A CLASS OF EFFICIENT ITERATIVE SOLVERS FOR THE STEADY STATE INCOMPRESSIBLE FLUID FLOW: A UNIFIED APPROACH.

By

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Abstract

The methods of solving the steady state incompressible fluid flow problems, in particular the Stokes partial differential equations generally require the solution of a large linear system of equations. The numerical treatment of the Stokes equations through the mixed finite element discretization gives rise to sparse, symmetric system that is indefinite, a feature that degrades the performance of conventional solution methods. It is amenable to solve by a special class of solution methods based on Krylov subspace solvers such as minimum residual (MINRES) and variants of conjugate gradients and other methods like Uzawa and multigrid. These methods require good preconditioning strategies and a careful choice of smoothers. The main focus of this thesis is to develop fast, efficient iterative solvers for the resulting symmetric indefinite system through their unified combination as preconditioners and smoothers. A class of solvers considered in this work are a unified combination of the Uzawa solvers, Krylov subspace solvers: preconditioned minimum residual (PMINRES) and preconditioned conjugate gradient (PCG), and the multigrid solvers. This entails that the multigrid solver is applied to the discrete system in combination with the iterative solvers like Uzawa, Braes-Sarazin, distributed Gauss Seidel (DGS) and pminres as smoothers. On the other hand the conventional multigrid, pcg, Chebyshev methods are used as preconditioner approximations for Uzawa, PCG, and PMINRES to improve their performance. The objective of this thesis is to develop new efficient solvers through this unified approach. The convergent behaviour of the unified solvers is shown numerically that the number of iterations to obtain the approximate solution is independent of the mesh size. The results in this work show that the multigrid method is an

appropriate preconditioner approximation of all the solvers studied in this thesis. It enhances their performance more than other preconditioner approximations. The other interesting observation from the numerical experiments is that Braess-Sarazin and inexact Uzawa smoothers produce a good performance of the multigrid solver. Comparing all efficient solvers the multigrid method (MGM) outperformed all other solvers. In addition the memory requirements and the computing times are at an acceptable level for all the solvers through applying suitable preconditioners and using appropriate smoothers. All the numerical tests are done in a 2 dimensional traditional lid driven cavity domain applying stable Hood-Taylor $Q_2 - Q_1$ pair of finite elements.