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HIERARCHICAL FORECASTING OF ELECTRICITY DEMAND IN SOUTH AFRICA

By

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Abstract

The study focuses on the application of hierarchical time series in forecasting electricity demand using South African data. The methods used are top-down, bottom-up and optimal combination. The top-down method is based on the disaggregation of the forecasts of the total series and distribute these down the hierarchy based on the historical proportions of the data. The bottom-up approach aggregates the individual forecasts at the lower levels, while the optimal combination technique optimally combines the bottom forecasts. Out-of-sample forecast performance evaluation was conducted to get some indication of the forecasting performance of the models. MAPE was used to determine the best model. Bottom–up approach is found to be the best approach compared to optimal combination and top–down approaches. In order to combine forecasts and compute the prediction intervals for the developed models the quantile regression averaging (QRA) and linear regression (LR) is used. The best set of forecasts is selected based on the prediction interval normalised average width (PINAW) and pinball loss. The best model based on pinball loss is QRA and the best model based on PINAW at 95 % is QRA.

Keywords: Modelling framework, disaggregation, hierarchical time series, top-down method, bottom-up method, optimal combination method, upper levels and lower level forecast.

Declaration

I, Rofhiwa Netshiomvani [student number 11625718], hereby declare that the dissertation titled: “Hierarchical forecasting of electricity demand in South Africa” for the Master of Science degree in Statistics at the University of Venda, hereby submitted by me, has not been submitted for any degree at this or any other university, that it is my own work in design and in execution, and that all reference material contained therein has been duly acknowledged.

Signature: Date: 11 August 2020

Dedication

This work is dedicated to my parents Radzilani M.P and Netshiomvani A, my daughter Netshiomvani A and my little brother Radzilani R.

Table of contents

List of Figures

List of Tables

Abbreviations

CI	Confidence Interval.
ENV	Envelop
GEFCom	Global Forecasting Competition
LR	Linear Regression
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MASE	Mean Absolute Scaled Error
MIDAS	Mixed-Data Sampling
GWh	Gigawatt-hours.
PINAW	Prediction Interval Normalized Average Width
PINC	Prediction Interval with Nominal Confidence
QCA	Quantile Combination Approach
QR	Quantile regression
QRA	Quantile Regression Averaging
RMSE	Root Mean Square Error
SA	Simple Averaging
STL	Seasonal and Trend Decomposition using Loess

Chapter 1

Introduction

1.1 Hierarchical Forecasting

Forecasts of electricity demand are required for economical and secure management of power grids. Accurate electricity demand forecasts play a key role in sustainable power systems. Electricity demand is the amount of electricity being consumed at any given time. The electricity consumption of a whole country could be disaggregated by states, cities, and households. Due to the prevailing downside of inconvenience electricity storage, excess electricity would cause unneeded waste. Thus electricity demand forecasting is one of the crucial issues within the field of electrical power management. With the rise of responsive grid, more meter information is available, which brings with more delicacy the possibility of predicting power demand.

Hierarchical forecasting is based on a strategy of aggregating items into groups. Hierarchical forecasting systems are capable of providing forecasts for items and respective groups (Fliedner, 2001). It has long been established that electricity demand forecasting is important for electricity utility planning. Electricity demand forecasting is divided into short-term forecasting which covers hourly to weekly forecasting, medium-term forecasting from on month to a year and lastly, long-term which is from a year to several decades. Total consumption within the whole geographical area may be geographically disaggregated into many sub-regions, and these sub-regions may be more disaggregated into regions at lower level. As an example, electricity consumption in countries may be disaggregated into provinces, cities, districts, etc. Electricity demand forecasting is a challenging problem, because it is subject to a variety of uncertainties, including underlying population growth, changing technology, economic conditions and prevailing weather conditions. (Hyndman, 2014)

Generating forecasts to support decision-making in a very hierarchical data structure is very important in several different applications, like retail (Kremer et al., 2016) and tourism (Wickramasuriya et al., 2015). Projected ways involve 2 stages, with forecasts produced severally for every series within the hierarchy then combined to modify a synthesis of accessible data, and to confirm compliance with the aggregation constraint. There is lack of probabilistic predictions in the hierarchical forecasting literature. Notable contributions are by (Taieb et al., 2017). " In the electrical demand context, probabilistic predictions of the full system load are used for random unit commitment models, electricity demand coming up with, setting operational reserve, worth statement, and electrical demand market commerce (Hong and Fan, 2016). According to the geographic disaggregate strategy, the statistic in numerous levels should adapt the aggregation constraints, i.e.

the demand in numerous levels ought to be summed systematically. Most of the progressive hierarchical declaration strategies estimate initial forecasts and then reconcile them to make sure mixture constraints.

The most common approaches in hierarchical forecasting are the top-down method and also the bottom-up methodology Top-down method involves forecasting the aggregated series, and then disaggregated the forecasts based on the historical or forecast proportions (Gross and Sohl, 1990). The bottom-up method involves forecasting each of the disaggregated series at the lowest level of the hierarchy, and then using aggregation to obtain forecasts at higher levels of the hierarchy, (Kahn, 1998). Most studies have centered on scrutiny the performance of those 2 approaches, with some favouring the top-down approaches (see as an example Fliedner (1999), Fogarty, Blackstone, and Homan (1990), Grunfeld and Griliches (1960) and Narasimhan et al. (1994)), whereas others favour the bottom-up approaches (see as an example Dangereld and Morris (1992), Edwards and Orcutt (1969), Kinney (1971), Orcutt et al. (1968) and Zellner and Tobias (2000), and still others find neither methodology to be uniformly superior (see as an example Fliedner and Mabert (1992), Shing (1993) and Weatherby (1984). But none of these methods take the correlation among the series at each level into account. Therefore a statistical method for optimal hierarchical forecasting was proposed by Hyndman, Ahmed, and Athanasopoulos (2009). The optimal combination approach relies on prognostication all series in the least levels, and so employing a regression model to get the minimum variance unbiased combination of the forecasts. The optimal combination approach because unlike any other existing method, this approach uses all the information available within a hierarchy. It allows for correlations and interactions between series at each level of the hierarchy, it accounts for ad hoc adjustments of forecasts at any level, and provided the base forecasts are unbiased, it produces unbiased revised forecasts.

1.2 Problem Statement

There is lack of probabilistic methods in the hierarchical forecasting literature. In electricity demand context, probabilistic predictions of the entire system load are used for random unit commitment models, power provide designing, setting operational reserve, value forecasting, and electricity market commercialism. Thanks to the prevailing downside of inconvenience electricity storage, excess electricity would cause supernumerary waste. Accurate forecasting is useful to guide the electrical power firms to create call. Thus, electricity demand forecasting is one in every of the foremost necessary issues within the field of electrical power management.

1.3 Aim and Objectives

1.3.1 Aim

The aim of this study is to apply a modelling framework for forecasting electricity demand using hierarchical time series.

1.3.2 Objectives:

The objectives of the study are to:

- apply a top-down, bottom-up and optimal combination methods in forecasting electricity demand in South Africa,
- compare the performance of the developed models.

1.4 Significance of the study

This study is significant since electricity demand forecasting is important for electricity utility planning. Forecasting of power demand plays an essential role in electric industry, as it provide the basis for making decisions in power system planning and operation. Most important for achieving more accurate forecasts for the electricity demand.

1.5 Scope

Monthly provincial electricity demand data was obtained from Stats SA. The performance of the models was evaluated using the accuracy measures root mean square error (RMSE) and mean absolute error (MAE), and the best models were selected based on the Mean Absolute Percentage Error (MAPE). Prediction intervals from top-down, bottom-up, optimal combination were compared with those from quantile regression averaging and linear regression to determine the model yielding narrower prediction interval widths.

1.6 Layout of the study

The rest of this study is organised as follows. Chapter 2 provides an overview on hierarchical forecasting using bottom–up, top–down and optimal combination approaches. In Chapter 3 research methodology is discussed. A discussion of the empirical results is showed in Chapter 4 and the conclusion is discussed in Chapter 5.

Chapter 2

Literature review

2.1 Introduction

This chapter presents an overview of forecasting hierarchical time series and it includes summaries of other studies that used the proposed methodology to forecast electricity demand in South Africa.

2.2 An overview of hierarchical forecast

Athanasopoulos et al. (2009), investigated hierarchical forecasts for Australian domestic tourism. They considered five approaches for hierarchical forecasts, two variations of the top-down approach, the bottom-up approach, a newly proposed top-down approach where top-level forecasts are disaggregated, and optimal combination approach. The study shows that the top-down approach and optimal combination approach perform best for the business hierarchies thought of. A detailed forecast for the Australian domestic tourism market was produced using the methods.

Amusa et al. (2009), studied aggregate demand for electricity in South Africa: An analysis using the bounds testing approach to cointegration. South Africa's demand for electricity has grown at a very rapid rate during the past decade. Any effects on aggregating electricity consumption in the pricing policy will depend on factors which influence the demand for electricity to have a useful understanding, and the degree to which the market for electricity reacts to changes in those variables. The research applied bounds testing approach to cointegration within an autoregressive distributed lag system to analyze the aggregate demand for electricity in South Africa during the period from 1960 – 2007. The findings show that income is the principal determinant of demand for electricity in the long run. Pricing of electricity with insignificant effect on the aggregate demand for electricity.

Inglesi and Pouris (2010), studied electricity demand forecasting in South Africa: a criticism of the Eskom projections. Their study aimed to add some new ideas and insights to the current claims Eskom had regarding demand of electricity in South Africa. When forecasting electricity demand, Eskom did not take into account the effects of electricity prices. Prices were expected to have a strong impact on electricity demand. Using similar assumptions for Eskom's economic growth in the region, the forecast results indicated a significant decline in electricity demand.

Sbrana and Silvestrini (2013), provide the analytical prediction properties of top-down (TD) and bottom-up (BU) approaches when forecasting the aggregated demand using a multivariate exponential smoothing as demand planning framework. The employed an unrestricted multivariate framework allowing for interdependency between its variables. The study establish the necessary and sufficient condition for the equality of mean squared errors (MSEs) of the two approaches. The study showed the relative forecasting accuracy of TD and BU depends on the parametric structure of the underlying framework. The results confirm the study theoretical findings, which is indeed the ranking of TD and BU forecasts is led by the parametric structure of the underlying data generation process, regardless of possible misspecification issues.

Athanasopoulos et al. (2011), studied optimal combination forecasts for hierarchical time series. They propose a new approach to hierarchical forecasting which provides optimal forecasts that are better than forecasts produced by either a top-down or bottom-up approach. They used a method based on independently forecasting all series at all levels of the hierarchy and using regression model to optimally combine and reconcile ties that the new methods perform better compared to the top-down approach and the bottom-up approach.

Wickramasuriya et al.(2015), studied the forecasting of hierarchical and grouped time series through trace minimization. A combination forecasting approach that incorporates the information from full covariance matrix of forecast errors in obtaining a set of aggregated consistent forecasts was used in the study. The approach used minimizes the mean squared error of the aggregate consistent forecasts across the entire collection of time series under the assumption of unbiasedness. The proposed method is compared to alternative methods using a series of simulation designs.

Hyndman et al. (2016), studied the fast computation of reconciled forecasts for hierarchical and grouped time series. The least squares approach to reconciling hierarchical time series forecasts can be extended to much more general collections of time series with aggregation constraints time series. The method for optimally reconciling forecasts of all series in a hierarchical to ensure they add up was proposed. The study show that this result is easily extended to cover non-hierarchical groups of time series, and groups with a partial hierarchical structure. The optimal reconciliation method involves fitting a linear regression model where the designed matrix has one column for each of the series at the most disaggregated level. The study also show that the computations involved can be handed efficiently by exploiting the structure of the associated design matrix, or by using sparse matrix routines. The algorithms proposed make forecast reconciliation feasible in business applications involving very large numbers of time series.

Taieb et al. (2017), proposed a study regarding hierarchic probabilistic forecasting of electricity demand with good meter knowledge. An algorithmic rule for manufacturing a chance density forecasts for every series at intervals a large-scale hierarchy was used. The study targeted on getting probabilistically coherent forecast distributions. the sentence was rephrased to “No conclusions were made about the underlying distributions, mistreatment conditional KDE for good meter time series, and mistreatment of rapid time series models that can manage the various seasonal trends ascertained within the collective knowledge. It was common for Empirical copulas to place dependencies between forecast distributions.

Taieb et al. (2017), considered the situation where probabilistic forecasts are needed for each series of the hierarchy. They defined forecast coherency a proposed an algorithm to computes predictive distributions under the form of random samples for each series in the hierarchy. The probabilistic forecasts are independently computed for all series in the hierarchy, and samples are computed from the associated predictive distributions. The algorithm computes sparse forecasts combinations for all series in the hierarchy. They evaluated the accuracy of our forecasting algorithm on both simulated data and large scale electricity smart meter data. The results show consistent performance gains compared to state-of-the art methods.

Taieb et al. (2017), considered regularization in hierarchical time series forecasting with application to electricity smart meter data. In order to provide more robustness to estimation errors in the adjustments, they proposed a new hierarchical forecasting algorithm that computes sparse adjustments while preserving the aggregation constraints. The problem was formulated as a high-dimensional penalized regression, which can be efficiently solved using cyclical coordinate descent methods. A large-scale hierarchical electricity demand data was used to conduct experiments. The proposed new approach proved to be effectiveness compared to state-of-the-art hierarchical forecasting methods, in both the sparsity of the adjustments and the prediction accuracy, according to the results.

Athanasopoulos et al. (2017), introduced the concept of temporal hierarchical for time series forecasting. A temporal hierarchy can be constructed for any time series by means of non-overlapping temporal aggregation. The objective of the study was to introduce a novel approach for time series modelling and forecasting temporal hierarchies. The study proposed methodology is independent of forecasting models. It can embed high level complex and unstructured information with lower level statistical forecasts. The results show that forecasting with temporal hierarchies increases accuracy over conventional forecasting particularly under increased modelling uncertainty.

Taieb (2017), studied the sparse and smooth adjustments for coherent forecasts in temporal aggregation of time series. State-of-the-art forecasting methods usually apply adjustment on the individual level forecasts to satisfy the aggregation constraints. A new forecasting algorithm that provides sparse and smooth adjustments while still preserving the aggregation constraints, was proposed, in order to keep a maximum number of individual forecasts which are not affected by estimation errors. The algorithm computes the revised forecasts by solving a generalized lasso problem. The experiments performed and a large-scale smart meter dataset confirm the effectiveness of the proposed algorithm compared to the state-of-the-art methods.

De Hoog and Adodulla (2019), studied data visualization and forest combination for probabilistic load forecasting in GEFCom2017 final match. This study describes the methods used by Team Cassandra, a joint effort between IBM Research Australia and University of Melbourne, in the GEFCom2017 load forecasting competition. Several data visualisation techniques was applied to understand the nature and size of gabs, outliers, the relationships between different entities in the dataset, and the relevance of custom date ranges. Cleaned data was used to train multiple probabilistic forecasting models. Model selection and forecast combination were used to choose a custom forecasting model for every entity in the dataset.

Smly and Hua (2019), studies the preprocessing and forecasting methods used by team ”arbuculum” during the qualifying match of the GEFCom2017. Tree-based algorithms (gradient boosting and quantile random forest) associated neural networks created up an ensemble. The ensemble prediction quantile were obtained by a straightforward averaging of the ensemble members prediction quantiles. This study shows a sturdy performance per the game loss metric, with the ensemble model achieving third place within the qualifying match of the competition.

Roach (2019), presented a new methodology for forecasting quantiles in hierarchy which outperform a commonly-used benchmark model. To generate demand forecasts, a simulation based approach was used. To ensure that all zonal forecasts reconciled appropriately, and a weighted approach was implemented to ensure that the bottom-level zonal forecast summed correctly to the aggregated zonal forecasts adjustments was made to each of the demand simulations. The study showed that reconciling in improves the forecast accuracy. Review of hierarchical time series forecasting and gradient boosting was also included.

Lima et al. (2019), analysed the quantile combination approach(QCA) of Lima and Meng (2017) in situations with mixed frequency data. MIDAS and soft(hard) thresholding methods towards quantile regression was used to address parameter proliferation problem from the estimation of quantile regression with mixed-frequency data. To forecast the growth rate of the industrial production index the proposed approach was used. The study shows that including high-frequency information in the QCA achieves substantial gains in terms of forecasting accuracy.

Kanda and Veguillas (2019), studied data preprocessing and quantile regression for probabilistic load forecasting in the GEFCom2017 final match. Quantile regression method was competed within the GEFCom2017 final match of hierarchical probabilistic load forecasting, using R package "quantreg".

2.3 Conclusion

This chapter provide summaries of some studies that used the proposed methodology to forecast electricity demand. There is lack of studies using bottom–up, top-down and optimal combination approaches to forecast electricity demand. Quantile regression models was also not used mostly for electricity demand. This chapter shows that the use of bottom–up, top-down and optimal combination approaches in electricity demand has not been done to date in South Africa.

Chapter 3

Methodology

3.1 Introduction

This chapter discussed approaches that will used in this study to forecast electricity demand. Bottom–up approach, Top–down approach based on historical proportions and Top–down approach based on forecasted proportions and optimal combination approach were discussed.

3.2 Hierarchical time series

Consider the hierarchical data structure of Figure 3.1. We tend to denote the fully mass total series as level zero, the first level of disaggregation as level one, then on down to the underside level K, that contains the foremost disaggregated series. Hence, the hierarchy delineate in Figure 3.1 may be a $K =$ one level hierarchy. Let $y_{x,t}$ be the t^{th} observation $(t = 1, ..., n)$ of series

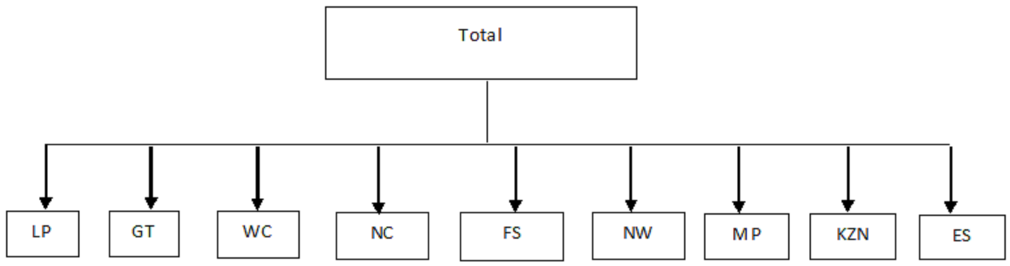

Figure 3.1: A two level hierarchical tree diagram.

Y_x, that corresponds to node X on the hierarchical tree.

where each province is going to be presented, WC = Western Cape, EC=Eastern Cape, NC=Northern Cape, FS=Free State, KZN=Kwazulu-Natal, NW=North West, GT=Gauteng, ML=Mpumalanga and LP=Limpopo. We use a sequence of letters to denote the individual nodes, as represented in Figure 3.1. As an example, $Y_{WC,t}$ denotes the t_{th} observation of the series node WC at level one. Notice that the actual letter sequence indicates the individual node and the length of the letter sequence denotes the level. For the totally aggregated level, the t^{th} observation is denoted by Y^t. We let m_i denote the total number of series for level i, and $m = m_0 + m_1 + ... + m_k$ is the total number of series in the hierarchy, (Athanasopoulus et al., 2009).

3.3 Alternative approaches to hierarchical forecasting

In order to generalise the notation for the varied approaches to hierarchical forecasting, we let vector $Y_{i,t}$ contain all the observations in level i at time t. The vector is defined as $Y_t = [Y_t, Y'_{,t}, ..., Y'_{K,t}]$. Now we can write

$$Y_t = SY_{K,t}, \tag{3.3.1}$$

where S is a “summing” matrix of order $m \times m_K$ that aggregates the bottom level series all the way up the hierarchy. For example, for the hierarchy of Figure 3.1 we have

$$\begin{bmatrix} Y_t \\ Y_{WC,t} \\ Y_{EC,t} \\ Y_{NC,t} \\ Y_{FS,t} \\ Y_{KZN,t} \\ Y_{NW,t} \\ Y_{GT,t} \\ Y_{ML,t} \\ Y_{LP,t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_{WC,t} \\ Y_{EC,t} \\ Y_{NC,t} \\ Y_{FS,t} \\ Y_{KZN,t} \\ Y_{NW,t} \\ Y_{GT,t} \\ Y_{ML,t} \\ Y_{LP,t} \end{bmatrix}$$

where I_k denotes an identity matrix of order kk.

3.3.1 The bottom-up approach

The most commonly applied methodology in hierarchal forecasting is that the bottom-up approach (Dangereld and Morris (1992), Dunn et al. (1976), Orcutt et al. (1968), Shlifer and Wol (1979), Theil (1954) and Zellner and

Tobias (1998)). The approach is depicted by exploitation the sort of equation (3.3.2), therefore

$$P = [0_{m_K \times (m-m_K)} | I_{m_K}], \tag{3.3.2}$$

where $0_{i \times j}$ is the $i \times j$ null matrix. The role of P here is to extract the underside level forecasts, that are then collective by the summation matrix S to supply the revised forecasts for the entire hierarchy. The best advantage of this approach is that, by modelling the information at the foremost disaggregated bottom level, we have a tendency to don't lose any information thanks to aggregation. Hence, we are able to higher capture the dynamics of the individual series. However, bottom level knowledge are often quite missing, and is more so challenging to model (Athanasopoulus, et al., 2009).

3.3.2 Top-down approaches based on historical proportions

The other normally applied methodology in hierarchical forecasting is that the top-down approach (see for instance Fliedner (1999), Grunfeld and Griliches (1960), Lutkephol (1984) and Narasimhan et al. (1985)). The foremost common type of the topdown approach is to disaggregate the forecasts of the full series and distribute these down the hierarchy supported the historical proportions of the info. In terms of the general type of equation (3.3.3), we write

$$P = [p | 0_{m_k \times (m-1)}], \tag{3.3.3}$$

where $p = [p_1, p_2, ..., p_{m_k}]'$ are a collection of proportions for the lowest level series. So the role of P here is to distribute the highest level forecasts to forecasts for the lowest level series, (Athanasopoulus, et al., 2009). In this dissertation we take into account two versions of this approach that performed quite well within the study by Gross and Sohl (1990). For the first one

$$P_i = \sum_{t=1}^{n} \frac{Y_{i,t}}{Y_t} / n \tag{3.3.4}$$

for $i = 1, ..., m_k$. We label this "Top-down HP1" within the tables that follow. Each proportion p_i reflects the average of the historical proportions of the bottom level series $Y_{i,t}$ over the period $t = 1, ..., n$ relative to the total aggregate Y_t; i.e., vector p reflects the average historical proportions. In the second version we consider

$$P_i = \sum_{t=1}^{n} \frac{Y_{i,t}}{n} / \sum_{t=1}^{n} \frac{Y_t}{n} \tag{3.3.5}$$

for $i = 1, ..., m_K$. We tend to this "Top-down HP2" in the tables that follow. Every p_i proportion here captures the typical historical worth of the bottom level series $Y_{i,t}$ relative to the typical worth of the full aggregate Y_t; i.e., vector p reflects the proportions of the historical averages. The simplicity of the applying of those top-down approaches is their greatest attribute. One solely must model and manufacture forecasts for the foremost mass high level series. These approaches appear to produce quite reliable forecasts for the mixture levels, and that they are terribly useful with low count knowledge. On the opposite hand, their greatest disadvantage is the loss of data because of aggregation. With these top-down approaches, we are unable to capture and profit of individual series characteristics like time dynamics, special events, etc. for instance, in the empirical application with commercial enterprise knowledge that follows, the info are extremely seasonal. The seasonal pattern of commercial enterprise arrivals might vary across series depending on the commercial enterprise destination. A resort hotel can have a really totally different seasonal pattern to a beach resort. this can not be captured by disaggregating the full of those destinations supported historical proportions. Finally, with these strategies we tend to base the disaggregation of the "Total" series forecasts on historical and static proportions, and these proportions can miss any trends within the knowledge, (Athanasopoulus et al., 2009).

3.3.3 Top-down approach based on forecasted proportions

In order to improve the historical and static nature of the proportions used to disaggregate the highest level forecasts, we tend to introduce a top-down methodology whereby the proportions to break down the highest level forecasts are supported by the predicted proportions of the lowest level series. To demonstrate the intuition of this methodology, consider a one level hierarchy and solely 1-step-ahead forecasts, that we tend to at first manufacture for all series severally. At level one we have a tendency to calculate the proportion of every individual forecast to the mixture of all the individual forecasts at this level. we have a tendency to check with these as the forecasted proportions, and use them to disaggregate the highest level forecast. For a K-level hierarchy, this method is continual for every node, going from the highest level to the terribly bottom level, (Athanasopoulus et al., 2009). We label this "Top-down FP" within the tables that follow. Because the results can show, this methodology has worked well with the tourism hierarchies that were considered in this paper. The best disadvantage of this methodology, that is indeed a disadvantage of any top-down approach, is that these approaches don't produce unbiased revised forecasts, though the bottom forecasts are unbiased (refer to the discussion of equation (5) in Hyndman et al. (2007)), (Athanasopoulus et al., 2009). Like the 2 previous top-down approaches,

$$P = [p|0_{m_k \times (m-1)}], \tag{3.3.6}$$

where $p = [p1, p2, ..., p_{m_K}]'$ are a collection of proportions for the lowest level series. In order to present a general form for the lowest level proportions we would like to introduce some new notation. Let $\hat{Y}^{\ell}_{j,n}(h)$ be the h-step-ahead forecast of the series that corresponds to the node that is ℓ levels on top of i. Also let $\hat{S}_{i,n}(h)$ be the addition of the h-step-ahead forecasts below node i that are directly connected to node j. The two notations will be combined. As an example, in Figure 3.1, $\hat{S}^{(2)}_{AA,n}(h) = \hat{S}_{Total,n}(h) = \hat{Y}_{A,n}(h) + \hat{Y}_{B,n}(h) + \hat{Y}_{C,n}(h)$. If we tend to generate h-step-ahead forecasts for the series of the hierarchy in Figure 3.1, the revised final forecasts moving down the farthest left branch of the hierarchy are

$$\begin{aligned}\bar{Y}_{A,n}(h) &= \left(\frac{\tilde{Y}_{A,n}(h)}{\hat{S}^{(1)}_{A,n}(h)}\right)\hat{Y}_{Total,n}(h)\\ &= \left(\frac{\hat{Y}^{(1)}_{AA,n}(h)}{\hat{S}^{(2)}_{AA,n}(2)}\right)\hat{Y}_{Total,n}(h)\end{aligned}$$

and

$$\bar{Y}_{AA,n}(h) = \left(\frac{\tilde{Y}_{AA,n}(h)}{\hat{S}^{(1)}_{AA,n}(h)}\right)\tilde{Y}_{A,n}(h)$$

$$= \left(\frac{\tilde{Y}_{AA,n}(h)}{\hat{S}^{(1)}_{AA,n}(h)}\right)\left(\frac{\hat{Y}^{(1)}_{AA,n}(h)}{\hat{S}^{(2)}_{AA,n}(2)}\right)\tilde{Y}_{Total,n}(h)$$

Consequently,

$$p_1 = \left(\frac{\tilde{Y}_{AA,n}(h)}{\hat{S}^{(1)}_{AA,n}(h)}\right)\left(\frac{\hat{Y}^{(1)}_{AA,n}(h)}{\hat{S}^{(2)}_{AA,n}(2)}\right)$$

.

The other proportions are obtained similarly. The general result can be written as follows:

$$p_i = \prod_{\ell=0}^{k-1}\frac{\hat{Y}^{\ell}_{i,n}(h)}{\hat{S}^{\ell+1}_{i,n}(h)} \tag{3.3.7}$$

for $i = 1, 2, ...m_K$.

3.3.4 The optimal combination approach

For the final approach to hierarchical forecasting, we have considered optimal combination approach introduced by Hyndman et al. (2007). This approach optimally combines the base forecasts to provide a collection of revised forecasts that are as shut as attainable to the univariate forecasts, however additionally meet the need that forecasts at higher levels in the hierarchy are the total of the associated lower level forecasts. In contrast to the other existing technique, this approach uses all of the data accessible inside a hierarchy; allows for correlations and interactions between series at every level of the hierarchy; accounts for circumstantial changes of forecasts at any level; and, as long as the base forecasts are unbiased, produces unbiased forecasts that are consistent across all levels of the hierarchy. That is more, this approach may also turn out estimates of forecast uncertainty that are consistent across levels of the hierarchy (forecast intervals made by the optimum combination approach are the topic of our current research), (Athanasopoulus et al., 2009).

The overall plan springs from the illustration of the h-step-ahead base forecasts of a hierarchy by the simple regression model

$$\hat{Y}_n(h) = S\beta_h + \varepsilon_h, \tag{3.3.8}$$

where $\beta_h = E[\hat{Y}_{K,n}(h)|Y_1, ..., Y_n]$ is the unknown mean of the bottom forecasts of the bottom level K, and ε_h has zero mean and covariance matrix $V[\varepsilon_h] = \Sigma_h$. The term ε_h represents the error within the higher than regression, and may not be confused with the h-step-ahead forecast error. If $\sum_h$ is known then we will use generalized method of least squares estimation to get the minimum variance unbiased estimate of β_h. In general, this is not glorious, however Hyndman et al. (2007) show that below the affordable assumption that $\varepsilon_h \approx S\varepsilon_{k,h}$, where $\varepsilon_{K,h}$ contains the forecast errors within the bottom level, the best linear unbiased estimator for β_h is $\hat{\beta}_h = (S'S)^{-1}S'\hat{Y}_n(h)$. This ends up in the revised forecasts given by $\hat{Y}_n(h) = S\hat{\beta}_h$, and hence, within the general form of equation (2), (Athanasopoulus et al., 2009),

$$P = (S'S)^{-1}S'. \tag{3.3.9}$$

In some circumstances, less complicated forecasting equations will be obtained. Note that hierarchy one is balanced, which implies that identical degree of disaggregation takes place at every node inside a level; i.e., the amount of series at every node varies across levels, however not inside grade. So the easy analysis of variance technique conferred in equations (12) and (13), respectively of Hyndman et al. (2007) will be applied to produce the revised forecasts for the optimal combination approach, (Athanasopoulus et al., 2009).

3.4 Forecast combination and prediction intervals

Prediction interval (PI), is an estimate of an interval, with a certain probability in which future observation fall. A prediction interval bears the same relationship to a future observation that a frequentist confidence intervals bears to an unobservable population parameter. The difference between the confidence interval and prediction interval is that the prediction interval refers to the uncertainty of an estimate, while the confidence interval refers to the uncertainty associated with the parameters of a distribution.

3.4.1 Linear regression

$$y_t = \beta_0 + \beta_1 x + \varepsilon_t, \tag{3.4.10}$$

where y_t are the actual values observed, β_0 and β_1 are intercept and parameters, x are fitted values and ε is the error term.

3.4.2 Quantile regression averaging

Quantile Regression Averaging(QRA) refers to forest combination method to computation of prediction intervals QRA involves applying quantile regression to the point forecasts of a small number of individual forecasting models (Nowotarski and Weron, 2015). Independent variables are used for the individual point estimates and dependent variables as the corresponding observed target variables in a standard quartile regression setting. The Quantile Regression Averaging method yields an interval forecast of the target variables, but does not use the prediction intervals of the individual methods. One of the reasons for using point forecasts and not interval forecasts is their availability.

The quantile regression problem can be written as follows:

$$Q_y(q|X_t) = X_t\beta_q, \tag{3.4.11}$$

where $Q_y(q|.)$ is the conditional $q-th$ quantile of the dependent variables(y_t), $X_t = [1, \hat{y}_{1,t}, ..., \hat{y}_{m,t}]$ is a vector of point forecasts of m individual models(independent variables) and β_q is a vector of parameters(for quantile q). The parameters are estimated by minimizing the loss function for a particular q^{th} quantile:

$$\min_{\beta_q}[\sum_{\{t:y_t \geq X_t\beta_q\}} q|y_t - X_t\beta_q| + \sum_{\{t:y_t \geq X_t\beta_q\}} (1-q)|y_t - X_t\beta_q|] = \min_{\beta_q}[\sum_t (q - 1_{y_t < X_t\beta_q})(y_t - X_t\beta_q)] \tag{3.4.12}$$

QRA assigns weight to individual forecasting methods and combines them to yield forecast of chosen quantiles (Elamin, 2018).

3.5 Prediction Intervals

3.5.1 Prediction Interval Widths

Prediction interval width (PIW) is the difference of the estimated upper and lower limit values (Mpfumali, 2019; Sun et al., 2017). The general equation is given by:

$$PIW = UL_i - LL_i \qquad i = 1, ..., t \tag{3.5.13}$$

where UL_i and LL_i is the upper limit and lower limit of the prediction interval respectively. The PI normalised Average Width (PINAW) is used to evaluate the performance of prediction intervals (PIs) in this study. The PINAW describes the width of the PIs and is given as: equation (3.5.14):

$$\text{PINAW} = \frac{1}{m}\sum_{i=1}^{m}\frac{UL(X_i) - LL(X_i)}{y_{max} - y_{min}}, \tag{3.5.14}$$

where y_{min} and y_{max} represent the minimum and the maximum values of PIW, respectively.

3.5.2 Performance of estimated prediction intervals

Research has shown that combining point forecasts can improve the accuracy of the forecasts (Bates and Granger, 1969; Gaillard and Goude, 2015). This is then extended to combining prediction limits of the combined forecasts. In this section we present prediction interval combinations from the models GRA and LR and compare the combined intervals from the intervals of the individual models.

Simple average

This method takes the simple arithmetic means of the prediction limits given as $L_{Av} = \frac{1}{K}\sum_{k=1}^{K} L_k$ and $U_{Av} = \frac{1}{K}\sum_{k=1}^{K} U_k$. This approach is fairly simple and is known to produce intervals which are robust (Gaba et al. (2017)).

Envelop

Let $[L_k, U_k], k = 1, ..., K$ be the $100(1-\alpha)\%$ prediction intervals given by K forecasting methods. Let $[L_C, U_C]$ denote the $100(1-\alpha)\%$ combined prediction intervals using the interval combining method C (Gaba et al. (2017). In this paper we are going to use the simple average (Av), median (Md) and the probability averaging of endpoints and simple averaging midpoints (PM) as discussed in Gaba et al. (2017).

3.6 Evaluation of forecasts

3.6.1 Mean absolute percentage error

The mean absolute percentage error (MAPE) will be used to evaluate the accuracy of our forecasts. The equation of MAPE measure is given by equation (3.6.15).

$$MAPE = \frac{1}{m}\sum \frac{[y_t - \hat{y}_t]}{y_t}, \qquad (3.6.15)$$

where y_t are the actual values observed, $\hat{y}$ is a predicted value by the model, and m is the number of predictions (Adhikari and Agrawal, 2013).

3.6.2 Mean absolute error

MAE, known as the mean absolute error, is the absolute value of the difference between the forecasted value and the actual value. It indicates the magnitude of an error that can be expected between the forecasted average and the actual average. The equation of MAE is given by equation (3.6.16)

$$MSE = \frac{1}{m}\sum_{t=1}^{m}|y_t - \hat{y}_t|, \tag{3.6.16}$$

where y_t are the actual values observed, $\hat{y}$ is a predicted value by the model, and m is the number of predictions (Adhikari and Agrawal, 2013).

3.6.3 Mean absolute scaled error

Mean Absolute Scaled Error (MASE) is a scale-free error metric that gives each error as a ratio compared to a baseline's average error.

$$q_j = \frac{e_t}{\frac{1}{T-m}\sum_{t=m+1}^{T}|y_t - y_{t-m}|}, \tag{3.6.17}$$

where m is the seasonal period, y_t is the actual observations time series, e_t is the forecast error for a given period (Adhikari and Agrawal, 2013). Therefore the mean absolute scaled error is simply

$$MASE = mean(|q_j|). \tag{3.6.18}$$

3.6.4 Pinball loss function

The pinball loss function is relatively easy to use and is given as:

$$L(q_\tau, y_t) = \begin{cases} \tau(y_t - q_\tau), & \text{if } y_t > q_\tau, \\ (1-\tau)(q_\tau - y_t), & \text{if } y_t \leq q_\tau, \end{cases} \tag{3.6.19}$$

where q_τ is the quantile forecast and y_t is the median forecast which represents the observed value.

Chapter 4

Empirical results and discussion

4.1 Introduction

In this chapter the methodology discussed in Chapter 3 is going to be used to presents data analysis and discussion. This chapter is going to compare Bottom–up, top-down and optimal approaches for forecasting monthly electricity demand.

4.2 Exploratory Data Analysis

Electricity demand(GWh) in South Africa from 2002 to 2018 is used. The data was obtained from Stats SA. The data is disaggregated according to nine provinces of South Africa of to come up with a hierarchy of levels 0 and level 1, respectively. A summary of the descriptive statistics of the electricity demand are summarized in Table 4.1.

Table 4.1: Summary statistics for electricity demand in Gigawatt-hours (GWh).

Provinces	Mean	Median	Max	Min	St.Dev	Skewness	Kurtosis
WC	1892.083	1713.5	1577	2102	98.70495	-0.5429085	3.419669
EC	635.6324	630	434	840	92.01321	0.2492269	2.431469
NC	452.8333	446	330	592	49.71229	0.2587879	2.896151
FS	855.0196	860.5	678	1018	62.47506	0.0490752	2.83662
KZN	3649.618	3670.5	2917	4139	223.7042	-0.282069	2.775754
NW	2406.779	2415	1915	2737	163.5153	-0.6697007	3.454785
GT	4902.525	4807	3902	6194	488.7616	0.5366371	2.58735
ML	2760.583	2774.5	2095	3382	196.5153	0.08442	3.869602
LP	1030.529	1062.5	620	1305	155.8244	-0559052	2.311214

Table 4.1 shows that the mean electricity demand range from 452.8333 to 4906.779 with Gauteng (GT) exhibiting a highest mean of 4906.779. Electricity demand for Gauteng (GT) are the highest and North West (NW) have the lowest value of 592 in one of the periods. The skewness values of electricity demand in South Africa under Eastern Cape (EC), Northern Cape (NC), Free State (FS), Gauteng (GT) and Mpumalanga (MP) are positive which implies that they are positively skewed and Western Cape (WC), Kwazulu-Natal (KZN), North West (NW) and Limpopo (LP) are negative which implies that they are negatively skewed, meaning their distributions are non-normal. The kurtosis for all the cases of Western Cape, North West and Mpumulanga (MP) are greater than three which shows that their distributions are leptokurtic. This implies that the data sets can be moded

by heavy tailed distributions. Eastern Cape (EC), North West (NW), Free State (FS), Kwazulu-Natal (KZN), Gauteng (GT) and Limpopo (LP) are less than 3 which shows that their distribution is platykurtic. Implying that the dataset has lighter tails than a normal distribution (less in the tails).

Konarasinghe and Abeynayake (2014) indicated that electricity demand pattern can be recognized by box plots hence, a graphical representation in the form of box plots of the monthly electricity demand in South Africa was done. The box plots for the monthly electricity demand in South Africa are illustrated in Figure 4.1.

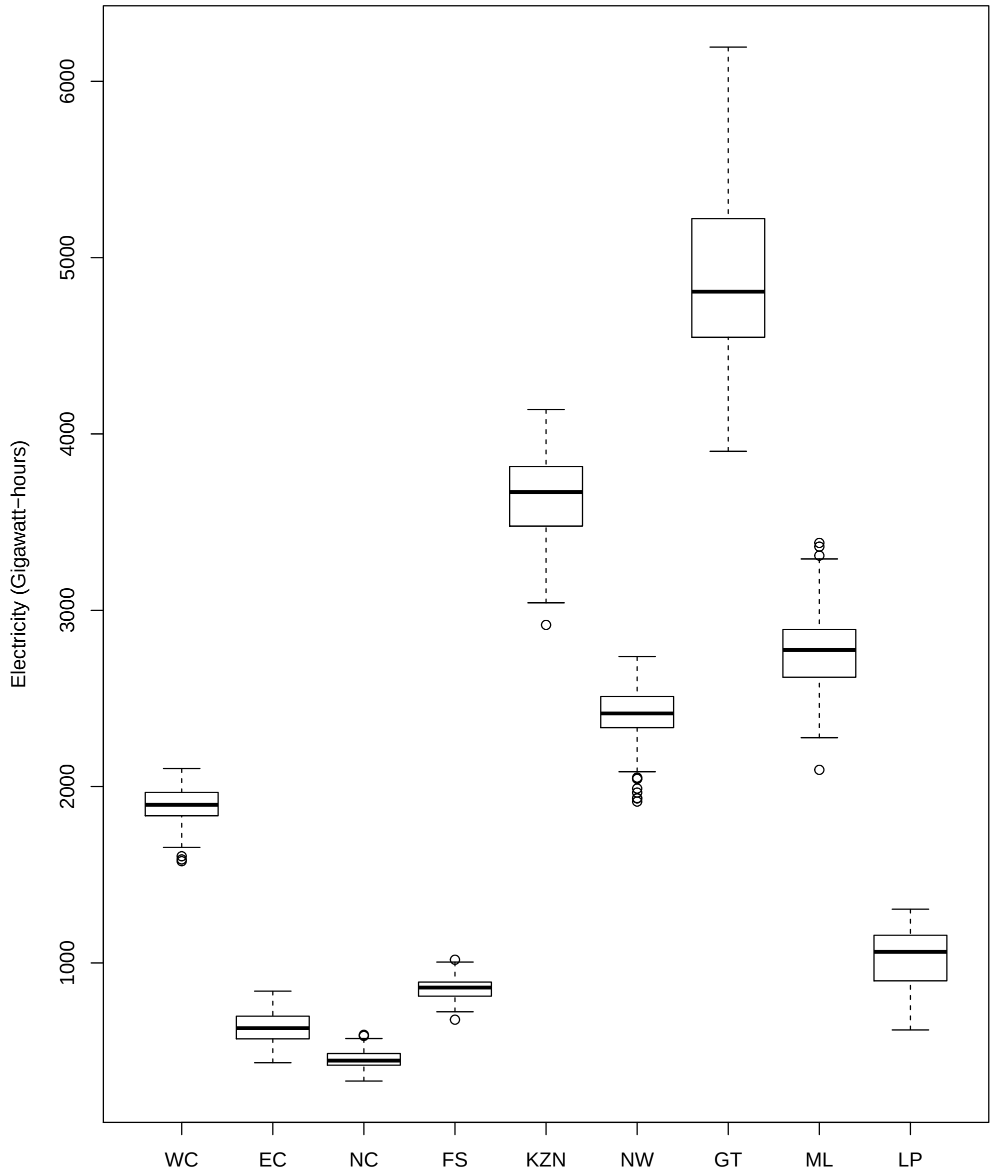

Figure 4.1: Boxplots of electricity generated and available for distribution from 2000 to 2018.

The box plots give summaries of the minimum, lower quartile, median, upper quartile and maximum number of electricity demand in each provinces. The pattern of electricity demand is noted and the box plots also indicate that the electricity demand of South Africa is dominated by Gauteng (GT), then Kwazulu Natal (KZN), Mpumulanga (MP), North West (NW), Western Cape (WC), Limpopo (LP), Free State (FS), Eastern Cape (EC) and Northern Cape (NC) respectively. Figure 4.2 shows the map of South Africa indicating all nine provinces the country has.

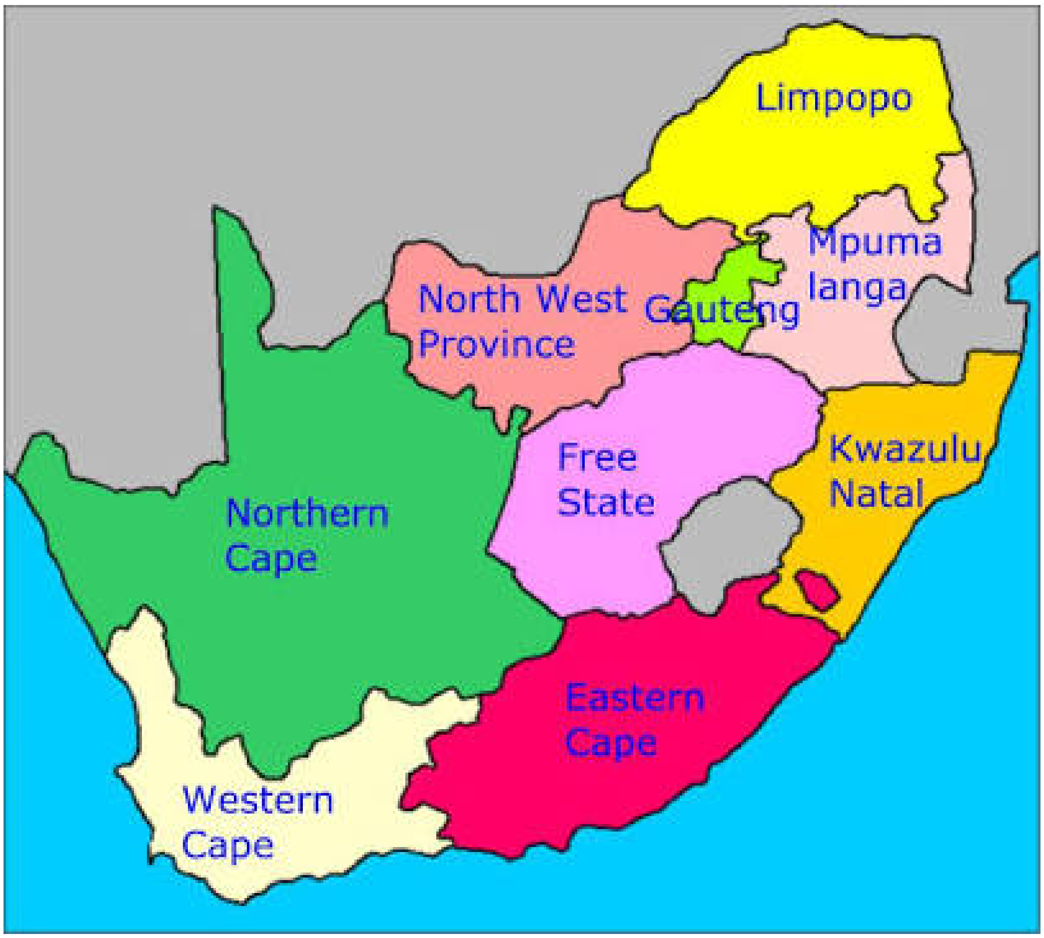

Figure 4.2: Map of South Africa provinces (source: https://www.southafrica.to/provinces/ provinces.htm).

4.3 Forecasting hierarchical time series

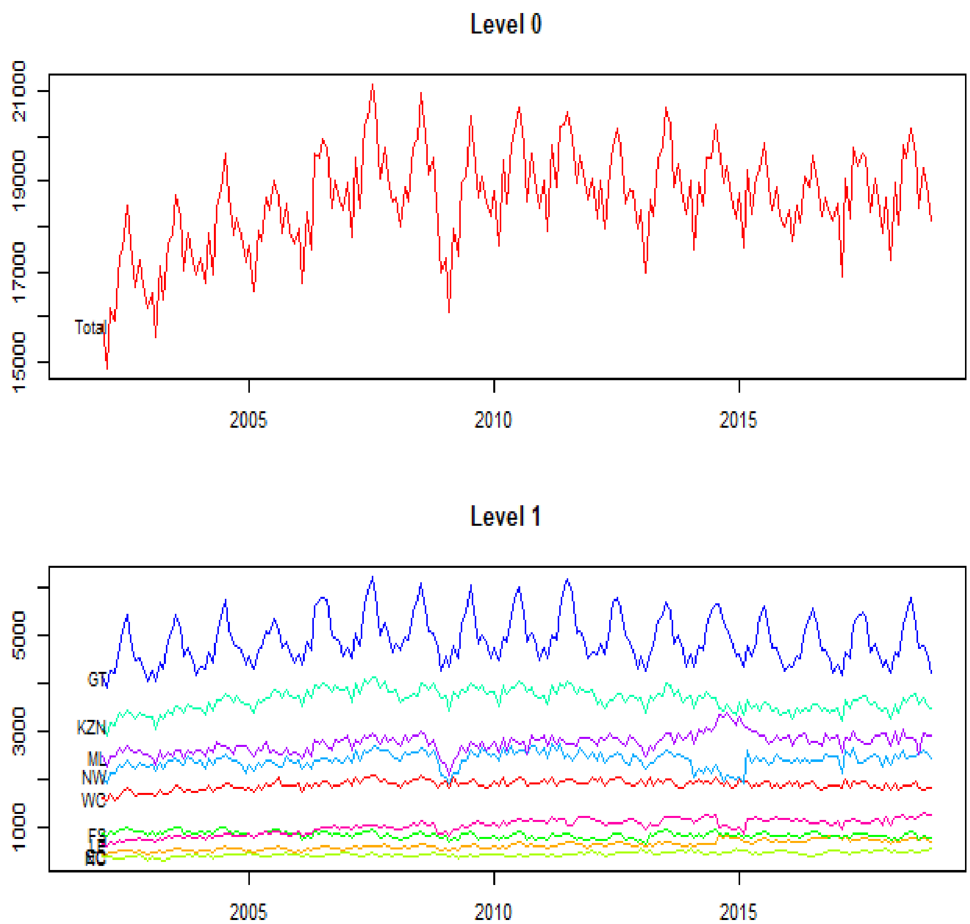

Figure 4.3: Time series plots of electricity generated and available for distribution from 2002 to 2018 in Gigawatt-hours (GWh).

The colours for level 1, royal blue line represents Gauteng (GT), Turquoise line represents Kwazulu-Natal, Purple line represents Mpumalanga, Sky blue represents North-West, Red line represents Western-Cape, Green line represents Free State, Pink line represents Limpopo, Orange line represents Northern Cape and Lime green line Eastern Cape. At level 1, it can be noted that Gauteng electricity demand is higher than most followed by Kwazulu-Natal, Mpumulanga, North-West, Western-Cape, Free State, Limpopo, Northern Cape and then Eastern Cape.

Time series decomposition using STL was done as showed in figure A.5 in the appendix. STL is an algorithm that was developed to help to divide up a time series into three components namely: the trend, seasonality and remainder(Clveland et al., 1990). The plot in Figure A.5 shows that the data have an overall upward trend (it increases at the beginning, then stationary in the middle and decrease at the end) and seasonal fluctuations. The seasonality decreases at the beginning of each year and increases toward the end of each year.

4.4 Forecasting hierarchical electricity demand series

This study employs the exponential smoothing based on innovations state space model in forecasting the South Africa's electricity demand across the nine provinces. Athanasopoulos et al. (2007) used the same model to forecast the Austrian tourism demand who state that this method include additive models that result in better forecast.

4.5 Forecasting accuracy of the models

Out-of-sample forecast performance evaluation was conducted to get some indication of the forecasting performance of the models. Forecast accuracy is a measure of how close the actual values are to the forecasted values (quantity). The data employed for training the models runs from January 2002 to December 2018. Electricity demand from January 2017 to December 2018 is used for validation. Electricity demand forecasting is done using the bottom-up approach; top-down approach which is the average of historical proportions, the proportion of historical averages and forecast proportions; and the optimal combination approach. MAPE, known as mean average percentage error, is measure of prediction accuracy of a forecasting method in statistics. It allows us to compare forecasts of different approaches in different scales. MAE, known as the mean absolute error, is the absolute value of the difference between the forecasted value and the actual value. It indicates the magnitude of an error that can be expected between the forecasted average and the actual average. Large errors are rare, and are adjusted using the RMSE. The RMSE, known as the root mean square error, is used to measure the differences between values predicted by a model or an estimator and the values observed. To measure the accuracy of forecasts, MASE is used. MASE is known as the mean absolute scaled error. All these methods are simple tools for evaluating forecast accuracy, however this study employs the MAPE method to determine the best performing approach. The MAPE approach is chosen because of its popularity across literature and it's easier to understand. The approach which has the lowest 'average' MAPE is determined as the best performing approach.

The table shows the MAPE of each model, averaged across the five methods.

Table 4.2: Comparative analysis of the fitted models.

MAPE	WC	EC	NC	FS	KZN	NW	GT	ML	LP	Average MAPE
Bottom up	1.43	13.51	4.06	3.01	2.07	3.29	1.62	2.67	4.37	**4.00**
Top—down HP1	3.51	16.81	11.68	5.19	4.63	4.61	6.12	4.77	11.75	7.67
Top—down HP2	3.45	16.68	11.82	4.91	4.56	4.64	6.18	4.78	11.48	7.61
Top—down FP	1.35	13.46	4.11	2.96	2.11	3.27	1.68	2.71	4.44	4.01
Optimal	1.39	13.47	4.11	2.97	2.09	3.28	1.66	2.70	4.43	4.01

Table 4.2. shows the forecasting performance results for each province based on each approach using the MAPE method. From the three alternative approaches, seemingly the overall best performing is the bottom-up approach since it has the lowest 'Average' MAPE equal to 4. It is followed by the optimal combination and Top-Down based on forecasted proportions approaches which are both equal to 4.01, however bottom-up approach is deemed the overall best performing. This implies that this method is capable of generating accuracy electricity demand forecasts.

4.6 Electricity demand forecasts

To predict the electricity demand for South Africa over the 60-months period, the bottom-up approach is applied across all nine provinces. Table 4.3 summarises the forecasts produced using the bottom-up approach and Figure 4.4 is the graphical picture of the forecasts and the original data series.

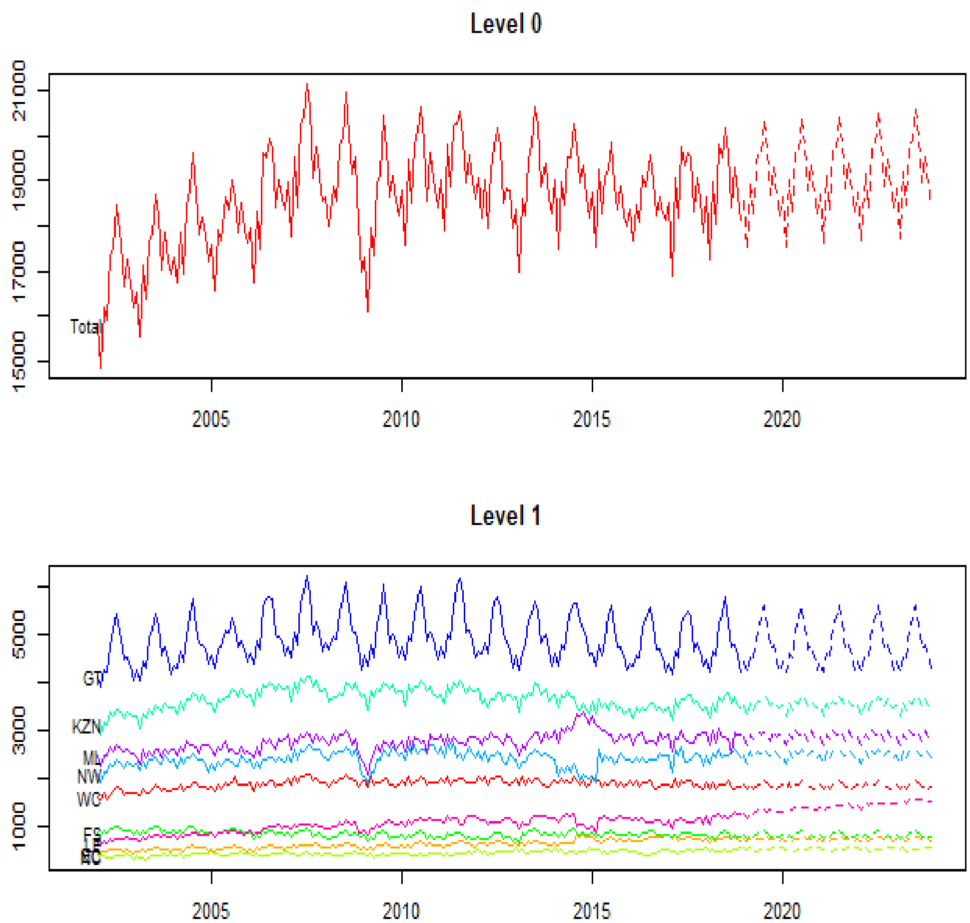

Figure 4.4: Bottom-up forecast in Gigawatt-hours (GWh).

From figure 4.4, the top part represents the total electricity demand in South Africa. On the bottom part, the solid lines represent the historical electricity demand data and the dashed lines represent the projected electricity demand forecasts for each of the nine provinces in South Africa. Furthermore, on bottom part of figure 4.4, the colour royal blue represents Gauteng province (GT), turquoise represents Kwazulu-Natal (KZN), purple represents Mpumalanga (MP), sky blue represents North-West (NW), red represents Western-Cape (WC), green represent Free State (FS), Pink represents Limpopo (LP), Orange represents Northern Cape (NC) and lime green represents Eastern Cape (EC). At level 1, it can be noted that Gauteng electricity demand is higher than most followed by Kwazulu-Natal (KZN), Mpumulanga (MP), North-West (NW), Western-Cape (WC), Free State (FS), Limpopo(LP), Northern Cape (NC) and then Eastern Cape (EC). According to the bottom-up approach, electricity demand is seasonal as high peaks are observed towards winter season. Gauteng is topping because the province have the largest city in the country and all top industries in the country are based there. Free State, Limpopo, Northern Cape and then Eastern Cape, have lowest electricity demand respectively and the reason is the provinces consist of rural areas compared to others.

Table 4.3: Out-of-sample future electricity demand forecasts (GWh).

Time period	WC	EC	NC	FS	KZN	NW	GT	ML	LP	Total
Jun 2019	1871.9	748.2	519.8	877.2	3566.3	2506.2	5342.2	2893.7	1319.8	19645.5
Dec 2019	1825.1	697.7	569.5	794.9	3476.6	2476.7	4229.2	2890.9	1311.2	18217.6
Jun 2020	1872.2	748.9	520.2	877.3	3568.3	2506.2	5344.9	2893.7	1376.1	19707.9
Dec 2020	1825.3	698.2	569.9	794.9	3478.3	2422.5	4231.1	2890.9	1367.4	18278.7
Jun 2021	1872.4	749.4	520.5	877.3	3569.8	2506.4	5347.1	2893.7	1432.3	19768.9
Dec 2021	1825.4	698.7	570.2	794.9	3479.6	2422.6	4232.6	2890.9	1423.7	18338.7
Jun 2022	1872.5	749.8	520.7	877.3	3571.0	2506.4	5348.8	2893.7	1488.6	19828.8
Dec 2022	1825.5	699.1	570.4	794.9	3480.6	2422.6	4233.8	2890.9	1479.9	18397.8
Jun 2023	1872.6	750.1	520.9	877.3	3571.9	2506.4	5350.1	2893.7	1544.8	19887.9
Dec 2023	1825.6	699.3	570.6	794.9	3481.4	2422.6	4234.7	2890.9	1536.1	18456.32

4.7 Bottom–up approach forecast

Figure 4.5 shows the time series plot of monthly electricity demand forecast of bottom–up approach for the next 60 months with density, normal quantile to quantile (QQ) and box plots. Figure 4.6 presents the monthly electricity demand forecast trend, with smoothing spline fitted with lambda value estimated.

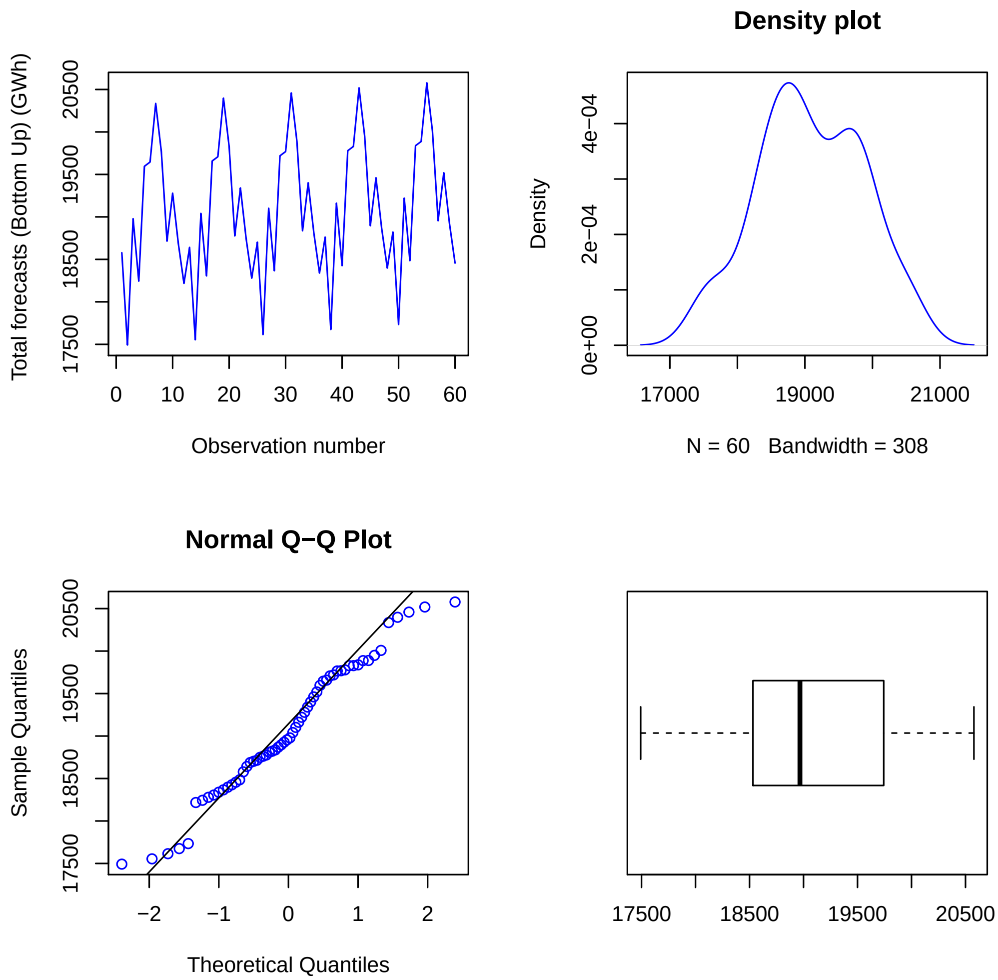

Figure 4.5: Buttom–up forecast diagnostic plots for monthly electricity demand.

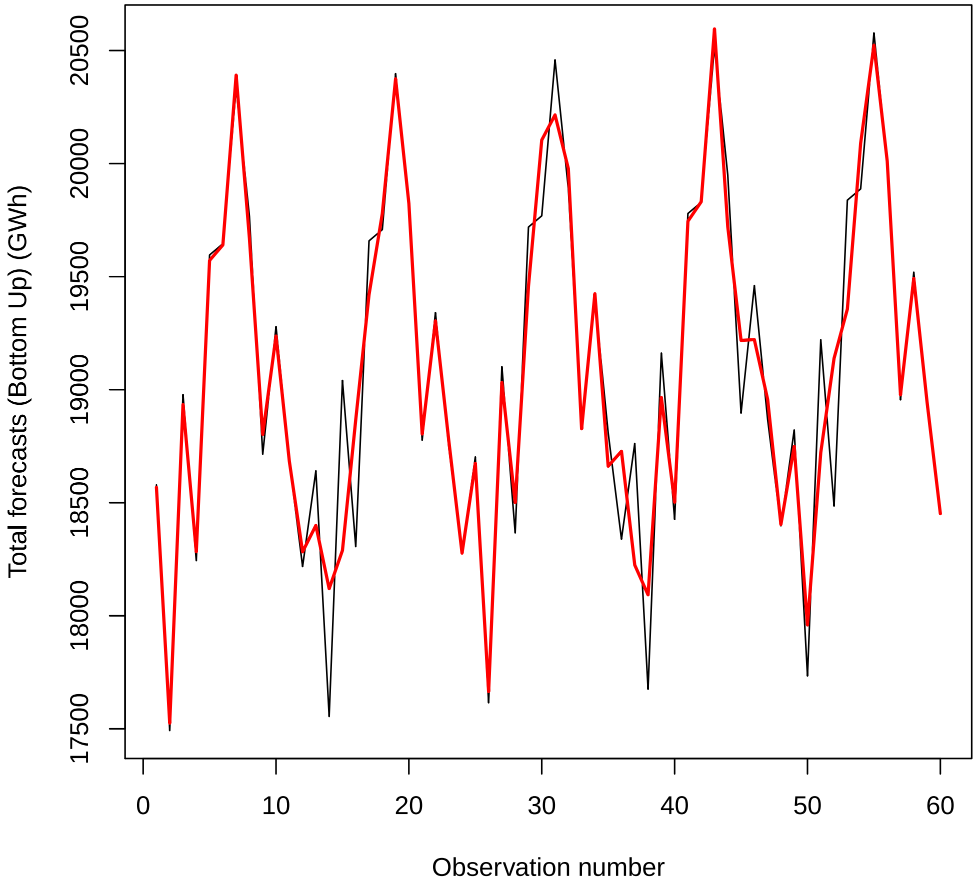

Figure 4.6: Buttom–up forecast plot of monthly electricity demand from January 2019 to December 2023 superimposed with a fitted smoothing spline trend.

4.8 Prediction interval for buttom–up forecasts

The importance of prediction interval is that they express the uncertainty in forecast. When prediction interval is produced, it gives a clear image how much uncertainty is associated with each forecasts.

The forecasts are smoothed using the penalised cubic smoothing spline function. The function is given by:

$$\pi(t) = \sum_{t=1}^{m}(y_t - f(t))^2 + \alpha \int (f''(t))^2 dx \tag{4.8.1}$$

where α is the smoothing parameter. The α value based on the generalised cross validation (GCV) ($\alpha = 368588.8$) as shown in Figure 4.6.

4.8.1 Prediction intervals based on linear regression

In Figure 4.7 upper limit prediction interval is given by the red line and black line present bottom–up approach forecast for the next 60 months. Figure 4.7 shows upper limit prediction interval for 95 % confident interval. The pattern of the forecast and prediction lines are similar. Bottom–up forecast line does not cross the prediction interval. Bottom–up forecast line does not cross the prediction interval, therefore we are 95 % confident that the bottom up approach forecasts falls within upper limit.In figure 4. red line present lower limit prediction interval and black line present bottom–up approach forecast for the next 60 months. Figure 4.8 shows lower limit prediction interval for 5

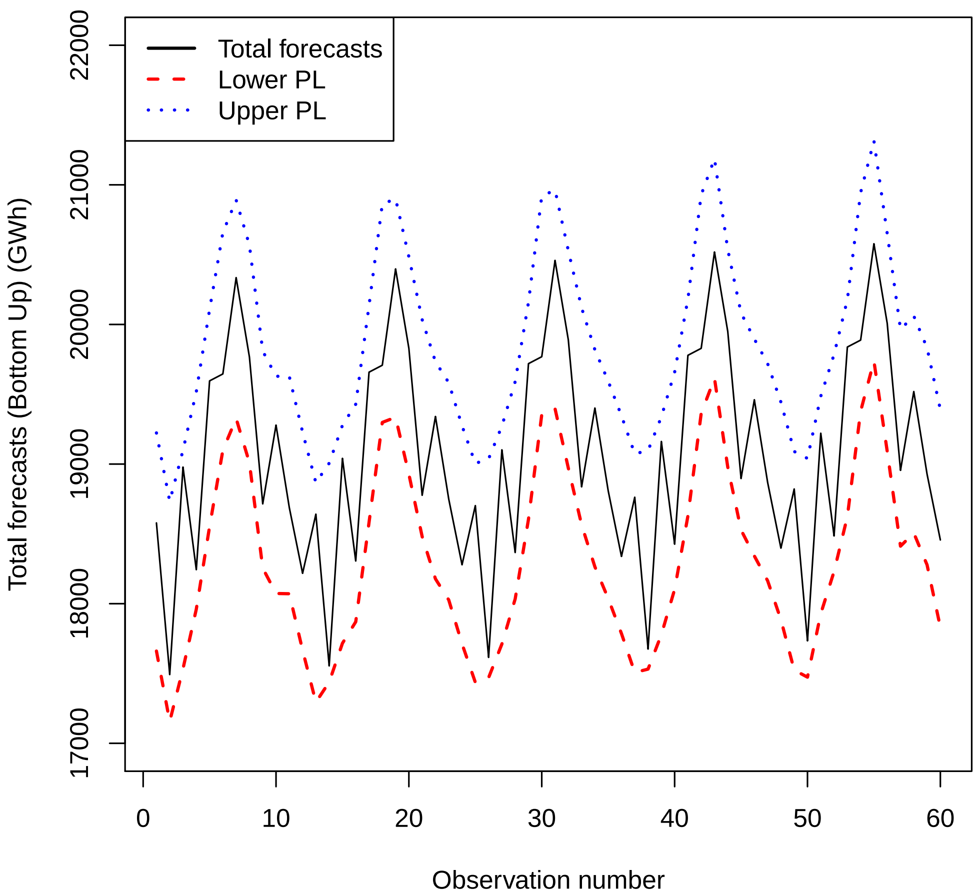

Figure 4.7: Total forecasts with 95 % prediction interval.

% confident interval. Bottom–up forecast line does not cross the prediction interval, therefore we are 5 % confident that the bottom up approach forecasts falls within lower limit.

4.8.2 Prediction intervals based on quantile regression

Figure 4.8 presents the lower and upper limits prediction intervals for 95 % confidence interval of quantile regression model.

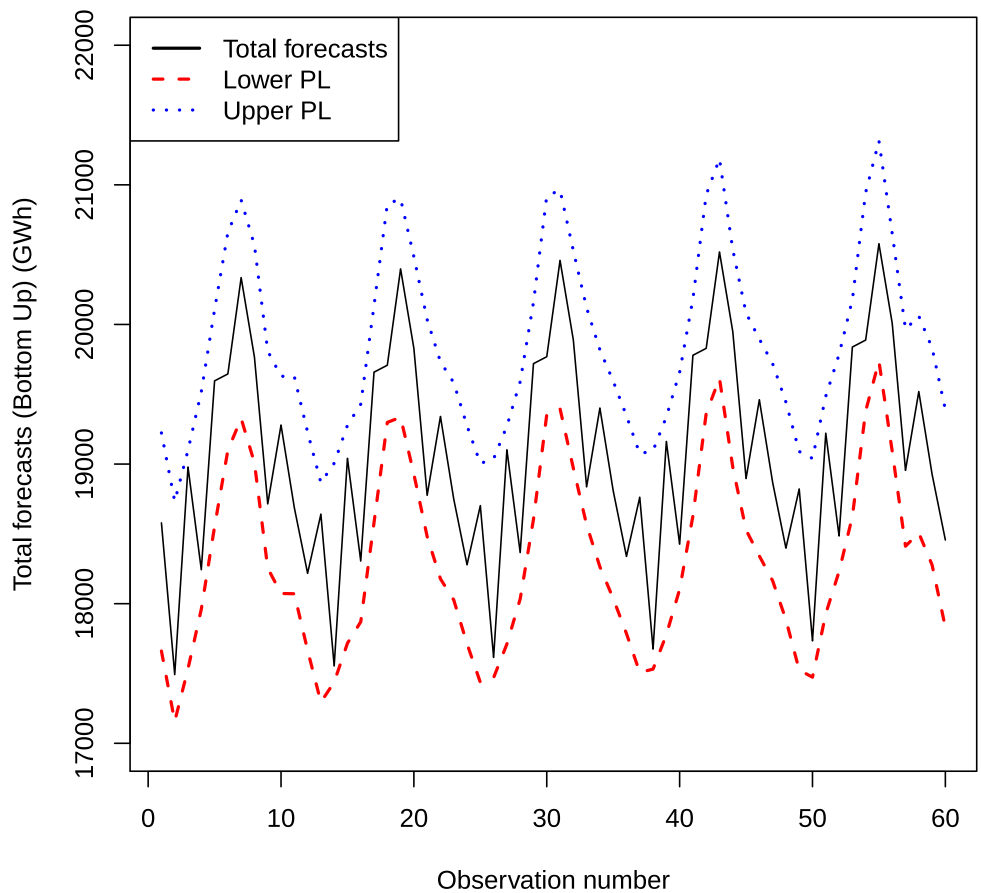

Figure 4.8: Total forecasts with 95 % prediction interval.

4.9 Comparative Analysis of the Models

4.9.1 Evaluation of prediction intervals

Figure 4.4 shows pinball loss box plots for LR, QRA, SA and EN. Table 4.4 shows a comparative evaluation of the models using PI indices for 95 % PINC value and pinball loss. The best model based on PINAW at 95 % is Envelop(ENV) with 0.27 %, followed by Simple averaging (SA) with 0.38 % and QRA with 1.21 %. The best model based on pinball loss is QRA with 13.81, followed by SA with 16.42, LR and ENV both with 19.50.

Table 4.4: Model comparisons.

	FBU	FHP1	FOPT	QRA	LR	SA	ENV
PINAW	89.38 %	26.86 %	28.26 %	1.21 %	2.62 %	0.38 %	0.27 %
Pinball loss	142.95	177.95	159.64	13.81	19.50	16.42	19.50

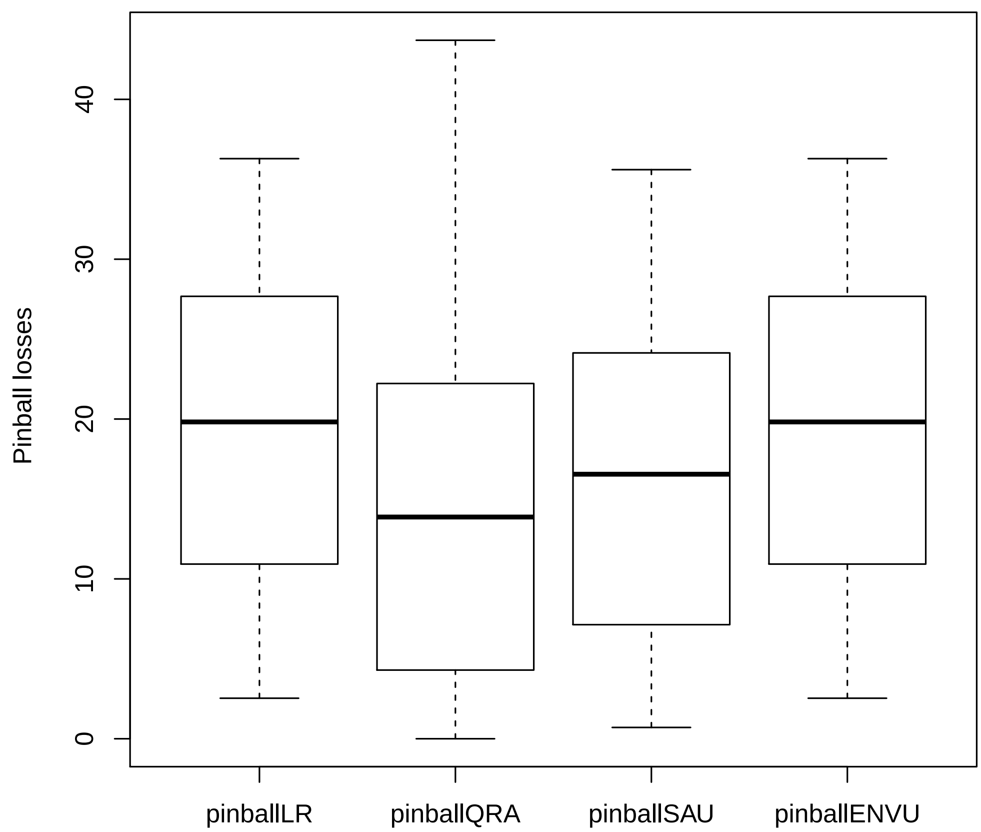

Figure 4.9: Box plots for pinball loss.

Chapter 5

Conclusion

5.1 Introduction

This dissertation discussed the application of top-down, bottom-up, optimal combination models for electricity demand in South Africa.The models were developed based on provincial monthly electricity demand in South Africa data from Stats SA. The fitted models were compared with linear regression and quantile regression averaging.

5.2 Summary

Chapter 1 discussed the main aim of the study which was to apply a modelling framework for forecasting electricity demand using hierarchical time series. The chapter also discussed the main objectives of the study which were to: apply a top-down, bottom-up and optimal combination methods in forecasting electricity demand in South Africa and compare the performance of the developed models.

Chapter 2 provide summaries of some studies that used the proposed methodology to forecast electricity demand. Chapter 3 discussed the general theory of the proposed methods, which are top-down, bottom-up, optimal combination, quantile regression (QR) and Linear regression (LR).

Chapter 4 presented a discussion of the empirical results. Section 4.3 shows a time series plots of electricity demand. The performance of the models was evaluated using the accuracy measures root mean square error (RMSE) and mean absolute error (MAE), and the best models were selected based on the Mean Absolute Percentage Error (MAPE).In order to combine forecasts and compute the prediction intervals for the developed models the quantile regression averaging (QRA) and linear regression (LR) was used in section 4.8.

5.3 Research Findings

The hierarchical forecasting methodology is applied to the electricity demand in South Africa. MAPE was used to determine the best performance of the approaches. The bottom-up, optimal combination and three versions of the top-down approaches were considered. The approach with lower MAPE was considered the best approach among others. The bottom-up approach was considered the best performing approach for the electricity demand in South Africa according to the forecasting performance evaluation method because it had the lowest MAPE which was 4.00%. Top-down approach based on forecasted proportions and optimal combination approaches were both second best approach having a MAPE of 4.01%. Top-down approaches based on historical proportions 1 and 2 were the last best performing approaches with 7.61% and 7.61% respectively Our aggregate electricity demand in South Africa forecasts exhibited a mixed pattern with seasonality over the 60 months forecasted. Disaggregated series allowed us to recognise the status of electricity demand in South Africa.

The best set of forecasts was selected based on the prediction interval normalised average width (PINAW) and pinball loss. The best model based on PINAW at 95 % is Envelop(ENV) with 0.27 %, followed by Simple averaging (SA) with 0.38 % , QRA with 1.21 % and LR with 2.62 %. The best model based on pinball loss is QRA with 13.81, followed by SA with 16.42, LR and ENV both with 19.50. The results showed that QRA is the best since it produces robust prediction intervals as compared to the other models.

Gauteng have the highest demand of electricity compared to the rest of the provinces. The reason been the number of industrial factories that require lot of electricity to operate, it has the highest population in the country and the province is more occupied by suburb regions where they tend to use electricity for everything. While Eastern Cape and Northern Cape have the lowest electricity demand in the country, because they are less populated. And most areas in these provinces are still developing so they use of electricity is less. The most cause of higher electricity demand is the dramatically changes in the climate, high humidity can lead to higher electricity usage either from running an air conditioner or turning on additional fans.

5.4 Limitations of the dissertation

This dissertation used provincial data which only give the electricity demand for each province but it is impossible to know what causes the electricity demand to be low or higher for each provinces. it is sometimes difficult to accurately fit numerous complex factors that affect demand of electricity into forecasting models.

5.5 Recommendations

Many people in our society lack the awareness of high electricity demand in South Africa, the government should introduce programs that teach or educate people about solar energy, wind mill, generators and other climate friendly energy sources. The electricity must be used more efficiently, the society should be encouraged to use more efficient appliances, avoid wasting electricity and reduce the amount of electricity used in the country. This study could be useful to system operators, including decision-makers in power utility companies.

Appendix A

Future forecast of all three top–down approaches and optimal combination approach

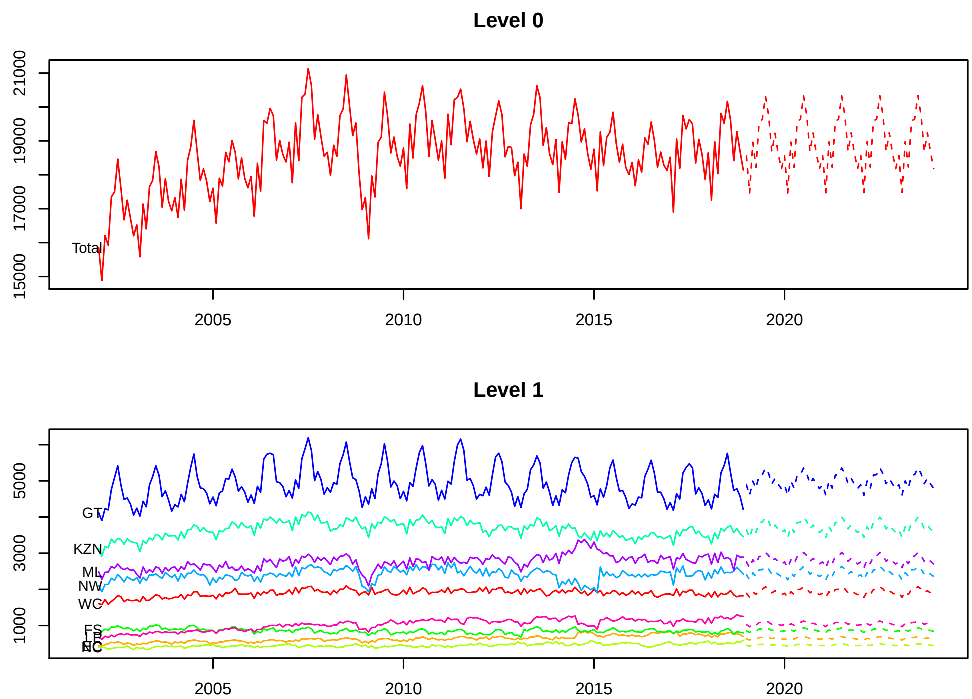

Figure A.1: Top–down PH1 forecasts in Gigawatt-hours (GWh).

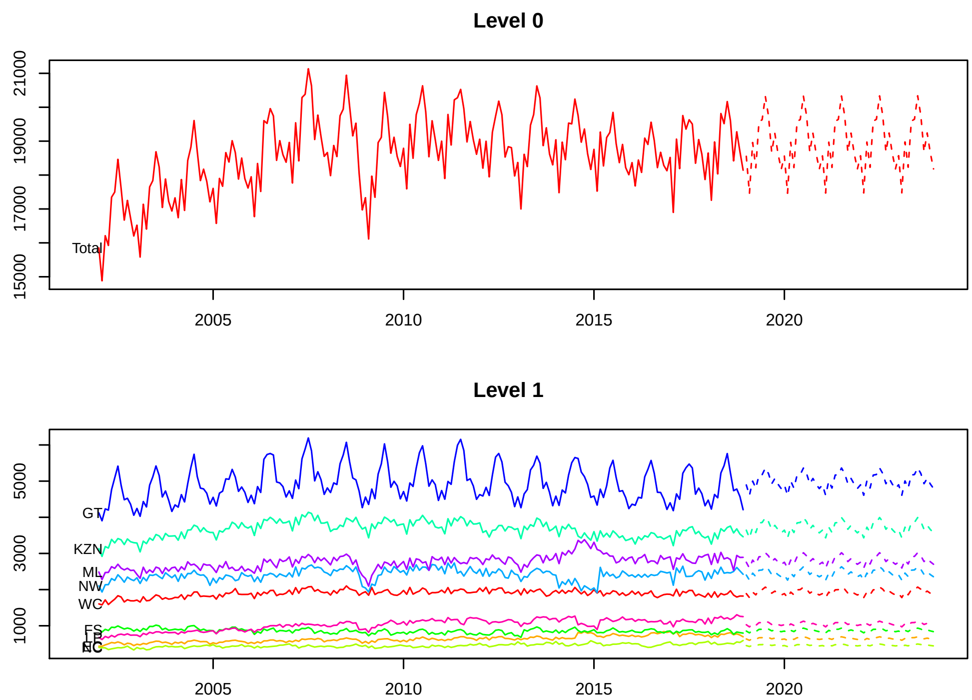

Figure A.2: Top–down PH2 forecast in Gigawatt-hours (GWh).

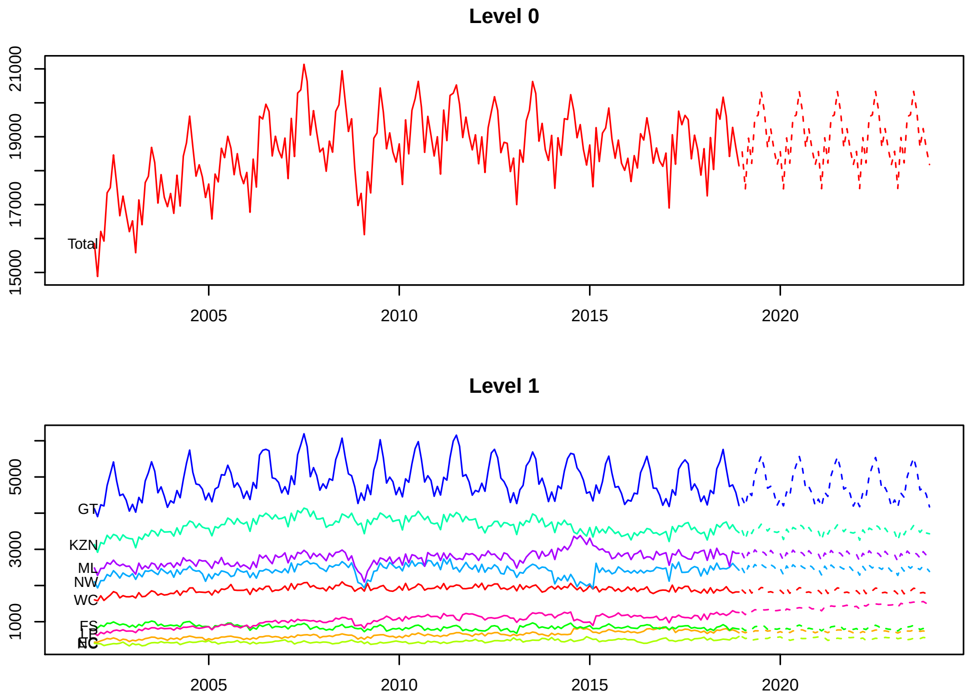

Figure A.3: Top–down FP forecast in Gigawatt-hours (GWh).

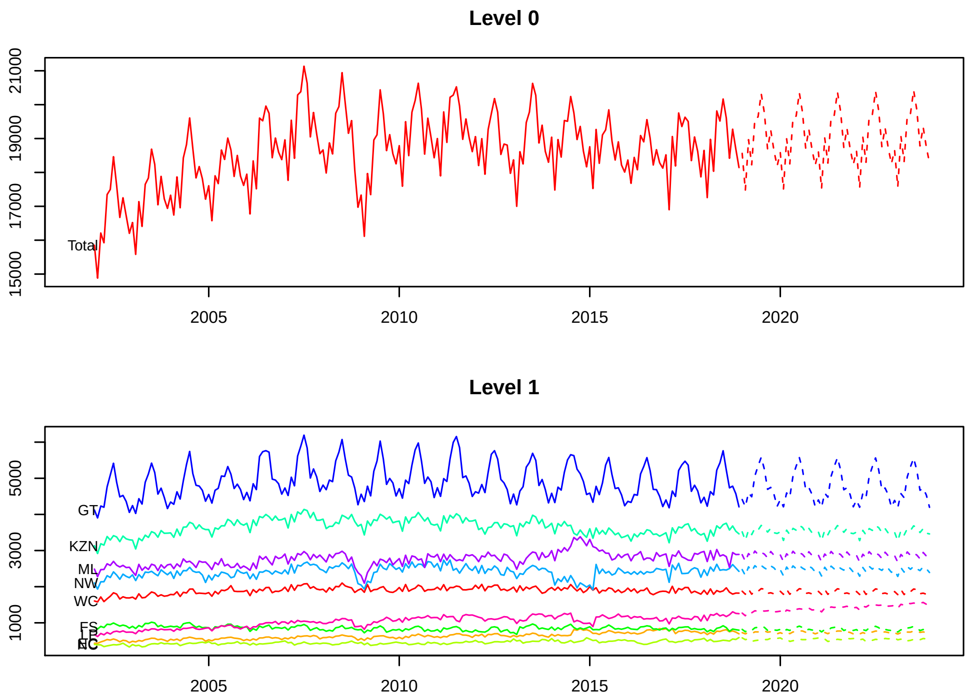

Figure A.4: Optimal combination forecast in Gigawatt-hours (GWh).

Table A.1: Out-of-sample future electricity demand forecasts in Gigawatt-hours (GWh).

Month	Year	Total	WC	EC	NC	FS	KZN
Jan	2019	18578.69	1876.084	728.6953	583.8206	819.4399	3518.984
Feb	2019	17492.21	1791.615	695.0500	535.1481	753.0912	3283.921
Mar	2019	18978.49	1929.734	734.2645	554.0353	818.5157	3568.067
Apr	2019	18243.68	1793.557	701.3496	486.2513	791.1997	3442.819
May	2019	19595.82	1882.508	749.4521	507.1767	863.0126	3617.766
Jun	2019	19645.53	1871.986	748.2025	519.8369	877.2921	3566.324
Jul	2019	20335.42	1947.190	776.1672	547.4918	910.5990	3697.412
Aug	2019	19766.37	1931.969	776.1563	536.3794	873.6671	3645.028
Sep	2019	18714.79	1817.604	741.6899	537.9435	793.0017	3513.582
Oct	2019	19279.15	1834.171	755.0629	572.2098	825.3513	3621.051
Nov	2019	18687.93	1804.441	737.0078	562.8548	804.8357	3497.968
Dec	2019	18217.55	1825.056	697.6632	569.5367	794.9741	3476.598
Jan	2020	18641.27	1876.364	729.4246	584.2792	819.4399	3521.188
Feb	2020	17554.37	1791.888	695.7647	535.5600	753.0912	3285.933
Mar	2020	19040.91	1929.999	734.9648	554.4532	818.5157	3570.204
Apr	2020	18305.79	1793.815	702.0359	486.6107	791.1997	3444.835
May	2020	19658.22	1882.760	750.1246	507.5441	863.0126	3619.837
Jun	2020	19707.92	1872.231	748.8614	520.2059	877.2921	3568.321
Jul	2020	20397.89	1947.428	776.8130	547.8726	910.5990	3699.437
Aug	2020	19828.52	1932.201	776.7890	536.7449	873.6671	3646.980
Sep	2020	18776.47	1817.830	742.3099	538.3028	793.0017	3515.422
Oct	2020	19340.81	1834.391	755.6705	572.5843	825.3513	3622.905
Nov	2020	18749.33	1804.656	737.6032	563.2158	804.8357	3499.720
Dec	2020	18278.67	1825.265	698.2467	569.8947	794.9741	3478.301
Jan	2021	18702.41	1876.567	729.9963	584.6390	819.4399	3522.874
Feb	2021	17615.17	1792.086	696.3249	535.8832	753.0912	3287.471
Mar	2021	19101.93	1930.192	735.5138	554.7812	818.5157	3571.838
Apr	2021	18366.56	1794.003	702.5739	486.8928	791.1997	3446.378
May	2021	19719.22	1882.943	750.6517	507.8324	863.0126	3621.422
Jun	2021	19768.91	1872.409	749.3780	520.4954	877.2921	3569.849
Jul	2021	20458.95	1947.602	777.3192	548.1714	910.5990	3700.985

Table A.2: Out-of-sample future electricity demand forecasts in Gigawatt-hours (GWh) conti...

Aug	2021	19889.31	1932.370	777.2851	537.0318	873.6671	3648.473
Sep	2021	18836.91	1817.994	742.7960	538.5847	793.0017	3516.829
Oct	2021	19401.24	1834.552	756.1468	572.8782	825.3513	3624.323
Nov	2021	18809.55	1804.812	738.0699	563.4991	804.8357	3501.059
Dec	2021	18338.68	1825.417	698.7041	570.1756	794.9741	3479.603
Jan	2022	18762.42	1876.715	730.4445	584.9214	819.4399	3524.164
Feb	2022	17674.93	1792.230	696.7641	536.1369	753.0912	3288.648
Mar	2022	19161.85	1930.332	735.9442	555.0385	818.5157	3573.088
Apr	2022	18426.29	1794.140	702.9956	487.1141	791.1997	3447.557
May	2022	19779.14	1883.075	751.0650	508.0586	863.0126	3622.634
Jun	2022	19828.83	1872.539	749.7830	520.7226	877.2921	3571.017
Jul	2022	20518.91	1947.728	777.7160	548.4059	910.5990	3702.170
Aug	2022	19949.08	1932.492	777.6740	537.2570	873.6671	3649.615
Sep	2022	18896.39	1818.114	743.1771	538.8060	793.0017	3517.905
Oct	2022	19460.72	1834.668	756.5203	573.1088	825.3513	3625.408
Nov	2022	18868.86	1804.925	738.4359	563.7214	804.8357	3502.084
Dec	2022	18397.83	1825.527	699.0626	570.3960	794.9741	3480.599
Jan	2023	18821.58	1876.822	730.7959	585.1430	819.4399	3525.150
Feb	2023	17733.89	1792.334	697.1085	536.3359	753.0912	3289.548
Mar	2023	19220.94	1930.434	736.2816	555.2405	818.5157	3574.045
Apr	2023	18485.23	1794.239	703.3263	487.2878	791.1997	3448.460
May	2023	19838.22	1883.172	751.3890	508.2361	863.0126	3623.562
Jun	2023	19887.91	1872.633	750.1005	520.9009	877.2921	3571.911
Jul	2023	20578.03	1947.820	778.0272	548.5899	910.5990	3703.076
Aug	2023	20008.04	1932.582	777.9789	537.4336	873.6671	3650.489
Sep	2023	18955.14	1818.201	743.4759	538.9796	793.0017	3518.729
Oct	2023	19519.46	1834.753	756.8130	573.2898	825.3513	3626.238
Nov	2023	18927.48	1805.008	738.7227	563.8959	804.8357	3502.868
Dec	2023	18456.32	1825.608	699.3438	570.5690	794.9741	3481.361

Table A.3: Out-of-sample future electricity demand forecasts in Gigawatt-hours (GWh) conti...

Month	Year	NW	GT	ML	LP
Jan	2019	2476.562	4415.812	2888.414	1270.874
Feb	2019	2311.583	4219.694	2704.655	1197.452
Mar	2019	2551.913	4602.472	2918.277	1301.210
Apr	2019	2431.286	4491.890	2825.931	1279.396
May	2019	2583.762	5068.692	2989.017	1334.433
Jun	2019	2506.174	5342.166	2893.697	1319.848
Jul	2019	2588.147	5581.870	2950.018	1336.521
Aug	2019	2522.027	5250.668	2904.124	1326.354
Sep	2019	2473.952	4699.546	2803.064	1334.412
Oct	2019	2580.705	4755.679	2978.736	1356.180
Nov	2019	2422.412	4568.749	2879.342	1305.508
Dec	2019	2476.696	4229.151	2890.966	1311.192
Jan	2020	2311.713	4418.354	2888.414	1327.112
Feb	2020	2552.038	4222.073	2704.655	1253.691
Mar	2020	2431.406	4605.015	2918.277	1357.448
Apr	2020	2583.878	4494.321	2825.931	1335.635
May	2020	2583.878	5071.379	2989.017	1390.672
Jun	2020	2506.286	5344.941	2893.697	1376.087
Jul	2020	2588.255	5584.711	2950.018	1392.760
Aug	2020	2522.131	5253.286	2904.124	1382.593
Sep	2020	2474.052	4701.842	2803.064	1390.651
Oct	2020	2580.802	4757.955	2978.736	1412.419
Nov	2020	2527.315	4570.89	2879.342	1361.747
Dec	2020	2422.502	4231.094	2890.966	1367.430
Jan	2021	2476.782	4420.342	2888.414	1383.351
Feb	2021	2311.796	4223.934	2704.655	1309.929
Mar	2021	2552.118	4607.003	2918.277	1413.687
Apr	2021	2431.483	4496.222	2825.931	1391.874
May	2021	2583.952	5073.481	2989.017	1446.910
Jun	2021	2506.358	5347.111	2893.697	1432.326
Jul	2021	2588.324	5586.932	2950.018	1448.999

Table A.4: Out-of-sample future electricity demand forecasts in Gigawatt-hours (GWh) conti...

Aug	2021	2522.198	5255.333	2904.124	1438.831
Sep	2021	2474.117	4703.637	2803.064	1446.890
Oct	2021	2580.864	4759.735	2978.736	1468.658
Nov	2021	2527.375	4572.566	2879.342	1417.986
Dec	2021	2422.560	4232.613	2890.966	1423.669
Jan	2022	2476.838	4421.896	2888.414	1439.590
Feb	2022	2311.850	4225.390	2704.655	1366.168
Mar	2022	2552.170	4608.559	2918.277	1469.926
Apr	2022	2431.533	4497.710	2825.931	1448.112
May	2022	2584.001	5075.125	2989.017	1503.149
Jun	2022	2506.404	5348.808	2893.697	1488.564
Jul	2022	2588.369	5588.670	2950.018	1505.237
Aug	2022	2522.241	5256.935	2904.124	1495.070
Sep	2022	2474.158	4705.041	2803.064	1503.129
Oct	2022	2580.904	4761.127	2978.736	1524.897
Nov	2022	2527.414	4573.876	2879.342	1474.225
Dec	2022	2422.597	4233.801	2890.966	1479.908
Jan	2023	2476.874	4423.112	2888.414	1495.829
Feb	2023	2311.884	4226.528	2704.655	1422.407
Mar	2023	2552.203	4609.775	2918.277	1526.165
Apr	2023	2431.565	4498.873	2825.931	1504.351
May	2023	2584.032	5076.411	2989.017	1559.388
Jun	2023	2506.434	5350.136	2893.697	1544.803
Jul	2023	2588.398	5590.029	2950.018	1561.476
Aug	2023	2522.269	5258.187	2904.124	1551.309
Sep	2023	2474.185	4706.139	2803.064	1559.367
Oct	2023	2527.439	4574.901	2978.736	1581.135
Nov	2023	2527.439	4574.901	2879.342	1530.464
Dec	2023	2422.621	4234.730	2890.966	1536.147

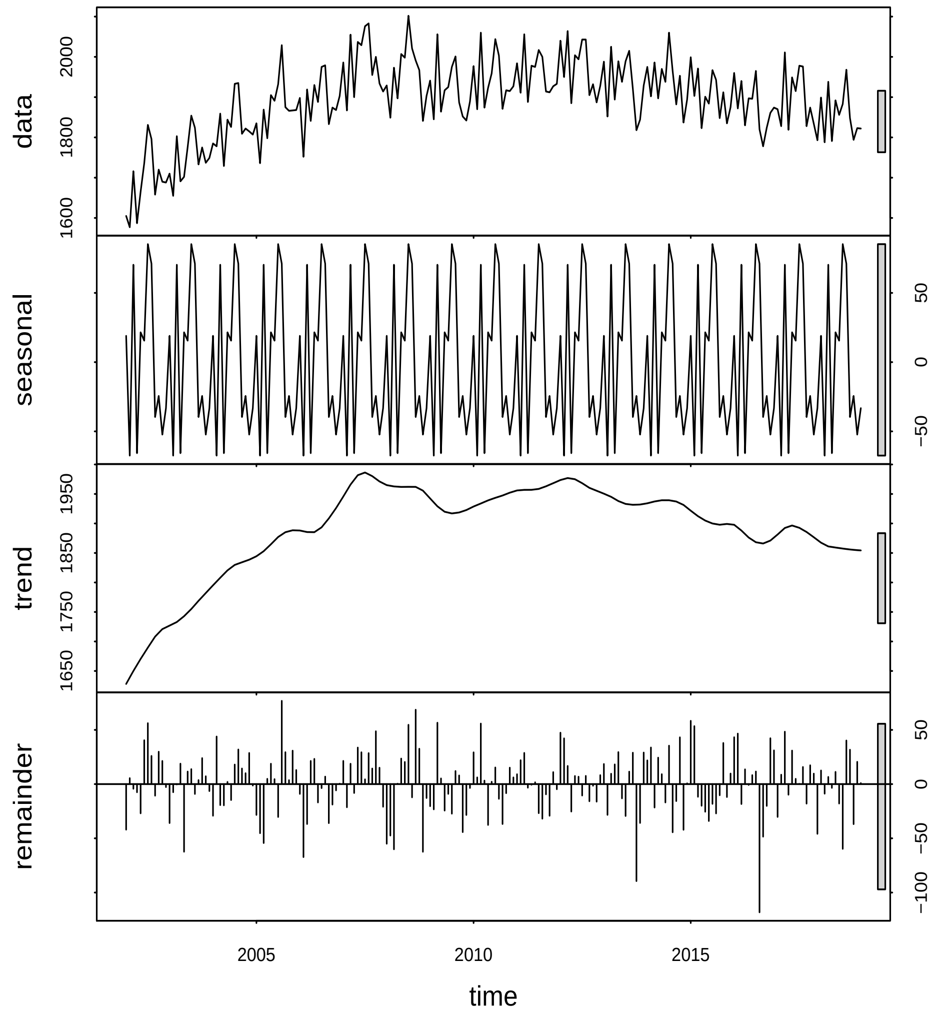

Figure A.5: Time series decomposition of total forecasts.

Table A.5: Forecast results

	Total	WC	EC	NC	FS	KZN	NW	GT	ML	LP
						Buttom up				
RMSE	231.66	35.85	105.40	26.09	30.45	89.36	109.25	100.91	96.69	60.72
MAE	186.71	26.80	100.46	21.37	24.80	74.95	78.11	80.64	74.21	51.93
MAPE	0.99	1.43	13.51	4.06	3.01	2.07	3.29	1.62	2.67	4.37
MASE	0.37	0.58	3.02	0.84	0.75	0.62	0.52	0.49	0.49	0.64
						Top down HP1				
RMSE	236.57	80.33	132.63	73.13	47.46	173.79	121.79	313.12	152.59	158.10
MAE	191.12	65.68	128.18	61.96	42.44	164.66	112.88	283.54	136.26	140.33
MAPE	1.02	3.51	16.81	11.68	5.19	4.63	4.61	6.12	4.77	11.75
MASE	0.37	1.42	3.86	2.43	1.28	1.36	0.75	1.73	0.90	1.74
						Top down HP2				
RMSE	236.57	79.14	131.69	73.71	45.17	171.39	122.11	317.19	153.21	155.31
MAE	191.12	64.58	127.21	62.66	40.06	162.14	113.57	285.83	136.71	137.16
MAPE	1.02	3.45	16.68	11.82	4.91	4.56	4.64	6.18	4.78	11.48
MASE	0.38	1.40	3.83	2.45	1.21	1.34	0.76	1.746177	0.90	1.70
						Top down FP				
RMSE	236.57	34.48	104.93	26.47	30.08	89.83	110.09	101.79	97.16	61.71
MAE	191.12	25.16	100.13	21.63	24.44	76.41	77.60	83.33	75.19	52.73
MAPE	1.02	1.35	13.46	4.11	2.96	2.11	3.27	1.68	2.71	4.44
MASE	0.37	0.54	3.01	0.85	0.74	0.63	0.52	0.51	0.50	0.65
						Optimal				
RMSE	234.38	35.16	105.09	26.46	30.13	89.51	109.96	101.39	97.02	61.64
MAE	189.55	25.99	100.24	21.62	24.48	75.56	77.73	82.25	74.85	52.67
MAPE	1.01	1.13	13.47	4.11	2.97	2.09	3.28	1.66	2.70	4.43
MASE	0.37	0.56	3.02	0.85	0.74	0.62	0.52	0.50	0.50	0.65

Appendix B

Some selected R Code

```
#######################################################################
### Rofhiwa et al. (2019)
###################################

attach(analyticdata)
head(analyticdata)
#win.graph()

result.mean <- mean(analyticdata)
print(result.mean)
median.result <- median(analyticdata)
##HIERACHICAL FORECASTING
library(forecast)
library(hts)
library(thief)
library(e1071)
win.graph(width=7,height=5,pointsize=8)

#analyticdata
energydata<-as.matrix(analyticdata)
energydata
y<-ts(energydata, start=c(2002, 01), end=c(2018,12), frequency=12)
y

# Convert the data into an hierarchy
y<- hts(y, nodes=list(9))

# Return the hierarchy
y
# Return the labels of the nodes at each level
y$labels
# Return how the nodes are organised
y$nodes

#Ploting a time series plot for all levels
plot(y)

ally <- aggts(y)
```

```
ally
somey <- aggts(y, levels = c(0, 1))
somey
S <- smatrix(y)
S



##FORECASTING ACCURACY
analyticdata<- window(y, start=c(2002,1), end=c(2016,12))
test <- window(y, start=c(2017,1), end=c(2018,12))

forecas <- forecast(analyticdata, h=24, method="bu", fmethod="ets")
accuracy.gts(forecas, test)

forecas<- forecast(analyticdata, h=24, method="comb", fmethod="ets")
accuracy.gts(forecas, test)

forecas<- forecast(analyticdata, h=24, method="tdgsa", fmethod="ets")
accuracy.gts(forecas, test)

forecas<- forecast(analyticdata, h=24, method="tdgsf", fmethod="ets")
accuracy.gts(forecas, test)

forecas<- forecast(analyticdata, h=24, method="tdfp", fmethod="ets")
accuracy.gts(forecas, test)

##FORECASTING 120 month ahead
## Bottom-up approach

allf <- forecast.gts(y, h=60,method="bu", fmethod="ets")

# Summary of the output
allf

# All-levels forecasts
allts(allf)
```

```
# Bottom-level forecasts
allf$bts
# Plot all forecasts
win.graph(width=7,height=5,pointsize=8)
plot(allf)
library("xlsx")

write.table(allts(allf),"Buforecasts.txt",sep="\t")

##FORECASTING 120 month ahead
## Topdown approach

allf <- forecast.gts(y, h=60,method="tdgsa", fmethod="ets")

# Summary of the output
allf

# All-levels forecasts
allts(allf)
# Bottom-level forecasts
allf$bts
# Plot all forecasts
plot(allf)
library("xlsx")

##FORECASTING 120 month ahead
## Topdown approach

allf <- forecast.gts(y, h=60,method="tdgsf", fmethod="ets")

# Summary of the output
allf

# All-levels forecasts
allts(allf)
# Bottom-level forecasts
allf$bts
```

```
# Plot all forecasts
win.graph(width=7,height=5,pointsize=8)
plot(allf)
library("xlsx")


##FORECASTING 120 month ahead
## Topdown approach

allf <- forecast.gts(y, h=60,method="tdfp", fmethod="ets")

# Summary of the output
allf

# All-levels forecasts
allts(allf)
# Bottom-level forecasts
allf$bts
# Plot all forecasts
win.graph(width=7,height=5,pointsize=8)
plot(allf)
library("xlsx")


##FORECASTING 120 month ahead
## Optimal combination approach

allf <- forecast.gts(y, h=72,method="comb", fmethod="ets")

# Summary of the output
allf

# All-levels forecasts
allts(allf)
# Bottom-level forecasts
allf$bts
# Plot all forecasts
win.graph(width=7,height=5,pointsize=8)
```

```r
plot(allf)


###################################################################################
###################  BOX PLOTS  ##############
##################################################

energy <- c("WC","EC","NC","FS","KZN","NW","GT","ML","LP")
boxplot(WC,EC,NC,FS,KZN,NW,GT,ML,LP,names= energy, horizontal = FALSE,
        main="",ylab="Electricity (Gigawatt-hours)")


############################################################################################
###################################
## Quantile regression averaging ###
####################################
attach(Buforecasts)
head(Buforecasts)
length(Buforecasts)
win.graph()
par(mfrow=c(2,2))
TotalBufo<-ts(Total)
TotalBufo
plot(TotalBufo, type = "l", xlab="Observation number",ylab="Total forecasts (Bottom Up)",
     col="blue")
plot(density(TotalBufo),col="blue")
qqnorm(TotalBufo, col="blue")
qqline(TotalBufo)
boxplot(TotalBufo, horizontal = T,xlab="Total forecasts (Bottom Up)")
win.graph()
plot(TotalBufo, type = "l", xlab="Observation number",ylab="Total forecasts (Bottom Up)")
z = smooth.spline(time(TotalBufo), TotalBufo)
z

lines(smooth.spline(time(TotalBufo), TotalBufo, spar=0.04),col="red",lwd=2) #spar=0.3792046
Bufitted = fitted((smooth.spline(time(TotalBufo), TotalBufo, spar=0.04)))
write.table(Bufitted,"~/nolfits.txt",sep="\t")
```

```r
plot(density(Bufitted),col="blue")

library(quantreg)
win.graph()
plot(TotalBufo, type = "l", xlab="Observation number",ylab="Total forecasts (Bottom Up)")
qr.g = rq(TotalBufo~ Bufitted,data= Buforecasts, tau=0.975) #tau = 0.025, 0.5, 0.975

summary.rq(qr.g,se="boot") # can use se = "nid" or se="ker"
lines(qr.g$fit, col="red")
fQRA1 = fitted(qr.g)
plot(fqr)
write.table(fQRA1,"~/QRA05.txt",sep="\t") #LL0025, QRA05, UL0975
accuracy(fQRA1,TotalBufo)
############################################################################
## POINTWISE BOOTSTRAP CI ##############
#####################################

library(extremefit)
win.graph()
length(X)
X <- abs(rcauchy(400))
#X <- TotalBufo

hh <- hill.adapt(X)
probs <- probgrid(0.1, 0.999999, length = 100)
B <- 200
## Not run: #For computing time purpose
bootCI(X, weights = rep(1, length(X)), probs = probs, B = B, plot = TRUE)
xgrid <- sort(sample(X, 100))
bootCI(X, weights = rep(1, length(X)), xgrid = xgrid, type = "survival", B = B, plot = TRUE)


################################################################
```

References

R. Adhikari and R.K. Agrawal. An introductory study on time series modeling and forecasting. *arXiv preprint arXiv:1302.6613*, 2013.

H. Amusa, K. Amusa, and R. Mabugu. Aggregate demand for electricity in south africa: An analysis using the bounds testing approach to cointegration. *Energy policy*, 37(10):4167–4175, 2009.

G. Athanasopoulos, R.A. Ahmed, and R.J. Hyndman. Hierarchical forecasts for australian domestic tourism. *International Journal of Forecasting*, 25 (1):146–166, 2009.

G. Athanasopoulos, N. Kourentzes R.J. Hyndman, and F. Petropoulos. Forecasting with temporal hierarchies. *European Journal of Operational Research*, 262(1):60–74, 2017.

J.M. Bates and C.W.J. Granger. The combination of forecasts. *Journal of the Operational Research Society*, 20(4):451–468, 1969.

R.B. Cleveland, W.S. Cleveland, J.E. McRae, and I. Terpenning. Stl: A seasonal-trend decomposition. *Journal of official statistics*, 6(1):3–73, 1990.

B.J. Dangerfield and J.S. Morris. Top-down or bottom-up: Aggregate versus disaggregate extrapolations. *International Journal of Forecasting*, 8(2): 233–241, 1992.

D.M. Dunn, W.H. Williams, and T.L. DeChaine. Aggregate versus subaggregate models in local area forecasting. *Journal of the American Statistical Association*, 71(353):68–71, 1976.

J.B. Edwards and G.H. Orcutt. Should aggregation prior to estimation be the rule? *The Review of Economics and Statistics*, pages 409–420, 1969.

B. Efron. The jackknife, the bootstrap, and other resampling plans. *Siam*, 38, 1992a.

B. Efron. Bootstrap methods: another look at the jackknife. *In Breakthroughs in statistics*, pages 569–593, 1992b.

B. Efron and R.J. Tibshirani. *An introduction to the bootstrap*. CRC press, 1994.

B.T. Efron and R. Tibshirani. An introduction to the bootstrap. *Chapman & Hall*, 1993.

N. Elamin. Quantile regression model for peak load demand forecasting with approximation by triangular distribution to avoid blackouts. 2018.

E.B. Fliedner and V.A. Mabert. Constrained forecasting: some implementation guidelines. *Decision Sciences*, 23(5):1143–1161, 1992.

G. Fliedner. An investigation of aggregate variable time series forecast strategies with specific subaggregate time series statistical correlation. *Computers & Operations Research*, 26(10-11):1133–1149, 1999.

G. Fliedner. Hierarchical forecasting: Issues and use guidelines. *Industrial Management and Data Systems*, 101:5–12, 02 2001. doi: 10.1108/02635570110365952.

D. W. Fogarty, J.H. Blackstone, and T.R. Hoffman. Production and inventory management. *South-Western Publication Co.*, 2nd ed, 1990.

A. Gaba, I. Tsetlin, and R.L. Winkler. Combining interval forecasts. *Decision Analysis*, 14(1):1–20, 2017.

P. Gaillard and Y. Goude. Forecasting electricity consumption by aggregating experts; how to design a good set of experts. In *Modeling and stochastic learning for forecasting in high dimensions*, pages 95–115. Springer, 2015.

C.W. Gross and J.E. Sohl. Disaggregation methods to expedite product line forecasting. *Journal of Forecasting*, 9(3):233–254, 1990.

Y. Grunfeld and Z. Griliches. Is aggregation necessarily bad? *The Review of Economics and Statistics*, pages 1–13, 1960.

D.F. Hendry and K. Hubrich. Combining disaggregate forecasts or combining disaggregate information to forecast an aggregate. *Journal of business & economic statistics*, 29(2):216–227, 2011.

T. Hong and D. Dickey. Electric load forecasting: fundamentals and best practices. *OTexts*, 2014.

T. Hong and S. Fan. Probabilistic electric load forecasting: A tutorial review. *International Journal of Forecasting*, 32(3):914–938, 2016.

J. De Hoog and K. Abdulla. Data visualization and forecast combination for probabilistic load forecasting in gefcom2017 final match. *International Journal of Forecasting*, 35(4):1451–1459, 2019.

R.J. Hyndman and Y. Khandakar. *Automatic time series for forecasting: the forecast package for R.* Number 6/07. Monash University, Department of Econometrics and Business Statistics, 2007.

R.J. Hyndman and A.B. Koehler. Another look at measures of forecast accuracy. *International journal of forecasting*, 22(4):679–688, 2006.

R.J. Hyndman, G. Athanasopoulos R.A. Ahmed, and H.L. Shang. Optimal combination forecasts for hierarchical time series. *Computational Statistics & Data Analysis*, 55(9):2579–2589, 2011.

R.J. Hyndman, A.J. Lee, and E. Wang. Fast computation of reconciled forecasts for hierarchical and grouped time series. *Computational Statistics & Data Analysis*, 97:16–32, 2016.

R. Inglesi and A. Pouris. Forecasting electricity demand in south africa: A critique of eskom's projections. *South African Journal of Science*, 106(1-2): 50–53, 2010.

K.B. Kahn. Revisiting top-down versus bottom-up forecasting. *The Journal of Business Forecasting*, 17(2):14, 1998.

I. Kanda and J.M.Q. Veguillas. Data preprocessing and quantile regression for probabilistic load forecasting in the gefcom2017 final match. *International Journal of Forecasting*, 35(4):1460–1468, 2019.

W.R. Kinney. Predicting earnings: entity versus subentity data. *Journal of Accounting Research*, pages 127–136, 1971.

W.G.S. Konarasinghe and N.R. Abeynayake. Modeling stock returns of individual companies of colombo stock exchange. In *Conference Proceedings of the 1st International Forum for Mathematical Modeling*, 2014.

L.R. Lima and F. Meng. Out-of-sample return predictability: A quantile combination approach. *Journal of Applied Econometrics*, 32(4):877–895, 2017.

L.R. Lima, F. Meng, and L. Godeiro. Quantile forecasting with mixed-frequency data. *International Journal of Forecasting*, 2019.

H. Lütkepohl. Forecasting contemporaneously aggregated vector arma processes. *Journal of Business & Economic Statistics*, 2(3):201–214, 1984.

P. Mpfumali, C. Sigauke, A. Bere, and S. Mulaudzi. Day ahead hourly global horizontal irradiance forecasting—application to south african data. *Energies*, 12(18):3569, 2019.

S.L. Narasimhan, D.W. McLeavey, and P. Billington. Production planning and inventory control. *Allyn & Bacon*, 2nd ed, 1994.

S.L. Narasimhan, D.W. McLeavey, and P. Billington. *Production planning and inventory control*. Prentice Hall Englewood Cliffs, 1995.

J. Nowotarski and R. Weron. Computing electricity spot price prediction intervals using quantile regression and forecast averaging. *Computational Statistics*, 30(3):791–803, 2015.

G.H. Orcutt, H. W. Watts, and J. B. Edwards. Data aggregation and information loss. *The American Economic Review*, pages 773–787, 1968.

A. Prat, A. Navarro, L. Paré, R. Noemí, P. Galván, P. Tomás, and A. Martínez et al. Immune-related gene expression profiling after pd-1 blockade in non–small cell lung carcinoma, head and neck squamous cell carcinoma, and melanoma. *Cancer research*, 77(13):3540–3550, 2017.

C. Roach. Reconciled boosted models for gefcom2017 hierarchical probabilistic load forecasting. *International Journal of Forecasting*, 35(4):1439–1450, 2019.

G. Sbrana and A. Silvestrini. Forecasting aggregate demand: analytical comparison of top-down and bottom-up approaches in a multivariate exponential smoothing framework. *International Journal of Production Economics*, 146(1):185–198, 2013.

N.K. Shing. *A study of bottom-up and top-down forecasting methods*. PhD thesis, M. Sc. thesis, Royal Melbourne Institute of Technology, 1993.

E. Shlifer and R.W. Wolff. Aggregation and proration in forecasting. *Management Science*, 25(6):594–603, 1979.

S. Smyl and N.G. Hua. Machine learning methods for gefcom2017 probabilistic load forecasting. *International Journal of Forecasting*, 35(4):1424–1431, 2019.

S.B. Taieb. Sparse and smooth adjustments for coherent forecasts in temporal aggregation of time series. In *NIPS 2016 Time Series Workshop*, pages 16–26, 2017.

S.B. Taieb, R. Huser, R.J. Hyndman, and M.G. Genton. Forecasting uncertainty in electricity smart meter data by boosting additive quantile regression. *IEEE Transactions on Smart Grid*, 7(5):2448–2455, 2016.

S.B. Taieb, J.W. Taylor, and R.J. Hyndman. Coherent probabilistic forecasts for hierarchical time series. In *International Conference on Machine Learning*, pages 3348–3357, 2017.

H. Theil. Linear aggregation of economic relations. 1954.

G. Weatherby. *Aggregation, disaggregation, and combination of forecasts*. PhD thesis, Georgia Institute of Technology, 1984.

S.L. Wickramasuriya, G. Athanasopoulos, and R.J. Hyndman. Forecasting hierarchical and grouped time series through trace minimization. *working paper 15/15, Department of Econometrics & Business Statistics, Monash University*, 2015.

A. Zellner and J. Tobias. A note on aggregation, disaggregation and forecasting performance. Technical report, 1998.

Y. Zhong, X. Xia, F. Shi, J. Zhan, J. Tu, and H.J. Fan. Transition metal carbides and nitrides in energy storage and conversion. *Advanced science*, 3(5):1500286, 2016.