Figures 7.1 to 7.8 also showed that non-linear patterns existed in the series.

Since nonlinearity existed in the data, the data was modelled under the SETAR model with 2-regimes, low and high. The 2-regime SETAR was chosen after observing the data and concluding that investors on these markets mostly traded in stocks (equity) and bonds. An autoregressive order of 2 for the low and high regimes was chosen because it gave the minimum AIC value after a grid search with different autoregressive orders. The findings after fitting a SETAR (2,2,2) to each of the data are presented below:

SETAR model-Botswana

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 15.03% High regime: 84.97%

Fit:

residuals variance = 6.025e-05, AIC = -38028, MAPE = 203.6%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	-0.00076033	0.00035977	-2.1134	0.03463	*
phiL.1	0.13623172	0.06924517	1.9674	0.04921	*
phiL.2	-0.01373384	0.03624262	-0.3789	0.70475	
const.H	0.00082280	0.00013801	5.9619	2.714e-09	***
phiH.1	0.00344975	0.01642538	0.2100	0.83366	
phiH.2	-0.00635298	0.01858255	-0.3419	0.73246	

Threshold

Variable: Z(t) = + (0) X(t) + (1) X(t-1)

Value: -0.0001436

The SETAR (2,2,2) model for Botswana is as thus:

$$y_t = \begin{cases} -0.00076033 + 0.13623172y_{t-1} - 0.01373384y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq -0.0001436 \\ 0.00082280 + 0.00344975y_{t-1} - 0.00635298y_{t-2}, & -0.0001436 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.13)$$

SETAR model - Egypt

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 84.56% High regime: 15.44%

Fit:

residuals variance = 0.0002765, AIC = -32063, MAPE = 156.8%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	0.00056801	0.00030099	1.8871	0.05922	.
phiL.1	0.18088177	0.01769691	10.2211	< 2e-16	***
phiL.2	0.00517104	0.02163782	0.2390	0.81113	
const.H	0.00214427	0.00152232	1.4086	0.15905	
phiH.1	0.06535656	0.03716910	1.7584	0.07876	.
phiH.2	-0.06334693	0.05480155	-1.1559	0.24778	

Threshold

Variable: $Z(t) = + (0) X(t) + (1) X(t-1)$

Value: 0.01433

The SETAR (2,2,2) model for Egypt is as thus:

$$y_t = \begin{cases} 0.00056801 + 0.18088177y_{t-1} + 0.00517104y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq 0.01433 \\ 0.00214427 + 0.06535656y_{t-1} - 0.06334693y_{t-2}, & 0.01433 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.14)$$

SETAR model - Kenya

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 19.01% High regime: 80.99%

Fit:

residuals variance = 7.5e-05, AIC = -37171, MAPE = 298.8%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	-0.00504608	0.00050553	-9.9817	< 2.2e-16	***
phiL.1	0.07354410	0.02633902	2.7922	0.005260	**
phiL.2	-0.20579638	0.03456380	-5.9541	2.845e-09	***
const.H	0.00045986	0.00016554	2.7780	0.005496	**
phiH.1	0.36162571	0.01984757	18.2202	< 2.2e-16	***
phiH.2	0.03018910	0.02293914	1.3161	0.188234	

Threshold

Variable: $Z(t) = + (0) X(t) + (1) X(t-1)$

Value: -0.00476

The SETAR (2,2,2) model for Kenya is as thus:

$$y_t = \begin{cases} -0.00504608 + 0.07354410y_{t-1} - 0.20579638y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq -0.00476 \\ 0.00045986 + 0.36162571y_{t-1} + 0.03018910y_{t-2}, & -0.00476 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.15)$$

SETAR model - Mauritius

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 17.74% High regime: 82.26%

Fit:

residuals variance = 4.105e-05, AIC = -39530, MAPE = 184.5%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	-0.00200680	0.00035320	-5.6818	1.430e-08	***
phiL.1	0.11009755	0.02500654	4.4027	1.097e-05	***
phiL.2	-0.17156558	0.03320427	-5.1670	2.498e-07	***
const.H	0.00048442	0.00012235	3.9594	7.646e-05	***
phiH.1	0.32885272	0.02080128	15.8093	< 2.2e-16	***
phiH.2	-0.05413442	0.02282509	-2.3717	0.01775	*

Threshold

Variable: Z(t) = + (0) X(t) + (1) X(t-1)

Value: -0.002745

The SETAR (2,2,2) model for Mauritius is as thus:

$$y_t = \begin{cases} -0.00200680 + 0.11009755y_{t-1} - 0.17156558y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq -0.002745 \\ 0.00048442 + 0.32885272y_{t-1} - 0.05413442y_{t-2}, & -0.002745 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.16)$$

SETAR model - Morocco

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 16.46% High regime: 83.54%

Fit:

residuals variance = 5.407e-05, AIC = -34994, MAPE = 197.5%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	-0.00230565	0.00055145	-4.1811	2.971e-05	***
phiL.1	0.31374992	0.03009814	10.4242	< 2.2e-16	***
phiL.2	-0.23495503	0.04328174	-5.4285	6.063e-08	***
const.H	0.00013249	0.00014748	0.8984	0.3690	
phiH.1	0.26239769	0.02006062	13.0802	< 2.2e-16	***
phiH.2	0.02991656	0.02480337	1.2061	0.2278	

Threshold

Variable: Z(t) = + (0) X(t) + (1) X(t-1)

Value: -0.005079

The SETAR (2,2,2) model for Morocco is as thus:

$$y_t = \begin{cases} -0.00230565 + 0.31374992y_{t-1} - 0.23495503y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq -0.005079 \\ 0.00013249 + 0.26239769y_{t-1} + 0.02991656y_{t-2}, & -0.005079 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.17)$$

SETAR model - Nigeria

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 84.82% High regime: 15.18%

Fit:

residuals variance = 0.0001548, AIC = -34334, MAPE = 2811%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	0.00047491	0.00022435	2.1168	0.03434	*
phiL.1	0.25693123	0.01935607	13.2739	< 2e-16	***
phiL.2	0.20726311	0.02176754	9.5217	< 2e-16	***
const.H	0.00683843	0.00073207	9.3412	< 2e-16	***
phiH.1	-0.44174126	0.02518455	-17.5402	< 2e-16	***
phiH.2	-0.03420134	0.03031423	-1.1282	0.25929	

Threshold

Variable: Z(t) = + (0) X(t) + (1) X(t-1)

Value: 0.008558

The SETAR (2,2,2) model for Nigeria is as thus:

$$y_t = \begin{cases} 0.00047491 + 0.25693123y_{t-1} + 0.20726311y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq 0.008558 \\ 0.00683843 - 0.44174126y_{t-1} - 0.03420134y_{t-2}, & 0.008558 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.18)$$

SETAR model - South Africa

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 30.95% High regime: 69.05%

Fit:

residuals variance = 0.0001447, AIC = -34598, MAPE = 119.2%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	0.00182733	0.00060257	3.0326	0.002441	**
phiL.1	0.00516987	0.02538010	0.2037	0.838600	
phiL.2	0.07776593	0.03992040	1.9480	0.051483	.
const.H	0.00028302	0.00028845	0.9812	0.326560	
phiH.1	0.06074657	0.02059621	2.9494	0.003203	**
phiH.2	-0.00976344	0.02761955	-0.3535	0.723735	

Threshold

Variable: $Z(t) = + (0) X(t) + (1) X(t-1)$

Value: -0.003741

The SETAR (2,2,2) model for South Africa is as thus:

$$y_t = \begin{cases} 0.00182733 + 0.00516987y_{t-1} + 0.07776593y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq -0.003741 \\ 0.00028302 + 0.06074657y_{t-1} - 0.00976344y_{t-2}, & -0.003741 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.19)$$

SETAR model - Tunisia

Non linear autoregressive model

SETAR model (2 regimes)

Proportion of points in low regime: 15.08% High regime: 84.92%

Fit:

residuals variance = 2.596e-05, AIC = -41324, MAPE = 246.5%

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t)	
const.L	-8.5174e-05	3.5466e-04	-0.2402	0.8102230	
phiL.1	9.6695e-02	2.8191e-02	3.4300	0.0006099	***
phiL.2	5.1073e-02	3.8953e-02	1.3111	0.1898850	
const.H	2.8319e-04	9.6376e-05	2.9384	0.0033185	**
phiH.1	3.2410e-01	1.9307e-02	16.7870	< 2.2e-16	***
phiH.2	7.8315e-04	2.3041e-02	0.0340	0.9728875	

Threshold

Variable: Z(t) = + (0) X(t) + (1) X(t-1)

Value: -0.003485

The SETAR (2,2,2) model for Tunisia is as thus:

$$y_t = \begin{cases} -8.5174e - 05 + 9.6695e - 02y_{t-1} + 5.1073e - 02y_{t-2}, & y_{t-1} \text{ and } y_{t-2} \leq -0.003485 \\ 2.8319e - 04 + 3.2410e - 01y_{t-1} + 7.8315e - 04y_{t-2}, & -0.003485 < y_{t-1} \text{ and } y_{t-2} \end{cases} \quad (7.20)$$

The results from the fitted SETAR (2,2,2) model showed stationarity because the coefficients in the low and high regimes are less than 1 and also the product of the coefficients in the low and high regimes were less than 1. Thus the necessary and sufficient condition as in *Chan et al.* (1985) is satisfied.

The SETAR (2,2,2) model results for Botswana, Kenya, Mauritius, Morocco, South Africa and Tunisia showed that the number of observations in the high regime were more than that in the low regime. The percentage of points in the high and low regimes were graphically shown in Figures 7.9, 7.11, 7.12, 7.13, 7.15 and 7.16, respectively. This meant that, for these countries, growth in returns were decreasing in the low regime with negative coefficients. Agents main aim is to make profit hence they are found more in the high regime because of the high opportunities. This is because the coefficients in the high regime for these countries are positive, an indication of an increasing rate of returns in the high regime.

The results for Egypt and Nigeria were however opposite to the other countries. The number of observations were higher in the low regime than in the high regime. Returns were increasing in the low regime because coefficients were positive, making it risky for agents to trade in the high regime. Regime switching plots for Egypt and Nigeria are presented in Figures 7.10 and 7.14 respectively.

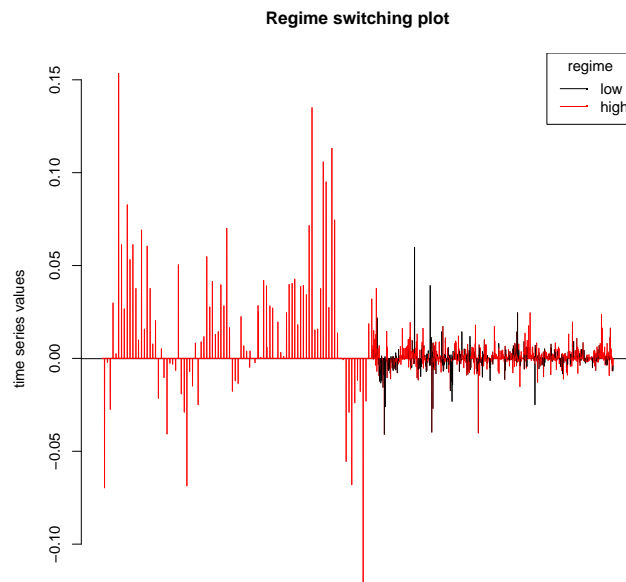


Figure 7.9: Regime Switching plot of Botswana

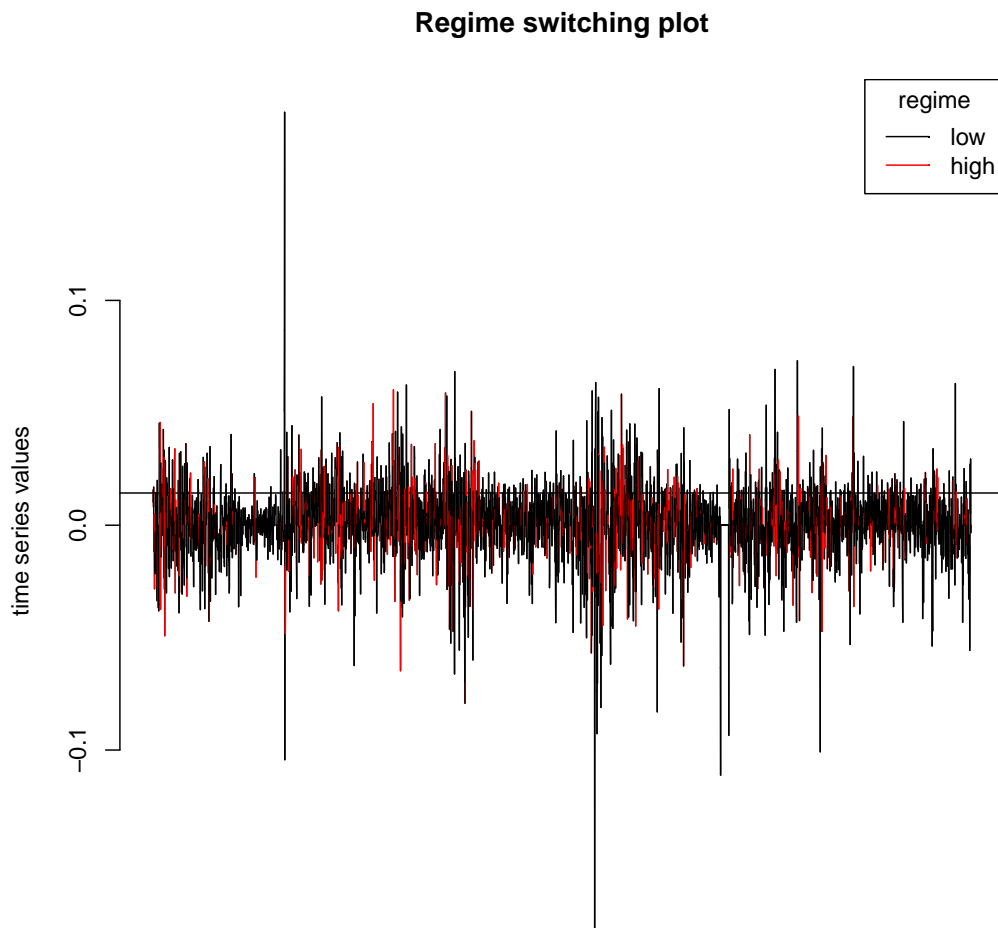


Figure 7.10: Regime Switching plot of Egypt

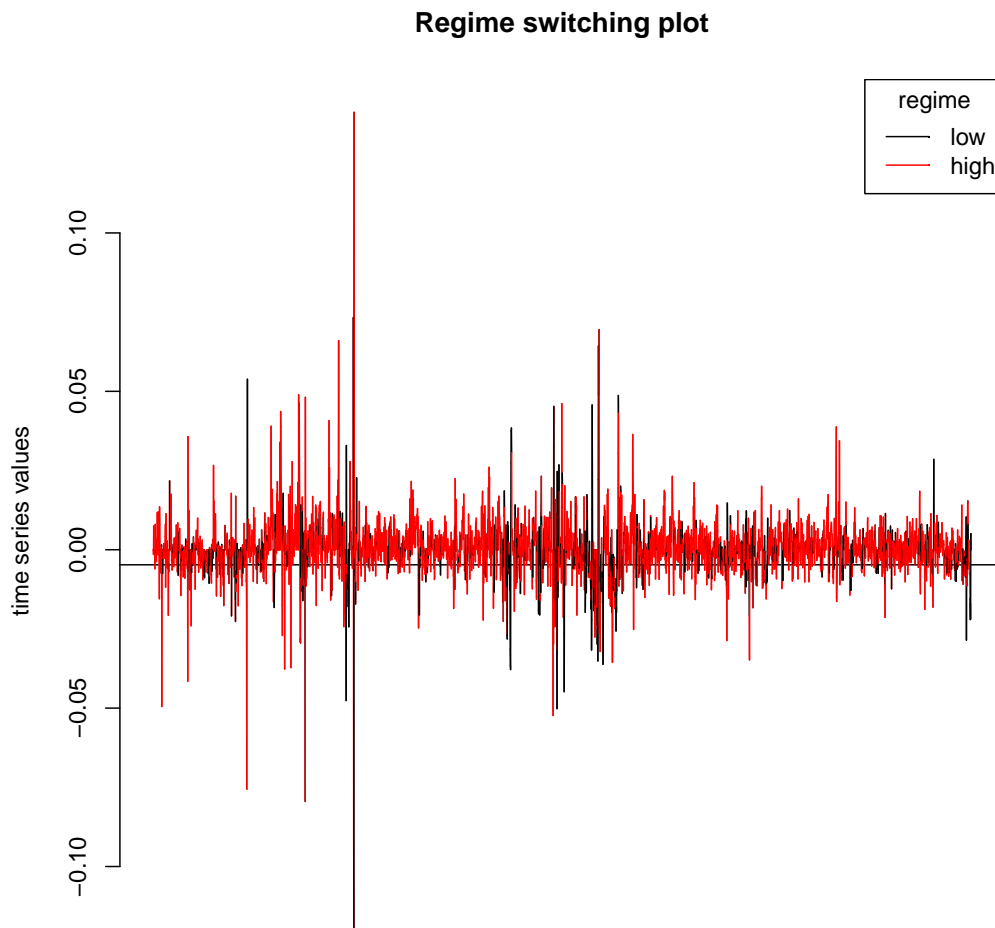


Figure 7.11: Regime Switching plot of Kenya

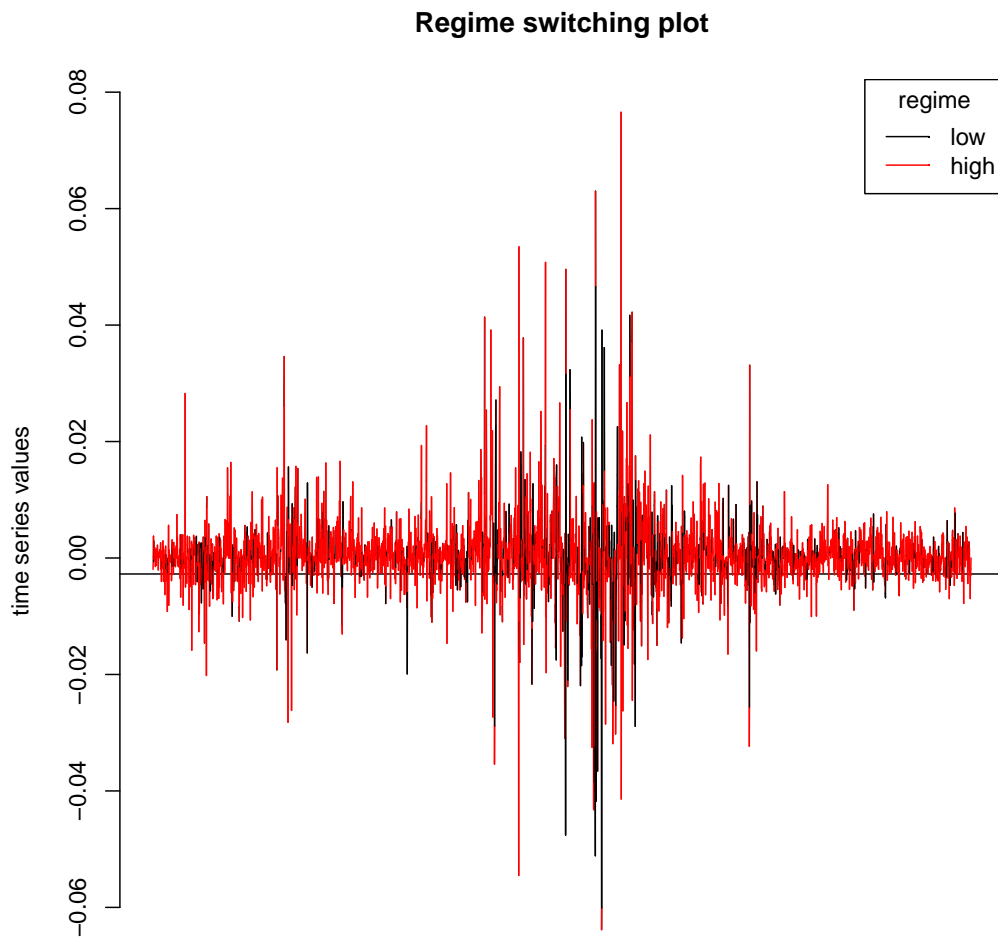


Figure 7.12: Regime Switching plot of Mauritius

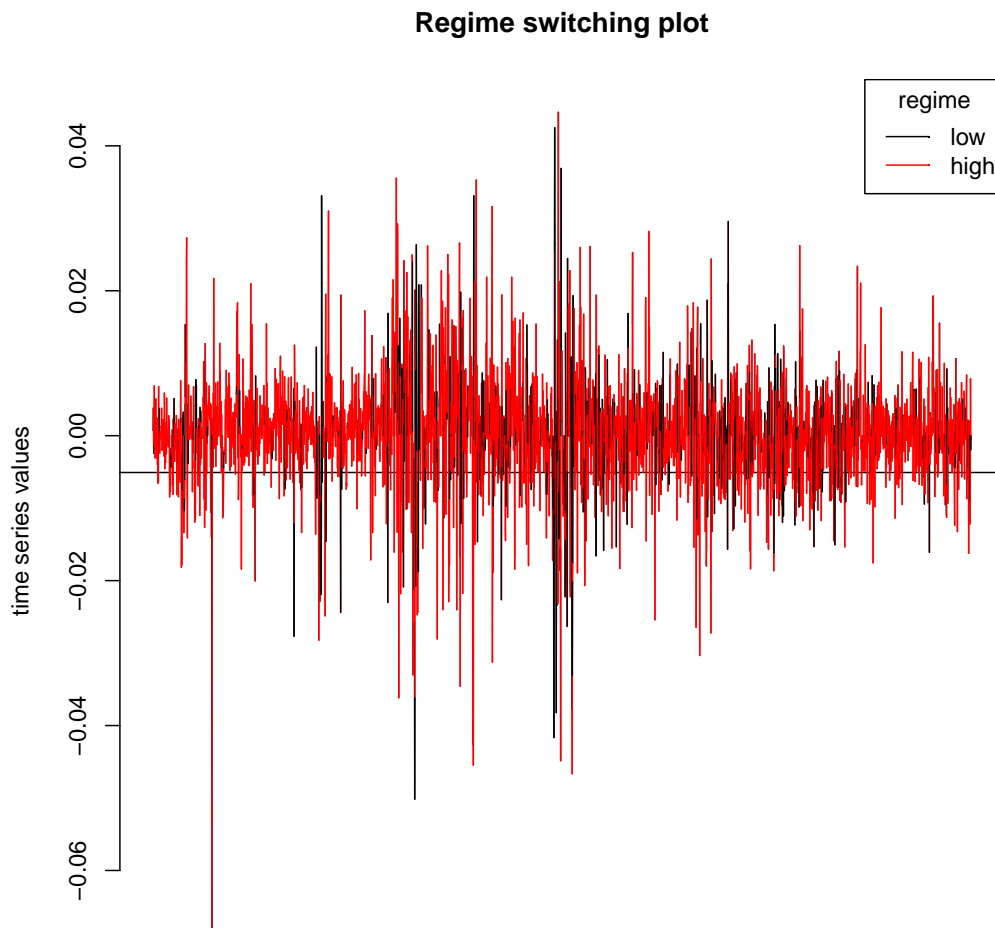


Figure 7.13: Regime Switching plot of Morocco

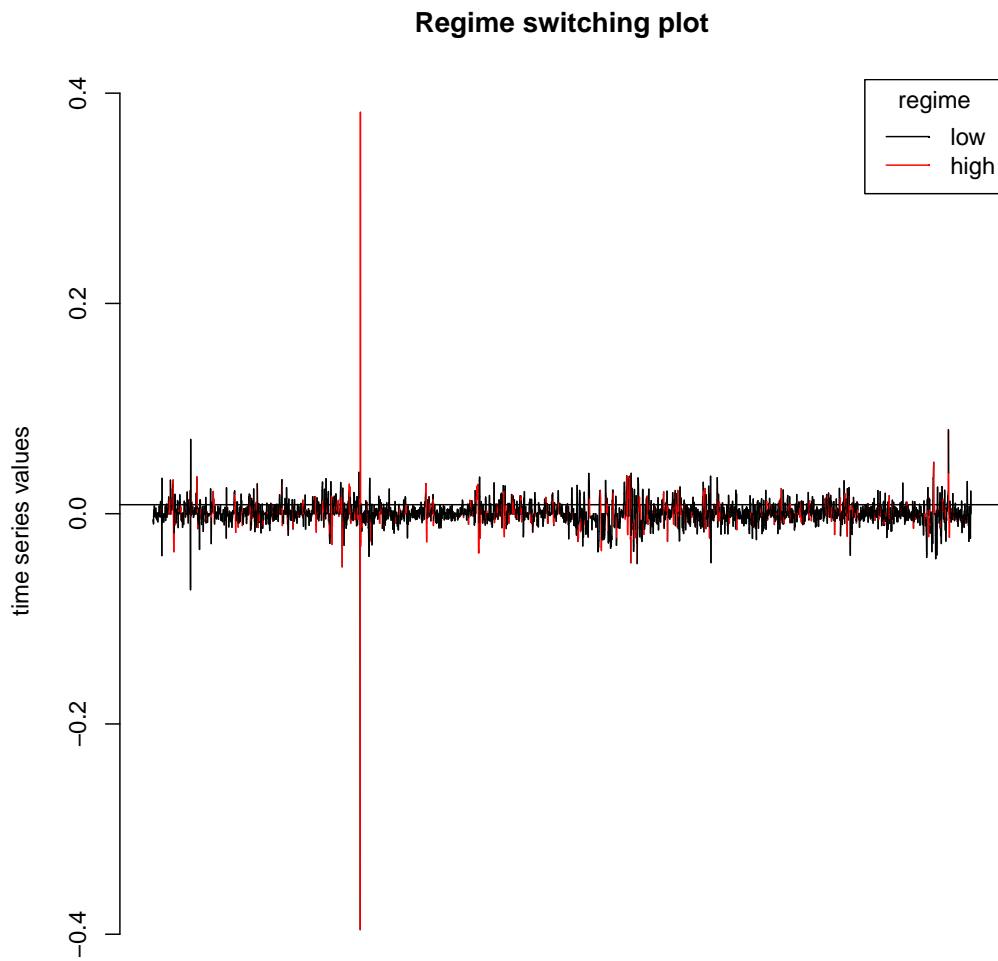


Figure 7.14: Regime Switching plot of Nigeria

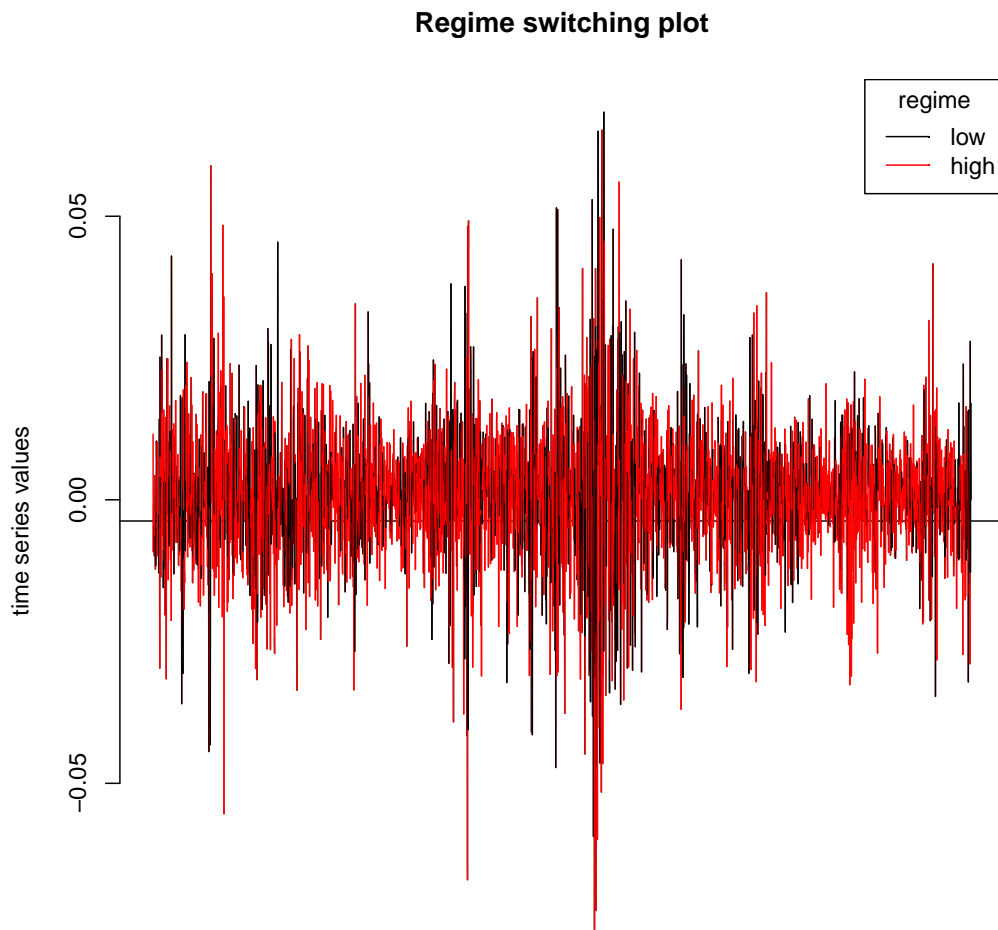


Figure 7.15: Regime Switching plot of South Africa

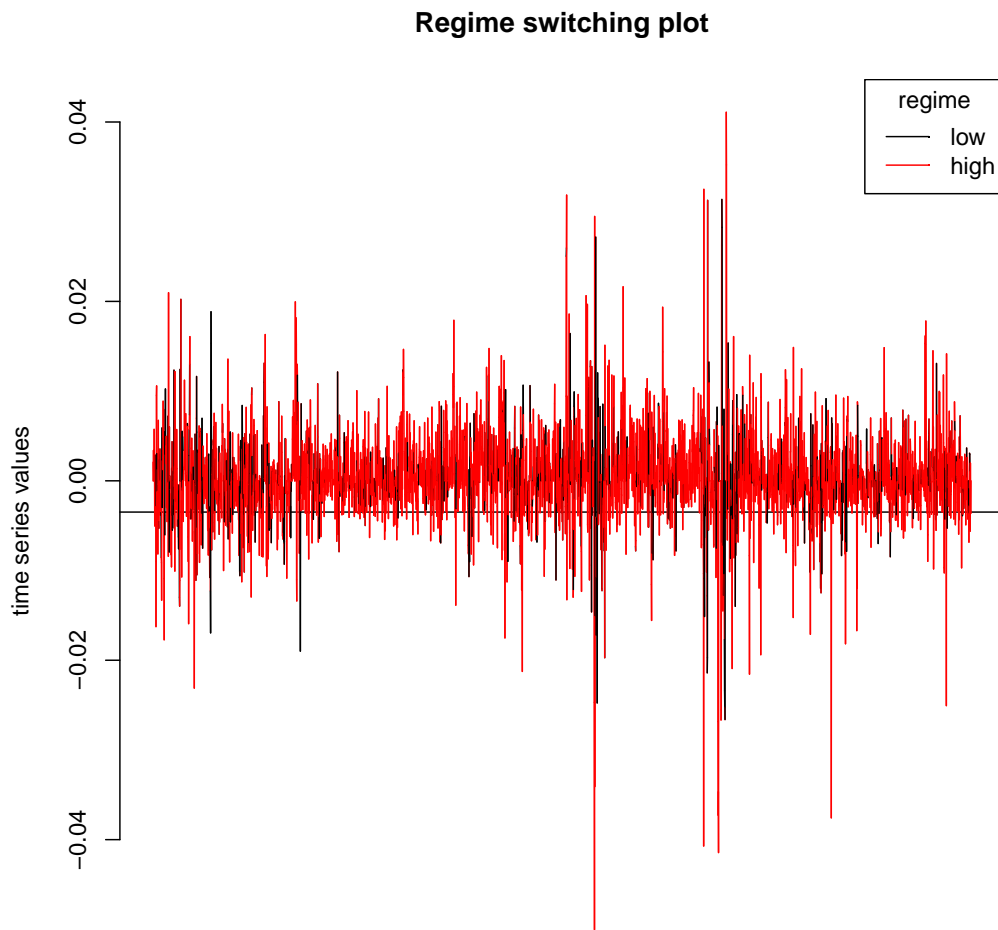


Figure 7.16: Regime Switching plot of Tunisia

Modelling Performance

The modelling performance of the SETAR (2,2,2) model with the standard AR(1) and AR(2) models were compared to confirm whether it was appropriate to model stock market returns from these eight African countries with the non-linear SETAR model. The AR(1) and AR(2) models were used as benchmarked models. This is because the conditional distribution of y_t is similar to the first AR (2) process if the lag 1 and lag 2 values of y_t are less than or greater than the threshold. Comparing AIC values as shown in Table 7.5, it was concluded that the SETAR (2,2,2) model performed and fitted the data better than the standard AR(1) and AR(2) models.

Market	SETAR(2,2,2)	AR(1)	AR(2)
Botswana	-44558.35	-44543.59	-44542.2
Egypt	-38593.26	-38590.85	-38591.53
Kenya	-43701.44	-43540.14	-43550.70
Mauritius	-46060.94	-45988.24	-45988.84
Morocco	-40936.85	-40913.05	-40915.28
Nigeria	-40864.39	-40328.03	-40378.11
South Africa	-41128.78	-41129.39	-41127.71
Tunisia	-47854.32	-47813.38	-47817.11

Table 7.4: AIC values

7.8 Discussion and Conclusions

It has been shown that non-linear patterns existed in the returns of the stock markets in Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Tunisia by employing four linearity tests- Keenan (1985), Tsay (1986), BDS (1987) and DVV (2004) tests. The BDS and DVV tests consistently detected nonlinearities in the returns series unlike the Keenan and Tsay tests which failed to reject the null of linearity in the returns of Mauritius, Morocco, Nigeria and South Africa (for Keenan test) and Botswana as well as Morocco (for Tsay test).

The returns series were modelled using the SETAR model. The AIC results of the SETAR model

were compared with that of the standard AR (1) and AR (2) models which were chosen as our benchmarked models. It was observed that, the SETAR model fitted the data better than the AR (1) and AR (2) models since the AIC values are largely negative. Therefore, the returns on the stock markets of these eight African countries are best modelled with non-linear models like the SETAR model, otherwise, the results might lead to spurious conclusions.

8. Market Efficiency of African Stock markets: A Meta-Analysis

Abstract

Investments on African stock markets have grown over the years. This is because stock markets in Africa have realized increase in market capitalization, membership, value and volume traded. Notwithstanding these increases, empirical studies on informational efficiency have had mixed conclusions about the markets necessitating the question: Are these mixed conclusions a result of variations in study characteristics? This chapter aims to find out if study characteristics have a probability of concluding efficiency of African stock markets. Previous studies of informational efficiency of African stock markets were quantitatively reviewed by means of meta-analysis. The mixed effect logistic meta-regression model was employed to examine which of the study characteristics is significant in concluding a market to be efficient. The mixed effect logistic model was chosen because it contains both fixed effects and random effects. The model explains an outcome as a linear combination of fixed and conditional random effects. The fixed effects assume equal influence of explanatory variables on an outcome whilst the random effects assume variations amongst observations when analyzing relationships between an outcome and explanatory variables. The results showed that only the indicator for publication bias is significant at the 5% level and that none of the study characteristics is significant in concluding efficiency of African stock markets. The indicator for publication bias being significant means the analysis suffers from publication bias. This implies that there has been a change in attitude in recent years towards studies on informational market efficiency whose results do not validate the Efficient Market Hypothesis (EMH), unlike the earlier years when the EMH was formulated and acclaimed to be one of the best propositions in economics. Although, none of the study characteristics was statistically significant in efficiency conclusions, it was observed that stock markets in Africa are about four times more likely to conclude that the markets are efficient if the study tested the weak-form rather than the semi-strong form of the EMH. The results have important implications in that, efficiency conclusions of African stock markets do not depend on any of these study characteristics, therefore, traders cannot devise strategies to outperform the markets based on any of these study characteristics. It was therefore concluded that informational efficiency conclusions

of African stock markets are context-specific which occurs randomly and not based on any of the study characteristics.

Keywords: Informational efficiency, African stock markets, Meta-analysis, Mixed Logistic model

8.1 Introduction

Market efficiency is in three forms. It can be informational, allocative or operational. A market is informationally efficient if all available information is instantaneously incorporated in setting prices. This means that all participants are privy to information on the market and therefore abnormal returns cannot be made using information on the market.

Operational efficiency is concerned with transaction costs on the market. It talks about how liquid the market is, that is, how easily it is for assets on the market to be bought and sold without their value being lost. Allocative efficiency is when investors provide funds for projects with the highest present value. Allocative efficiency depends on the degree of informational and operational efficiency.

This study focused on informational efficiency of African stock markets.

The Efficient Market Hypothesis (EMH) postulated by Fama (1970b) grouped information on the market into three: weak, semi-strong and strong forms. With weak-form efficiency, prices on the market reflect information on past prices. Thus the information set includes only the history of prices or returns themselves, hence, a market is said to be weak - form efficient if a participant cannot make abnormal returns by only analysing past prices.

Semi-strong form efficiency has information set which include past price history and public information, such as, interest rates, exchange rates, annual earnings and dividends announcements, among others. In this case, participants cannot earn abnormal profits by analysing macroeconomic and financial data or any other public information about the market.

The information set on the strong form efficiency is extensive to include all private information known to any market participant, therefore, even those with privileged or inside information cannot use it to make abnormal profits. This means that private or inside information is difficult to access for making abnormal returns because it is highly competitive to have such information, thus, there is perfect incorporation of all private information in market prices.

A review of the extant literature on informational efficiency on African stock markets shows mixed conclusions. The efficiency conclusion or otherwise of African stock markets are not consistent in the literature. For example, in a systematic review of informational efficiency of African stock markets by Afego (2015), it was observed that the South African stock market was concluded as efficient in the weak-form of the EMH by researchers such as Affleck-Graves & Money (1975), *Smith et al.* (2002) and Simons & Laryea (2005) but Smith (2008) concluded that the South African stock market is inefficient.

A question arising from the above conclusion is: Are the inconsistencies in the efficiency conclusions due to variations in characteristics of the studies? A review of the literature shows that there are variations in the method of analysis, data type, test type and the geographical location where a study was carried out. This chapter, therefore, aims to quantitatively review previous studies of informational efficiency of African stock markets through a meta - analysis. The mixed logistic meta - regression model will be employed in the analysis in order to find out which of the study characteristic is/are significant in concluding a market to be efficient. To the best of the researcher's knowledge, this chapter is the first to examine which study characteristics affect efficiency conclusions of African stock markets after the systematic review of Afego (2015).

The rest of the chapter is organised as follows:

In section 8.2, the sources of data were described, the inclusion criteria for the dataset and how the study characteristics were defined and grouped. In section 8.3, the mixed effect logistic model was described. The results of the study are presented in section 8.4 and the conclusion in section 8.5.

8.2 Data

In this section, a description of how the past studies were selected for the meta analysis was done and how the study characteristics were defined.

A search of empirical studies of the literature on informational market efficiency of African stock markets was done from research databases such as Academic Search Complete, Business Source Complete, Ebscohost and Econlit. A total 48 studies were obtained which generated 152 observations as a result of some studies examining two or more countries or some studies investigating the weak-form and semi-strong forms of the Efficient Market Hypothesis (EMH).

In order to obtain the dataset for the meta analysis, the study characteristics of each study were reported. Each study reported an outcome, a conclusion of either efficiency or inefficiency which then becomes the dependent variable. The explanatory variables, which include number of years since study was published, type of test investigated, nature of data employed, method of testing and the countries where the analysis were done, were reported for each study.

8.2.1 Variable Definition

The study characteristics observed were not balanced but had variations in them. These variations were grouped into homogenous groups which are defined as follows:

Variation in Method

There was a distinction between three methods of test. Studies that employed methods that are frequency domain denoted as (FD), those that are time domain denoted as (TD) and studies that used non-parametric methods denoted as (OMETHOD) in the meta regression.

Variation in Data

The variation in data was grouped into either daily (including intra-day) or non-daily (that is weekly, monthly and annually) data. Daily data is denoted as DAILY and non-daily denoted as NDAILY.

Variation in Test Type

The type of test was grouped into either weak-form (denoted WF) or semi-strong form (denoted SSF). This study did not include studies on the strong form of EMH on African stock markets as it is difficult to quantify private information.

Variation in Geographical Location

It was realised that most of the studies on market efficiency were biased towards southern and western parts of Africa especially the two largest economies of South Africa and Nigeria. There are institutional and market conditions differences amongst stock markets from different countries, hence, studies were grouped into those that were carried out in countries from the Southern part of Africa (denoted SOUTH), Western part of Africa (denoted WEST) and the rest of Africa (denoted OTHERAFRICA).

Study Conclusion

Studies were grouped according to efficient or inefficient conclusions.

Age

One of the limitations of meta analysis is publication bias. Therefore, to determine whether the study suffers from publication bias or otherwise, an age variable which indicated the number of years since a study was published was included in the model.

8.3 Methodology - Mixed Effect Logistic Model

The mixed effect logistic model was employed as the meta regression model. The mixed effect logistic model contains both fixed effects and random effects. The fixed effects assume equal influence of explanatory variables on an outcome while the random effects assume variations amongst observations when analysing relationships between an outcome and explanatory variables. The mixed effect model therefore explains an outcome as a linear combination of fixed and conditional random effects.

The model is thus shown:

$$\eta = x'\beta + y'b, \quad b \sim N(0, \Sigma) \quad (8.1)$$

where $\eta = \log\left(\frac{\pi}{1-\pi}\right)$ and π is the probability of efficiency for the dichotomous outcome variable, $x'\beta$ denotes the fixed effects and $y'b$ denotes conditional random effects.

The values of the explanatory variables for the fixed and random effects are contained in x' and y' , respectively. The coefficients of the fixed and random effects which will be obtained via Laplace approximation are contained in β and b . The Laplace approximation was used because it performs better than maximum likelihood or quasi-log likelihood when estimating the coefficients in terms of accuracy and computational time according to Harding and Hausman (2007).

8.4 Results

8.4.1 Summary Statistics

It is observed from the dataset that inefficiency conclusions of African stock markets have been 104 of the studies. Fifty-eight studies analysed stock market efficiency of Southern African countries followed by Western African and other African countries with 57 and 37 studies respectively.

The weak-form test of the EMH has been examined in 138 of the studies and 14 studies for the semi-strong form test of the EMH respectively. Studies that used non-daily data were 98 as compared to 54 studies that used daily data. Frequency domain methods have featured more on efficiency studies of African stock markets (88 studies) than those that are time-domain (17 studies) and non-distributional methods (47 studies).

8.4.2 Results - Fixed Effects Logistic Model

The explanatory variables; method of test, data type, test type and geographical location of study were treated as categorical variables. In the fixed effect logistic model, the reference categories for the dummy variables are time domain (TD) for method of test, NDAILY for data type, SSF for test type and OTHERAFRICA for geographical location.

The estimates of the saturated fixed effect logistic model, (Model 1) were presented in Table 8.1:

Coefficient	Estimate	Std. Error	z-value	Pr(> z)
Intercept	-2.70	1.29	-2.09	0.0362*
AGE	0.06	0.03	2.22	0.0267*
DAILY	0.23	0.39	0.57	0.5697
FD	-0.31	0.59	-0.51	0.6086
OMETHOD	-0.42	0.62	-0.68	0.4954
SOUTH	0.32	0.46	0.69	0.4891
WEST	-0.01	0.49	-0.03	0.9801
WF	1.48	1.10	1.34	0.1798
Deviance Residuals				
Min	1Q	Median	3Q	Max
-1.8413	-0.8818	-0.7483	1.2777	2.3258
Null deviance: 189.59 on 151 degrees of freedom				
Residual deviance: 179.69 on 144 degrees of freedom				
AIC: 195.69				
Number of Fisher Scoring iterations: 4				

Table 8.1: Estimates for saturated model - Model 1

The results of the saturated model (Model 1) presented in Table 8.1 showed that only the variable AGE was statistically significant. The estimated coefficient for AGE was positive and statistically significant at the 5% level. This observation is contrary to the findings of Shanaka and Gunther (2015) who did a meta-analysis on the informational efficiency of the real estate market. The variable AGE being positive and statistically significant at the 5% level meant the analysis suffered from publication bias and that there had been attitudinal change towards market efficiency. AGE being significant meant researchers who concluded inefficiency for African stock markets were likely to have had their papers (articles) rejected by publishers in the early 1970s when the EMH was formulated. This is because the EMH was argued to be one of the best propositions in economics (Jensen, 1978) unlike now that it has been criticised by researchers such as Dyckman and Morse (1986), *Beechey et al.* (2000), *Lee et al.* (2001) and Lo (2004). An example is a study of Affleck-Graves & Money (1975) on the South African stock market. They concluded

that the market was weak-form efficient unlike studies on the South African stock market by Bhana (1991), Smith (2008), Alagidede (2008), Hsieh & Hodnett (2011) and Gyamfi *et al.* (2016a, 2016b) who concluded that the market was inefficient in the weak-form. This means Affleck-Graves & Money (1975) conclusion on the South African market was published because the year of publication was close to the time the EMH was formulated and highly acceptable by publishers.

From Table 8.1, the rest of the variables were not significant. This meant that they had no effect in efficiency conclusions of African stock markets. Thus this study does not support differences in efficiency conclusions of African stock markets with respect to study characteristics such as method of test, test type, data type and geographical location of study.

Notwithstanding, a look at some of the estimates of the study characteristics to find out the effect they have on the outcome, relative to the reference category.

The variable WF (weak-form) had a positive estimate of 1.48 but not statistically significant to have an effect on efficiency conclusions. This means that if a market was to be analysed in the weak-form of the EMH rather than the SSF(reference category) of the EMH, the log odds of finding efficiency increased by 1.48 (as in Model 1). Studies on African stock markets thus were about four times more likely to conclude that the markets were efficient if the study tested the WF rather than SSF of the EMH.

This observation was expected because the WF test uses only prices on the markets unlike the SSF which uses past prices and public information such as dividend announcements, earnings announcements, among others. Prices are controlled by the forces of demand and supply. Information on price changes are available to all market participants, hence, can be deemed random and thus weak-form efficient. On the other hand, public information might not be available to all market participants which can cause some participants to have undue advantage over others. Abnormal profits therefore can be made by the participants who are privy to public information hence a conclusion of inefficiency in the SSF will be achieved. It is to be stated that the findings showing that the weak-form test increases the log odds of finding efficiency on African stock markets rather than the semi-strong form, resonates with the findings by Afego (2015) in his systematic review of efficiency of African stock markets. It was observed from studies by Bhana (1991), Olowe (1998), Adelegan (2003) and Afego (2012) that the markets were found to be inefficient when the semi-strong form of the EMH was tested.

Also, the log odds of finding efficiency increased by 0.32 if a study was done in the southern part of Africa rather than other parts of Africa while the log odds of finding efficiency decreased by -0.01 if a study was done in a country in western Africa. This means that although there are variations in public information and institutional organisation of stock markets between countries in the southern, western and other parts of Africa, however, where a country is located in Africa had no effect on efficiency conclusions of its stock market.

Next, a parsimonious model based on the minimum Akaike Information Criterion (AIC) was sought. The estimates of the best model (Model 2) based on the minimum AIC is as thus presented in Table 8.2:

Coefficient	Estimate	Std. Error	z-value	Pr(> z)
Intercept	-2.78	1.09	-2.53	0.0113*
AGE	0.06	0.03	2.21	0.0269*
WF	1.52	1.08	1.41	0.1585
Deviance Residuals				
Min	1Q	Median	3Q	Max
-1.7040	-0.8833	-0.7614	1.3727	2.2433
Null deviance: 189.59 on 151 degrees of freedom				
Residual deviance: 181.47 on 149 degrees of freedom				
AIC: 187.47				
Number of Fisher Scoring iterations: 4				

Table 8.2: Estimates for model of best fit - Model 2

The variable that was significant in the efficiency conclusions of African stock markets was AGE, the indicator for publication bias. Also, a significance test which included the Wald test, likelihood ratio test and the score test were performed on the saturated model (Model 1) to identify the variable(s) which affected the outcome. It was observed that only the variable AGE significantly affected the outcome. This showed that Model 2 appropriately fits the data.

8.4.3 Model Diagnostics

A goodness of fit test was conducted to find out how model 2 fits the data. It was observed that, the Pearson's test and the deviance D test had low p-values ($p\text{-values} < 0.05$), an indication of a good fit. Also, both the Hosmer and Lemeshow tests and the modified Hosmer and Lemeshow tests had high p-values; an indication of a model fitting the data well. Furthermore, the area under the receiver - operating curve (ROC) shown in Figure 8.1 was 64.2%. This made Model 2 acceptable as the best model that describes the data well.

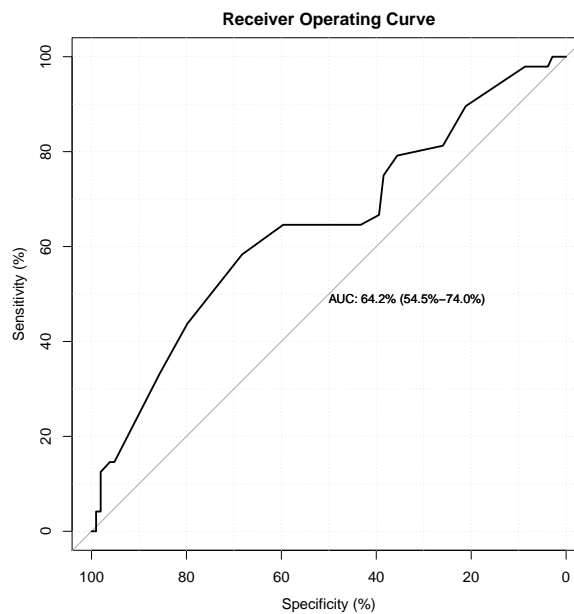


Figure 8.1: Area under the receiver-operating curve (ROC)

8.4.4 Results - Mixed Effects Logistic Model

The mixed effects model assumed heterogeneity in the variables- method of test, data type, test type and geographical location of study- unlike the fixed effects model assumption of constant effect on the outcome by the explanatory variables. The mixed effects model was considered as a better alternative to the fixed effects model according to Hunter and Schmidt (2004) and *Borenstein et al.* (2009).

In this chapter, several mixed effect models were sequentially estimated via maximum likelihood (Laplace Approximation). The study characteristics, method of test, data type and geographical

location of study were alternatively treated as random effects while AGE and WF remained fixed. The best mixed effect model based on the minimum AIC and BIC was the model with data type as random effect. The estimates of the model is as shown in Table 8.3: It was observed from

AIC	BIC	logLik	deviance	df.resid
191.5	206.6	-92.1	184.3	148
Scaled residuals				
Min	1Q	Median	3Q	Max
-1.8086	-0.6908	-0.5799	1.2512	3.3736
Random Effects				
Groups Name	Variance	Std Deviation		
DATA TYPE	1E-14	1E-07		
Fixed Effects				
Coefficient	Estimate	Std. Error	z-value	Pr(> z)
Intercept	-2.78	1.09	-2.53	0.0113*
AGE	0.06	0.03	2.21	0.0269*
WF	1.52	1.08	1.41	0.1586

Table 8.3: Estimates for mixed effects model

Table 8.3 that the variable AGE was significant as was the case in Tables 8.1 and 8.2 even after the researcher controlled for the variance associated within studies and within variables.

8.5 Conclusions

This chapter quantitatively reviewed published studies on informational efficiency of African stock markets between 1974 and 2016. There were variations in the study characteristics of the 48 peer-reviewed studies analysed with respect to method of test, data type, test type and the geographical location of a study. The aim was, first, to find out which of the study characteristics had an effect in concluding a market to be efficient informationally. Also, the researcher wanted to know if the attitudes of editors or publishers had changed over the years, thus, it was aimed at knowing if recent studies concluding efficiency of African stock markets had a better chance

or otherwise of being published than earlier studies.

The mixed effect logistic model was employed for this study. It was observed that in both the fixed-effect and mixed-effect models, only the indicator for publication bias was statistically significant; none of the study characteristics was significant and hence had no effect in efficiency conclusions of African stock markets. It was concluded that, but for the variable for publication bias, informational efficiency conclusions of African stock markets were not based on any of the study characteristics. The efficiency conclusions can be best described as context - specific which occurs randomly as observed in Shanaka and Gunther (2015).

9. General Conclusion and Recommendations

General Conclusion

In this chapter, the researcher provided general conclusion and recommendations on the weak-form market efficiency of eight African stock markets.

The weak-form market efficiency was examined using past prices on the markets. The eight markets were chosen because they are amongst the oldest and the largest stock markets in Africa. Also, these eight markets were chosen because of data availability for the 15 year period chosen. For example, the Johannesburg stock exchange (JSE) of South Africa is the oldest African stock market which was founded in 1883 and the largest with a market capitalization of 278.4% of GDP and 360 listed companies as at 2010. Also, each of the four regions of Africa were represented with at least a market. This made the analysis not biased towards a certain region. The southern and northern parts of Africa was represented by three stock markets each namely; Botswana, South Africa, Mauritius and Egypt, Morocco, Tunisia respectively. East Africa is represented by Kenya while Nigeria represented West Africa.

A battery of linear and non-linear methods on stock market returns were employed. The linear methods used in this thesis are as presented in Chapters 3, 4, and 5 gave mixed conclusions. For example, it was observed that all the eight stock market returns exhibited long-memory when whole samples were used in the analysis because each market reported a Hurst estimate $H > 0.5$. The presence of long-memory is a violation of the EMH hence the conclusion that the markets were weak-form inefficient, although, the time-varying method in analysing long-memory in the returns showed periods the markets were efficient and periods the markets were inefficient. There were periods of $H < 0.5$ and periods of $H > 0.5$. The foregone findings meant that African stock markets should be analysed through time to find the dynamics of the market and not to be analysed in absolute form using the whole sample.

Additionally, the return predictability using the rolling window approach showed periods of return predictability and periods of no return predictability. The researcher is therefore of the opinion that, markets in Africa should be examined for their efficiency taking into consideration the Adaptive Market Hypothesis (AMH) instead of the Efficient Market Hypothesis (EMH).

Furthermore, the non-linear unit root tests showed the returns on the markets were non-stationary; a condition supporting weak-form efficiency of the markets, thus, the markets were efficient when analysed using a non-linear method. To better understand the weak-form efficiency of markets in Africa, the researcher had to use non-linear methods so as to conform to the non-linear nature of the data-generating process. This observation meant that Africa's foreign direct investment which stood at \$ 54 billion as of 2015 would have been increased over the years if investors had a good knowledge about the stock markets. This is because, over the years, studies on the weak-form efficiency African stock markets have ended with mixed conclusions with most of the markets being concluded to be weak-form inefficient as a result of the use of linear methods in the analysis. This observation has had an effect on investors' commitment to Africa because the right methodology was not employed, painting a gloomy picture about the stock markets where most investors commit their resources.

Also, because of the presence of non-linear patterns in the returns as a result of non-linear data-generating process, the SETAR model performed better than the standard AR(1) and AR(2) models. This is because, with the SETAR (2,2,2) model, investors are able to move freely in search of higher opportunities between the low and high regimes. Investors aim to make profits hence the threshold model of SETAR (2,2,2) gives them the freedom to move to a regime where the rate of returns is increasing unlike the standard AR(1) and AR(2) linear models where there are no switching of regimes.

Finally, a meta-analysis which was done to quantitatively review previous studies on market efficiency of African stock markets showed that none of the study characteristics of, method of test, data type, test type and the geographical location of a study had an effect on describing efficiency conclusions on African stock markets. It was however found that the indicator variable for publication bias was significant in determining efficiency conclusions. Thus, the attitudes of editors or publishers have changed over the years as studies reporting a violation against the EMH are published without bias. This finding, in a way supports the assertion that African stock markets better support the Adaptive Market Hypothesis (AMH) than the Efficient Market Hypothesis (EMH).

Recommendations

1. Theory

From this thesis, African markets have shown to follow the Adaptive Market Hypothesis (AMH) rather than the Efficient Market Hypothesis (EMH). Therefore, more research work should be done using the AMH than the EMH. Also, the rolling window approach should be used in assessing whether African stock markets are weak-form efficient or not instead of using the absolute form.

2. Practice

Investors should research into the dynamics of a market before investing. Investments in Africa could have increased if the right methodologies were employed in analysing the efficiency of the stock markets.

3. Further studies

This thesis has not exhausted all robust methods in testing the weak-form efficiency of African stock markets. In the future, the Multi-fractal DFA and coupling DFA which have more power than the normal DFA should be considered for testing if African stock markets are becoming weak-form efficient with time. Also, a look at the semi-strong form of the EMH where we make use of public information such as macroeconomic indicators like inflation, interest rates and others could be studied. Similarly, since only the non-linear SETAR (2,2,2) model was considered when comparing the modelling performance with standard linear models of AR(1) and AR(2), extensions of nonlinear modelling could be made to include logistic smooth threshold autoregressive (LSTAR) model and Artificial Neural Networks (ANN) in modelling the returns of African stock markets.

Also, further studies could explore how to predict a stock market return using the improved Augmented regression method and the Empirical Mode Decomposition (EMD) method.

References

- [1] ABADIR K.M & DISTASO W. (2007), Testing Joint Hypotheses when one of the alternatives is one-sided, *Journal of Econometrics*, 140, 695-718.
- [2] ADELEGAN, O.J. (2003), Capital Market Efficiency and the Effects of Dividend Announcements on Share Prices in Nigeria, *African Economic Research Consortium (AERC)*, 15(2&3), 218-236.◆
- [3] ADDO, P. BILLIO, M. & GUEGAN, D. (2013A), Studies in Nonlinear Dynamics and Wavelets for Business Cycle Analysis. Documents de travail du Centre d'Economie de la Sorbonne 12023r, Universit Panthon-Sorbonne (Paris 1), Centre d'Economie de la Sorbonne.
- [4] ADDO, P. BILLIO, M. & GUEGAN, D. (2013B), Nonlinear dynamics and recurrence plots for detecting financial crisis. *The North American Journal of Economics and Finance*, 26(C): 416-435.
- [5] ADDO, P. BILLIO, M. & GUEGAN, D. (2013C), Understanding Exchange Rates Dynamics. Universit Panthon-Sorbonne (Paris 1) (Post-Print and Working Papers) halshs-00803447, HAL.
- [6] AFEGO, P. (2015), Market Efficiency in Developing African Stock Markets: What do we know?, *The Journal of Developing Areas*, 49(1), 243-266.
- [7] AFEGO, P. (2013), Stock Price Response to Earnings Announcements: Evidence from the Nigeria stock market, *Journal of African Business*, 14(3), 141-149.◆
- [8] AFEGO, P. (2012), Weak Form Efficiency of the Nigerian Stock Market: An Empirical Analysis (1984 - 2009), *International Journal of Economics and Financial Issues*, 2(3), 340-347.◆
- [9] AFFLECK-GRAVES, J.F. & MONEY, A.H. (1975), A Note on the Random Walk Model and South African Share Prices, *South African Journal of Economics*, 43(3), 382 - 338.◆
- [10] ALAGIDEDE, P. (2010), Equity market integration in Africa, *African Review of Economics and Finance*, 1(2), 88-119.

- [11] ALAGIDEDE, P. (2008), Day of the week seasonality in African stock markets, *Applied Financial Economics Letters*, 4, 115-120.◆
- [12] ALEXANDER, S. (1964), Price Movements in Speculative Markets: Trends or Random Walks, *Industrial Management Review*, 5: 25 - 46.
- [13] ANDERSEN, T. G, BOLLERSLEV, T., DIEBOLD, F. X. & LABYS, P. (2001), The distribution of realized exchange rate volatility, *Journal of the American Statistical Association*, 96 (453), 42-55.
- [14] APPIAH-KUSI, J. & MENYAH, K. (2003), Return predictability in African Stock Markets, *Review of Financial Economics*, 12(3), 247 - 270.◆
- [15] AYADI, F.O. (1994), The Random Walk Hypothesis and the Behaviour of Share Prices in Nigeria, *Nigeria Journal of Economic and Social Studies*, 26(1), 57-71.◆
- [16] BACHELIER, L. (1900), Theory of Speculation. In Cootner : 17 - 78.
- [17] BAILLIE, R. (1996), Long memory processes and fractional integration in econometrics, *Journal of Economics*, 73, 5-59.
- [18] BARKOULAS, J., BAUM, C. (1996), Long-term dependence in stock returns, *Economics Letters*, 53 (3), 253-259.
- [19] BARKOULAS, J., BAUM, C.& TRAVLOS, N. (2000), Long memory in the Greek stock market, *Applied Financial Economics*, 10 (2), 177-184.
- [20] BAUER, G. H. (2004), A taxonomy of Market Efficiency, *Bank of Canada Financial System Review*, (December), 37-40.
- [21] BEECHEY, M., GRUEN, D.& VICKERY, J. (2000), The efficient market hypothesis: A survey, *Research Discussion Paper 2000-01*, Economic Research Department, Reserve Bank of Australia.
- [22] BENTES, S., MENEZES, R.& MENDES, D. (2008), Long memory and volatility clustering: is the empirical evidence consistent across stock markets? *Physica A: Statistical and Theoretical Physics*, 387 (15), 3826-3830.

- [23] BHANA, N. (1991), Price adjustments on the JSE for announcements of share (stock) dividends, *Investment Analysts Journal*, 46, 35-44.◆
- [24] BLAKE, A.P. & KAPETANIOS, G. (2007), Testing for ARCH in the presence of nonlinearity of unknown form in the conditional mean, *Journal of Econometrics*, 137(2), 472-488.
- [25] BOLLERSLEV, T.& WRIGHT, J. (2000), Semiparametric estimation of long-memory volatility dependencies: the role of high-frequency data, *Journal of Economics*, 98, 81-106.
- [26] BONGA-BONGA, L. (2012), The Evolving Efficiency Of The South African Stock Exchange, *International Business & Economics Research Journal*, 11(9), 997-1002.◆
- [27] BORENSTEIN, M., HEDGES, L.V., HIGGINS, J.P.T.& ROTHSTEIN, H. R. (2009), *Introduction to Meta-Analysis*, John Wiley & Sons Ltd: West Sussex, UK.
- [28] BRILLINGER, D.R., & ROSENBLATT, M. (1976), Computation and interpretation of k-th order spectra. In B. Harris (Ed.), *Spectral Analysis of Time Series* (pp. 189-232). New York: John Wiley and Sons.
- [29] BROCK, W. A, DECHERT, W.& SCHEINKMAN, J. (1996), A test of independence based on the correlation dimension, *Econometric Review* , 15, 197 - 235.
- [30] BROOKS, R.D., MAHARAJ, E.A., & PELLEGRINI, B. (2008), Estimation and analysis of the Hurst exponent for Australian stocks using wavelet analysis, *Applied Financial Economics Letters*, 4(1), 41-44.
- [31] BUNDOO, S. K. (2000), The Mauritius Stock Exchange: An Assessment, *Social Sciences and Humanities and Law and Management Research Journal*, 3, 67 - 80.◆
- [32] CAJUEIRO, D.& TABAK, B. (2004A), Ranking efficiency for emerging markets, *Chaos, Solitons Fractals* 22, 349-352.
- [33] CAJUEIRO, D.& TABAK, B. (2004B), The Hurst exponent over time: testing the assertion that emerging markets are becoming more efficient, *Physica A: Statistical and Theoretical Physics*, 336 (3), 521-537.

- [34] CAMPBELL, K. & OHUOCHA, C. (2011), The stock market reaction to stock dividends in Nigeria and their information content, *Managerial Finance*, 37(3), 295-311. ♦
- [35] CAMPBELL, J.Y., LO, A. W. & MACKINLAY, A. C. (1997), *The Econometrics of Financial Markets*, Princeton, New Jersey, Princeton University Press.
- [36] CANER, M & HANSEN, B E. (2001),, Threshold Autoregression with a Unit Root, *Econometrica*, 69(6), 1555-1596.
- [37] CARAIANI, P. (2015), Testing for nonlinearity in Unemployment rates via delay vector variance, *Romanian Journal of Economic Forecasting XVIII* (1), 81-92.
- [38] CHAMBERS, D.& MANDIC, J. (2001), *Recurrent neural networks for prediction: learning algorithms architecture and stability*. Chichester, UK: Wiley.
- [39] CHAN, K.& HAMEED, A. (2006), Stock price synchronicity and analyst coverage in emerging markets, *J. Financ. Econ.* 80, 115-147.
- [40] CHAN, W.S. (2003), Stock price reaction to news and no-news: drift and reversal after headlines, *Journal of Financial Economics*, 70(2), 223-260.
- [41] CHAN, K.S. & TONG, H. (1990), On likelihood ratio tests for threshold autoregression, *Journal of the Royal Statistical Society B*, 52(3), 469-476, 1990
- [42] CHAN, K.S., PETRUCCELLI, J., WOOLFORD, S. & TONG, H. (1985), A multiple threshold AR(1) model, *Journal of Applied Probability*, 22(2) , 267-279.
- [43] CHARLES, A., DARNE, O. & KIM, J. H. (2010), Small sample properties of alternative tests for martingale difference. *Economics Letters*, 110, 151 - 154.
- [44] CHARLES, A., & DARN, O. (2009A), The random walk hypothesis for Chinese stock markets: evidence from variance ratio tests. *Economic Systems*, 33(2), 117-126.
- [45] CHEN, Y.T. (2003), Testing serial independence against time irreversibility. *Studies in Nonlinear Dynamics & Econometrics*, 7(3), Article 1.
- [46] CHEN, Y.T., CHOU, R.Y., & KUAN, C.M. (2000), Testing time reversibility without moment restrictions, *Journal of Econometrics*, 95(1), 199-218.

- [47] CHEUNG, Y.& LAI, S. (1995), A search for long memory in international stock returns, *Journal of International Money & Finance* 14 (4), 597-615.
- [48] CHEUNG, Y.W., CHINN, M.D. & MARSH, I. W. (2004), How do UK-based foreign exchange dealers think their market operates? *International Journal of Finance& Economics*, 9, 289-306.
- [49] CHOW, K. V., DENNING, K. C, FERRIS, S. & NORONHA, G. (1995), Long-term and short-term price memory in the stock market, *Economics Letters*, 49 (3), 287-295.
- [50] CHOW, K.V., & DENNING, K.C. (1993), A simple multiple variance ratio test, *Journal of Econometrics*, 58(3), 385-401.
- [51] COCHRANE, J. (1998), How Big is the Random Walk in GNP?, *Journal of Political Economy*, 96, 893-920, 1998.
- [52] COSTA, R.& VASCONCELOS, G. (2003), Long-range correlations and nonstationarity in the Brazilian stock market, *Physica A: Statistical Mechanics and its Applications*, 329, 231-248.
- [53] COUTTS, J.A & SHEIKH, M.A. (2002), The anomalies that aren't there: the weekend, January and pre-holiday effects on the all gold index on the Johannesburg Stock Exchange 1987-1997, *Applied Financial Economics*, 12:12, 863-871.◆
- [54] COWLES, A. (1934), Can Stock Market Forecasters Forecast?, *Econometrica*, 1;309 - 324.
- [55] CRATO, N. (1994), Some international evidence regarding the stockastic behavior of stock returns, *Applied Financial Economics*, 4, 33-9.
- [56] CRATO, N.& RAY, B.K. (2000), Memory in returns and volatilities of futures contracts, *The Journal of Futures Markets* 20, 525-543.
- [57] CRYER, J.D. & K.S. CHAN. (2008), *Time Series Analysis with Applications in R*, Springer Verlag.
- [58] DARRAT, A.F& LI, B. (2013), Seasonal Anomalies: A Closer Look at the Johannesburg Stock Exchange, *Contemporary Management Research*, 9(2), 155-168.◆

- [59] DARRAT, A.F., & ZHONG, M. (2000), On testing the random-walk hypothesis: a model comparison approach, *Financial Review*, 35(3), 105-124.
- [60] DICKINSON, J. P., & K. MURAGU. (1994), Market Efficiency in Developing Countries: A Case Study of Nairobi Stock Exchange, *Journal of Business Finance & Accounting*, 21(1), 133 - 150.◆
- [61] DI MATTEO, T., ASTE, T., & DACOROGNA, M.M. (2005), Long-term memories of developed and emerging markets:using the scaling analysis to characterize their stage of development, *Journal of Banking & Finance*, 29(4), 827-851.
- [62] DING, Z., GRANGER, C. AND ENGLE, R. (1993), A long memory property of stock returns and a new model, *Journal of Empirical Finance*, 1, 83-106.
- [63] DOMNGUEZ, M.A., & LOBATO, I.N. (2003), Testing the martingale difference hypothesis, *Econometric Reviews*, 22(4), 351-377.
- [64] DYCKMAN, THOMAS R. & DALE MORSE. (1986), *Efficient Capital Markets and Accounting: A Critical Analysis*, Prentice-Hall.
- [65] EKECHI, A.O. (1989), Weak-Form Efficiency in the Nigerian Stock Exchange, *African Review of Money, Finance and Banking*, 1, 5-16.◆
- [66] ELDER, J.& JIN, H.J. (2007),Long memory in commodity futures volatility: a wavelet perspective, *Journal of Futures Markets* 27, 411-437.
- [67] ENGLE, R.F. (1982), Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, *Econometrica*, 50(4), 987-1007.
- [68] ESCANCIANO, J.C., & VELASCO, C. (2006A), Generalized spectral tests for the martingale difference hypothesis. *Journal of Econometrics*, 134, 151 - 185.
- [69] FAMA, E. & FRENCH, K.(1998) , Permanent and Temporary Components of Stock Prices, *Journal of Political Economy*, 96(2), 246-273.
- [70] FAMA, E.F. (1970A), Efficient Capital markets: A Review of theory and empirical work, *Journal of Finance*, 25(1), 383 - 417.

- [71] FAMA, E.F. (1970B), Reply to Efficient Capital Market: A Review of Theory and Empirical Work, *Journal of Finance*, 25(2), 383-417.
- [72] FAMA, E.F. (1965), The Behaviour of Stock Market Prices, *The Journal of Business*, 18; 34 - 105.
- [73] GARCIA, R. & GENAY, R. (2000), Pricing and Hedging Derivative Securities with Neural Networks and a Homogeneity Hint, *Journal of Econometrics*, 94, 93-115.
- [74] GAUTAMA, T., MANDIC, D. P.& HULLE, M. M. V. (2003), A differential entropy based method for determining the optimal embedding parameters of a signal. In *Proceedings of ICASSP 2003, Hong Kong IV*, 29-32.
- [75] GAUTAMA, T., MANDIC, D. P.& HULLE, M. M. V. (2004A), The delay vector variance method for detecting determinism and nonlinearity in time series. *Physica D* 190 (34), 167-176.
- [76] GEWEKE, J., & PORTER-HUDAK, S. (1983), The estimation and application of long memory time series models. *Journal of Time Series Analysis*, 4(4), 221-238.
- [77] GILBERTSON, B.P. & ROUX, F.J.P (2015), The Johannesburg Stock Exchange as an efficient market, *Investment Analysts Journal*, 6:9, 21-27.◆
- [78] GIMBA, V. K. (2012), Testing the Weak-form Efficiency Market Hypothesis: Evidence from Nigerian Stock Market, *CBN Journal of Applied Statistics*, 3(1), 117-136.◆
- [79] GIRAITIS, L., KOKOSZKA, P., LEIPUS, R. & TEYSIRE, G. (2003), Rescaled variance and related tests for long memory in volatility and levels, *Journal of Econometrics*, 112(2), 265-294.
- [80] GONZLEZ, A. & TERSVIRTA, T. (2006), Simulation-based finite sample linearity test against smooth transition models, *Oxford Bulletin of Economics and Statistics*, 68(S1), 797-812.
- [81] GRANGER, C.W.J. & TERASVIRTA, T. (1993), *Modelling Nonlinear Economic Relationships*, Oxford UK, Oxford University Press.

- [82] GRANGER, C.W.J. & MORGENSTERN, O. (1963), Spectral analysis of New York stock market prices, *Kyklos*, 16(1), 1-27.
- [83] GRAU-CARLES, P. (2005), Tests of long memory: a bootstrap approach, *Computational Economics*, 25, 103-112.
- [84] GREENE, M.T. & FIELITZ, B.D. (1977), Long-term dependence in common stock returns, *Journal of Financial Economics*, 4(3), 339-349.
- [85] GROENEWOLD, N., TANG, S.H.K & WU, Y. (2003), The efficiency of the Chinese Stock market and the role of the banks, *Journal of Asian Economics* 14, 593-609.
- [86] GROSSMAN, S. & STIGLITZ, J. (1980), On the impossibility of informationally efficient markets, *American Economic Review*, 70, 393-408.
- [87] GYAMFI, E.N., KYEI, K. A. & GILL, R. (2016), Long-memory persistence in African Stock Markets, *EuroEconomica* 35 (1), 83-91.◆
- [88] GYAMFI, E.N., KYEI, K. A. & GILL, R. (2016), Long-memory in asset returns and volatility: evidence from West Africa, *Investment Management and Financial Innovations* 13 (2), 26-30.◆
- [89] HALL, S. & URGA, G. (2002), Testing for Ongoing Efficiency in the Russian Stock Market, Unpublished paper.
- [90] HANSEN, B.E. (1999), Testing for linearity, *Journal of Economic Surveys*, 13(5), 551-576.
- [91] HARDING, M.C. & HAUSMAN, J. (2007), Using a Laplace approximation to estimate the random coefficients logit model by non-linear least squares, *International Economic Review* 48 (4), 1311-1328.
- [92] HASANOV, M. (2009), A note on efficiency of Australian and New Zealand stock markets, *Applied Economics*, Taylor & Francis Journals, 41(2), 269-273.
- [93] HASANOV, M. & Omay, T. (2008), Nonlinearities in emerging stock markets: evidence from Europe's two largest emerging markets, *Applied Economics*, 40, 2645 - 2658.
- [94] HARVEY, C. (1994), Predictable risk and returns in emerging markets, *National Bureau of Economic Research*, 4621, 1-29.

- [95] HENRY, T. (2002), Long memory in stock returns: some international evidence, *Journal of Applied Financial Economics*, 12 (10), 725-729.
- [96] HINICH, M.J., & ROTHMAN, P. (1998), Frequency-domain test of time reversibility, *Macroeconomic Dynamics*, 2(1), 72-88.
- [97] HINICH, M. & PATERSON, D. (1985), Evidence of Nonlinearity in Stock Returns, *Journal of Business and Economic Statistics*, 3, 69-77.
- [98] HINICH, M.J., & PATTERSON, D.M. (1982), A new diagnostic test of model inadequacy which uses the martingale difference criterion, *Journal of Time Series Analysis*, 13(3), 233-252.
- [99] HO, A., MOODY, K., PENG, G., MIETUS, C., LARSON, J., LEVY, M. & GOLDBERGER, D. (1997), Predicting survival in heart failure case and control subjects by use of fully automated methods for deriving nonlinear and conventional indices of heart rate dynamics. *Circulation* 96, 842-848.
- [100] HONG, Y., & LEE, Y.J. (2005), Generalized spectral tests for conditional mean models in time series with conditional heteroscedasticity of unknown form, *Review of Economic Studies*, 72(2), 499-541.
- [101] HSIEH, H & HODNETT, H. (2011), Tests of the Overreaction Hypothesis and the Timing of Mean Reversals on the JSE Securities Exchange (JSE): the Case of South Africa, *Journal of Applied Finance & Banking*, 1(1), 107-130.◆
- [102] HSIEH, D.A. (1991), Chaos and Nonlinear Dynamics: Applications to Financial Markets, *Journal of Finance*, 46(5):1839-1877.
- [103] HUNTER, J.E. & SCHMIDT, F.L. (2004), *Methods of meta-analysis: Correcting error and bias in research findings*, Sage Publications: Thousand Oaks.
- [104] HURST, H. (1951), Long term storage capacity of reservoirs, *Transactions of the American Society of Civil Engineers*, 116, 770-808.
- [105] INANGA, E.L. & M.O. ASEKOMI (1992), The Random Character of Equity Share Prices in the Nigerian Stock Market, *Management and Development*, RVB Research Papers, 12(1), 12-17.◆

- [106] JAKATA, O., HLUPO, P.& GONDO, C. (2015), Random Walk Hypothesis: Evidence from Market Efficiency of the Zimbabwe Stock Exchange, *International Journal of Science and Research*, 4(9), 1460-1475.◆
- [107] JAMMINE, A.P. & HAWKINS, D.M. (1974), The Behaviour of some Share Indices: A Statistical Analysis, *South African Journal of Economics*, 42(1), 43 - 55.◆
- [108] JEFFERIS, K. & THUPAYAGALE, P. (2008), Long memory in Southern African stock markets, *South African Journal of Economics*, 76(3): 384-398.
- [109] JEFFERIS, K. & SMITH, G. (2005), Capitalisation and weak-form efficiency in the JSE Securities Exchange, *South African Journal of Economics*, 72(4), 684-707.
- [110] JEFFERIS, K. & SMITH, G. (2005), The Changing Efficiency of African Stock Markets, *South African Journal of Economics*, 73, 54 - 67.◆
- [111] JENSEN, M.C. (1978), Some anomalous evidence regarding market efficiency, *Journal of Financial Economics* 6 (2-3), 389-416.
- [112] KAPETANIOS, G., SHIN, Y. & SNELL, A. (2003), Testing for a unit root in the nonlinear STAR framework, *Journal of Econometrics*, 112, 359-79.
- [113] KAPETANIOS, G. (2001), Modelling Selection in Threshold Models, *Journal of Time Series Analysis*, 22(6), 733-754.
- [114] KEENAN, D.M. (1985), A Tukey nonadditivity-type test for time series Nonlinearity, *Biometrika*, 72: 39-44.
- [115] KENDALL, M. (1953), The Analysis of Economic Time Series, Part 1; Prices, *The Journal of Royal Statistical Society*, 116: 11 - 25.
- [116] KIM, H., STERN, L.V., & STERN, M.L. (2009), Nonlinear mean reversion in the G7 stock markets, *Applied Financial Economics*, 19(5), 347-355.
- [117] KIM, J.& SHAMSUDDIN, A. (2008), Are Asian stock markets efficient? Evidence from new multiple variance ratio tests, *Journal of Empirical Finance* 15(3), 518-532.
- [118] KIM, S. & WU, E. (2008), Sovereign credit ratings, capital flows and financial sector development in emerging markets, *Emerging Markets Review*, 9, 17-39.

- [119] KOUSTAS, Z., LAMARCHE, J.F., & SERLETIS, A. (2008), Threshold random walks in the US stock market. *Chaos, Solitons & Fractals*, 37(1), 43-48.
- [120] KRUSE, R. (2011), A new unit root test against ESTAR based on a class of modified statistics, *Statistical Papers*, 52, 71 - 85.
- [121] KUAN, C.M., & LEE, W.M. (2004), A new test of the martingale difference hypothesis, *Studies in Nonlinear Dynamics & Econometrics*, 8(4), Article 1.
- [122] KYAW, N.A., LOS, C.A., & ZONG, S. (2006), Persistence characteristics of Latin American financial markets, *Journal of Multinational Financial Management*, 16(3), 269-290.
- [123] LAURENCE, M., CAI, F., & QIAN, S. (1997), Weak-form efficiency and causality tests in Chinese stock markets. *Multinational Finance Journal*, 1(4), 291-307.
- [124] LAZAR, D., TODEA, A. & FILIP, D. (2012), Martingale Difference Hypothesis and Financial Crisis: Empirical Evidence from European Emerging Foreign Exchange Markets, *Economic Systems*, 36(3), 338 - 350.
- [125] LEAN, H.H., & SMYTH, R. (2007), Do Asian stock markets follow a random walk? Evidence from LM unit root tests with one and two structural breaks, *Review of Pacific Basin Financial Markets and Policies*, 10(1), 15-31.
- [126] LEE, C.F., CHEN, G.M., & RUI, O.M. (2001), Stock returns and volatility on China's stock markets, *Journal of Financial Research*, 24(4), 523-543.
- [127] LON, N. K. (2015), Forecasting Stock Return Volatility: Evidence from the West African Regional Stock Market, *Journal of Applied Finance & Banking*, 5(6), 25-35.◆
- [128] LIEN, D.& YANG, L. (2010), The effects of structural breaks and long memory on currency hedging, *The Journal of Futures Markets* 30, 607-632.
- [129] LIM K.P.& BROOKS, R. (2010), Why do emerging stock markets experience more persistent price deviations from a random walk over time? A country-level analysis. *Macroeconomic Dynamics*, 14(Suppl. 1), 3 - 41.

- [130] LIM, K.P., BROOKS, R.D., & HINICH, M.J. (2008A), Nonlinear serial dependence and the weak-form efficiency of Asian emerging stock markets, *Journal of International Financial Markets, Institutions and Money*, 18(5), 527-544.
- [131] LIM, K.P. (2007), Ranking of market efficiency for stock markets: a nonlinear perspective. *Physica A*, 376, 445 - 454.
- [132] LIMA, E.J& TABAK, B.M. (2004), Tests of the random walk hypothesis for equity markets: Evidence from China, Hong Kong and Singapore, *Applied Economics Letters* 11, 255-258.
- [133] LIU, C., LUO, Z. Q., MA, L. & PICKEN, D. (2008), Identifying house price diffusion patterns among Australian state capital cities, *International Journal of Strategic Property Management*, 12(4), pp. 237-250.
- [134] LIU, X., SONG, H., & ROMILLY, P. (1997), Are Chinese stock markets efficient? A cointegration and causality analysis, *Applied Economics Letters*, 4(8), 511-515.
- [135] LIU, C.Y& HE, J. (1991), A Variance-Ratio test of random walks in Foreign Exchange Rates, *The Journal of Finance* 46(2), 773-785.
- [136] LO, A. (2004), The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective, *Journal of Portfolio Management*, 30, 15 - 29.
- [137] LOS, C.A. (2003), *Financial Market Risk: Measurement and Analysis*, Routledge.
- [138] LO, A. (1991), Long term memory in stock market prices, *Econometrica* 59, 1279-1313.
- [139] LO, A. & MACKINLEY, A. (1998), Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test, *Review of Financial Studies*, 1, 41-66.
- [140] LONG, D.M., PAYNE, J.D., & FENG, C. (1999), Information transmission in the Shanghai equity market, *Journal of Financial Research*, 22(1), 29-45.
- [141] LUMSDAINE, R.L., & NG, S. (1999), Testing for ARCH in the presence of a possibly misspecified conditional mean. *Journal of Econometrics*, 93(2), 257-279.
- [142] LUUKKONEN, R., SAIKKONEN, P.,& TERASVIRTA, T. (1998), Testing linearity against smooth transition autoregressive models, *Biometrika* 75, 491-499.

- [143] MA, S. (2004), *The Efficiency of China's Stock Market*. Aldershot: Ashgate.
- [144] MAGNUSSON M.A. & WYDICK B. (2002), How efficient are Africa's emerging stock markets?, *Journal of Development Studies*, 38, 141 - 156.◆
- [145] MANDELBROT, B.B. (1971), When can price be arbitrated efficiently? A limit to the validity of the random walk and martingale models, *Review of Economics & Statistics*. 53, 225-236.
- [146] MATOME, T.T. (1998), The Price Discovery Process of the Namibian Stock Exchange and the Informational Content of the Share Index, *Savings and Development*, 22(3).◆
- [147] McMILLAN, D.G. & THUPAYAGALE, P. (2008), Efficiency of the South African equity market, *Applied Financial Economics Letters*, 4, 327-330.
- [148] McMILLAN, D. G. (2003), Non-linear predictability of UK stock market returns, *Oxford Bulletin of Economics & Statistics*, 65, 557-573.
- [149] MCPHERSON, M.Q. & PALARDY, J. (2007), Are international stock returns predictable? An examination of linear and non-linear predictability using generalized spectral tests, *Journal of International Financial Markets, Institutions and Money*, 17(5), 452-464.
- [150] MCPHERSON, M.Q., PALARDY, J. & VILASUSO, J. (2005), Are international stock returns predictable? An application of spectral shape tests corrected for heteroskedasticity. *Journal of Economics and Business*, 57(2), 103-118.
- [151] MENSAH, T. J, ADOM, P. K. & POMAA-BERKO, M. (2014), Does automation improve stock market efficiency in Ghana?, *African Review of Economics and Finance*, 6(1), 69-101.◆
- [152] MLAMBO, C. & BIEKPE, N. (2007), The efficient market hypothesis: Evidence from ten African stock markets, *Investment Analysts Journal*, 36(66), 5-17.◆
- [153] MOLLAH, S., & VITALI, F. (2011), Stock market efficiency in Africa: Evidence from Random Walk Hypothesis, paper submitted to the South Western Finance Conference, available at http://southwesternfinance.org/conf2011/swfa2011_submission_82.pdf.

- [154] MOOKERJEE, R. & YU (1999), An Empirical Analysis of the Equity Markets in China, *Review of Financial Economics* 8: 41-60, 1999
- [155] MORRIS,Q., VAN VUUREN, G & STYGER, P. (2009), Further Evidence of long memory in the South African Stock market, *South African Journal of Economics* 77(1), 81-101.◆
- [156] MUKHERJEE, I., SEN, C.,& SARKAR, A. (2011), Long memory in stock returns: insights from the Indian market, *The International Journal of Applied Economics & Finance* 5, 62-74.
- [157] NARAYAN, P.K. (2008), Do shocks to G7 stock prices have a permanent effect? Evidence from panel unit root tests with structural change, *Mathematics and Computers in Simulation*, 77(4), 369-373.
- [158] NARAYAN, P. K. (2007), Do health expenditures 'catch-up'? Evidence from OECD countries, *Health Economics*, 16, 993-1008.
- [159] NARAYAN, P.K., & NARAYAN, S. (2007), Mean reversion in stock prices: new evidence from panel unit root tests, *Studies in Economics and Finance*, 24(3), 233-244.
- [160] NARAYAN, P.K & SMYTH, R. (2007), Are Shocks To Energy Consumption Permanent Or Temporary? Evidence From 182 Countries, *Energy Policy*, Elsevier, 35(1), 333-341.
- [161] NARAYAN, P. K. (2005), Are the Australian and New Zealand stock prices nonlinear.
- [162] NNEJI, I. (2013), Efficiency of the Nigerian Capital Market; an Empirical Analysis, *Research Journal of Finance and Accounting*, 4(4), 69-77.◆
- [163] NTIM, C.G, OPPONG, K.K.& DANBOLT, J. (2011B), Testing the weak-form efficiency in African stock markets, *Managerial Finance*, 37(3), 195 - 218.
- [164] NTIM, G. C., OPONG, K. K. & DANBOLT, J. (2007), An Empirical Re-Examination of the Weak Form Efficient Markets Hypothesis of the Ghana Stock Market Using Variance-Ratios Tests, *African Finance Journal*, 9(2).◆

- [165] NWOSU, E. O., ORJI, A. & ANAGWU, O. (2013), African Emerging Equity Markets Reexamined: Testing the Weak Form Efficiency Theory, *African Development Review*, 25(4), 485-498.◆
- [166] OH, G., KIM, S. & EOM, C. (2008), Long-term memory and volatility clustering in high-frequency price changes. *Physica A*, 387(5-6), 1247-1254.
- [167] OKPARA, G. C. (2010), Stock market prices and the random walk hypothesis: Further evidence from Nigeria, *Journal of Economics and International Finance*, 2(3), 49-57.◆
- [168] OKPARA, G. C. (2010), Analysis of weak-form efficiency on the Nigerian stock market: Further Evidence from GARCH model, *The International Journal of Applied Economics and Finance*, 4(2), 62-66.◆
- [169] OLATUNDE, O. & MAKINA, D. (2010), Price Behaviour Of New Share Listings In Nigeria, *International Business & Economics Research Journal*, 9(4), 67-76.◆
- [170] OLOWE, R. A. (1999), Weak Form Efficiency of the Nigerian Stock Market: Further Evidence, *African Development Review*, 11(1), 54 - 68.◆
- [171] OLOWE, R.A. (1998), Stock Splits and the Efficiency of the Nigerian Stock Market, *African Review of Money, Finance and Banking*, 12, 97-125.◆
- [172] ONOH, J. O & NKAMA, N.O. (2016), Semi-Strong Market Efficiency Studies of the Nigerian Capital Market Using Dividend Announcements, *Journal of Business and African Economy*, 2(1), 22-42.◆
- [173] ONYUMA, S.O. (2009), Day-of-the-Week and Month-of-the-Year Effect on the Kenyan Stock Market Returns, *Eastern Africa Social Science Research Review*, 25(2), 53-74.◆
- [174] OSBORNE, M. (1959), Brownian Motion in the Stock Market, *Operation Research*, 7: 145- 173.
- [175] OSEI, K.A. (1998), Analysis of Factors Affecting the Development of an Emerging Capital Market: The Case of the Ghana Stock Market, *African Economic Research Consortium Research Paper* 76.◆

- [176] OZDEMIR, Z.A. (2008), Efficient market hypothesis: evidence from a small open-economy, *Applied Economics*, Taylor& Francis Journals, vol. 40(5), 633-641.
- [177] PANAS, E. (2001), Long memory and chaotic models of prices on the London Metal Exchange, *Resources Policy* 27, 235-246.
- [178] PENG, C.-K., BULDYREV, S. V., HAVLIN, S., SIMONS, M., STANLEY, H. E. & GOLDBERGER, A.L. (1994), Mosaic organization of DNA nucleotides, *Phys. Rev.*49, 1685-1689.
- [179] PERRON, P. & QU, Z. (2010), Long-Memory and Level Shifts in the Volatility of Stock Market Return Indices, *Journal of Business & Economic Statistics*, 28(2), 275-290 .
- [180] PETRUCCELLI, J.D., & DAVIES, N. (1986), A portmanteau test for self-exciting threshold autoregressive-type nonlinearity in time series, *Biometrika*, 73(3), 687-694.
- [181] PSARADAKIS, Z. (2008), Assessing time-reversibility under minimal assumptions, *Journal of Time Series Analysis*, 29(5), 881-905.
- [182] QIAN, X.Y., SONG, F.T., & ZHOU, W.X. (2008), Nonlinear behaviour of the Chinese SSE index with a unit root: Evidence from threshold unit root tests, *Physica A*, 387(2-3), 503-510.
- [183] RACINE, J.S., & MAASOUMI, E. (2007), A versatile and robust metric entropy test of time-reversibility, and other hypotheses, *Journal of Econometrics*, 138(2), 547-567.
- [184] RAJAN, R., ZINGALES, L. (2003), The great reversals: the politics of financial development in the twentieth century, *Journal of Financial Economics*, 69 (2), 5-50.
- [185] RAMSEY, J.B. & ROTHMAN, P. (1996), Time irreversibility and business cycle asymmetry, *Journal of Money, Credit and Banking*, 28(1), 1-21.
- [186] ROBINSON, P.M. (1995), Gaussian semiparametric estimation of long range dependence, *Annals of Statistics*, 23(5), 1630-1661.
- [187] RYDEN, T., TERSVIRTA, T. & ASBRINK, S. (1998), Stylized facts of daily return series and the hidden Markov model, *Journal of Applied Econometrics* 13: 217-244.

- [188] SADIQUE, S., SILVAPULLE, P. (2001), Long-term memory in stock market returns: international evidence, *International Journal of Finance & Economics*, 6, 59-67.
- [189] SAMUEL, J.M.& YACOUT, N. (1981), Stock Exchange in Developing Countries, *Savings and Development*, 5(4), 217-230.◆
- [190] SAMUELSON, P. (1965), Proof That Properly Anticipated Prices Fluctuate Randomly, *Industrial Management Review*, 6: 41-49.
- [191] SHANAKA, H.& GUNTER, M. (2015), Informational efficiency of real estate market: A meta-analysis, *Journal of Economic Research*, 20(2), 117-168.
- [192] SERLETIS, A. & ROSENBERG, A.A. (2009), Mean reversion in the US stock market. *Chaos, Solitons & Fractals*, 40(4), 2007-2015.
- [193] SERLETIS, A., URITSKAYA, O.Y. & URITSKY, V.M. (2008), Detrended fluctuation analysis of the US stock market, *International Journal of Bifurcation and Chaos*, 18(2), 599-603.
- [194] SERLETIS, A. & SHINTANI, M. (2003), No evidence of chaos but some evidence of dependence in the US stock market. *Chaos, Solitons & Fractals*, 17(2-3), 449-454.
- [195] SHINTANI, M. & LINTON, O. (2004), Nonparametric neural network estimation of Lyapunov exponents and a direct test for chaos, *Journal of Econometrics*, 120(1), 1-33.
- [196] SIMONS, D., & LARYEA, S. A. (2005), Testing the Efficiency of African Markets, *Finance India*, 20, 553 - 571.◆
- [197] SMITH, G. & DYAKOVA, A. (2014), African Stock Markets: Efficiency and Relative Predictability, *South African Journal of Economics*, 82(2), 258 - 275.◆
- [198] SMITH, G. (2008), Liquidity and the Informational Efficiency of African Stock Markets, *South African Journal of Economics*, 76(2), 161 - 175.◆
- [199] SMITH, G. (2007), Random Walks in middle eastern stock markets, *Applied Financial Economics*, 17, 589-596.
- [200] SMITH, G., JEFFERIS, K. & RYOO, H.J. (2002), African stock markets: Multiple variance ratio tests of random walks. *Applied Financial Economics*, 12, 475 - 484.◆

- [201] STARICA, C.& GRANGER, C. (2005), Nonstationarities in stock returns, *Review of Economics & Statistics*, 87 (3), 503-522, 2005
- [202] TACHIWOU, A. M. (2010), Day-of-the-Week-Effects in West African Regional Stock Market, *International Journal of Economics and Finance* 2(4), 167-173.◆
- [203] TAYLOR, S. (1986), *Modelling Financial Time Series*, 2nd ed. Wiley, New York.
- [204] TOLVI, J. (2003), Long memory and outliers in stock market returns, *Applied Financial Economics*, 13 (7), 495-502.
- [205] TONG, H. & LIM, K.S. (1980), Threshold Autoregression, Limit Cycles and Cyclical Data (with discussion), *Journal of the Royal Statistical Society Series B*, 42: 245-292.
- [206] TONG, H. (1978), On a threshold model. In *Pattern Recognition and Signal Processing*, In: C. H. Chen (ed.), NATO ASI Series E: Applied Sec. No. 29, Amsterdam:Sijthoff & Noordhoff, pp. 575-586.
- [207] TSAY, R. S. (1992), Nonlinear Time-Series Analysis of Stock Volatilities, *Journal of Applied Econometrics*, 7(S): S165-l85.
- [208] TSAY, R.S.(1989), Testing and modeling threshold autoregressive processes, *Journal of the American Statistical Association*, 84(405), 231-240.
- [209] TSAY, R. S. (1986), Nonlinearity test for time series, *Biometrika* 73 (2): 461-466.
- [210] VAN DIJK, D. J. C. (1999), *Smooth Transition Models: Extensions and Outlier Robust Inference*, Tinbergen Institute Research Series No. 200.
- [211] WORKING, H. (1934), A Random Difference Series for the Use in the Analysis of Time Series, *Journal of the American Statistical Association*, 29(185), 11-24. 521-537, 2004
- [212] WRIGHT, J.H. (2000A), Alternative variance-ratio tests using ranks and signs, *Journal of Business and Economic Statistics* 18(1), 1 - 9.
- [213] WRIGHT, J.H (2000B), Log-periodogram estimation of long memory volatility dependencies with conditionally heavy tailed returns, *International Finance Discussion Papers* 685, Board of Governors of the Federal Reserve System (U.S.).

-
- [214] ZHUANG, Y., GREEN, C.& MAGGIONI, P. (2000), The great rebound, the great crash, and persistence in British stock prices, Technical Report 00/11. Loughborough University, Loughborough.

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¹◆Studies used in meta-analysis