ANALYSIS OF A BOUNDARY VALUE PROBLEM FOR A SYSTEM OF NON-HOMOGENEOUS ORDINARY DIFFERENTIAL EQUATIONS (ODE), WITH VARIABLE COEFFICIENTS.

By

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Abstract

In this study we present a condition for the existence and uniqueness of the solution $y(x)$ for a system of nonhomogeneous linear first order Ordinary Differential Equations (ODE). The existence and uniqueness of the solution of $y(x)$ was confirmed through the Picard Lindelof Theorem. We then study the stability of matrix $A(x)$ using its spectrum, moreover, $A(x)$ is symmetric. This is a pre-condition for the application of Lefschetz direct stability method. We then modify the given Lefschetz system (Meyer, 1964) to suit the problem at hand. The direct method requires the construction of a suitable Lyapunov function; not easy for a time-independent (non-dynamic) problem. For a time-dependent problem the energy thereof becomes a suitable candidate for a Lyapunov function. For a non-dynamic problem it is harder to construct a Lyapunov function as there are no rules for that purpose. In our study we modified the Lefschetz system for the direct stability method and applied it to confirm the Lefschetz stability criterion using the modified systems of linear first order ODEs with variable coefficients. The Lefschetz method afforded us the construction of a credible Lyapunov function which enabled us to confirm the stability of the null solution to our problem. From our modified Lefschetz direct stability system, we solved the Makhabane / Hlomuka equation (5) for $B(x)$ (7) which we later confirmed as both symmetric and positive definite.