SCHOOL OF ENVIRONMENTAL SCIENCES

DEPARTMENT OF HYDROLOGY AND WATER RESOURCES

Development of risk-based groundwater operating rules: a case study of Siloam Village, South Africa

By

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A thesis submitted to the Department of Hydrology and Water Resources in fulfillment of the requirements for Doctor of Philosophy in Environmental Sciences (Hydrology) degree

March, 2019

DECLARATION

I, Rachel Makungo, hereby declare that this thesis submitted to the Department of Hydrology and Water Resources at the University of Venda, for the Doctor of Philosophy in Environmental Sciences (Hydrology), is my own work and has not been previously submitted, in whole or in part, to any university for any degree; and all reference materials contained herein have been duly acknowledged.

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We, the promoters, certify that this declaration is correct.

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DEDICATION

This thesis is dedicated to:

- My beloved father, Mr. Nkhumeleni Elias Makungo, who passed on before the completion of this study.
- My children who were able to bear with a busy mother all the times, throughout the duration of my studies.
- My mother, Mrs. Thizwikoni Mabel Makungo, for the motherly support she has provided throughout my entire life.

ACKNOWLEDGEMENTS

As I write these acknowledgements, I have a humble heartfelt gratitude towards Almighty God for giving me strength, as there were several times when I felt that this day would never come. This is such a blessed day.

I would like to express my sincere gratitude to the promoters of my thesis who are Prof J.O. Odiyo, Prof. J.G. Ndiritu and Prof B. Mwaka. Professor J.O. Odiyo is acknowledged for effectively promoting this thesis and providing valuable support from its inception to the end. His patience, understanding, tolerance and encouragement that it will finally happen even when things seemed unsurmountable are highly appreciated. His support and passion to mentor young academics as the Dean of the School of Environmental Sciences at the University of Venda is also appreciated. Prof J.G. Ndiritu from the University of the Witwatersrand provided highly valuable support in the development of this thesis. He provided guidance on conceptualisation of groundwater modelling using the GW-PITMAN model, stochastic analysis and generation of groundwater operating rules. Prof. Ndiritu also took time out of his busy schedule to assist in programing the models for groundwater levels generation and deriving operating rules. Without his support, the main objective of this study would never have been achieved. Prof B. Mwaka from the Department of Water and Sanitation is also acknowledged for the role he played in promoting this thesis. His mentorship particularly on improving the development of operating rules and shaping of chapter 9 of this thesis is highly appreciated. He also encouraged and urged me to remain patient and tolerant at getting the tough part of the thesis done at a time when I felt it was time to conclude the last stages of the thesis.

The Water Research Commission (WRC) is acknowledged for providing partial funding for this study. The reference group of the WRC Project titled “Groundwater yield-reliability analysis and operating rules for data constrained rural areas in South Africa” are acknowledged for valuable and constructive contributions particularly with respect to delineation of groundwater resource unit and development of hydrogeological conceptual model. Dr. Shafick Adams (Executive Manager of Water Resource Management and Ecosystems at WRC) is acknowledged for steering the project to its successful and logical end. Mr. Stephane Tsakou who is a PhD student at the University of Western Cape is acknowledged for assisting in the development of the hydrogeological conceptual model for the study area. Dr. Martin Holland from Delta H Water System Modelling Consulting is acknowledged for assisting with interpretation of linear features from magnetic data. The National Research Foundation is acknowledged for providing funding through student bursary and sabbatical leave grant. University of Venda's Research and Publications Committee is also acknowledged for providing funding for research running expenses.

Residents of Siloam Village who allowed installation of soil moisture monitoring probes and drilling of boreholes in their homesteads are also acknowledged. Members of the Royal House of Siloam Village (Chief Mugwena, Mrs. Selina Mugwena, Ms. Muvhulawa Mugwena and Mr. Ramuhageli) are acknowledged for giving permission to undertake this study and for encouraging community members to safeguard the research equipment installed in their village. Ms. Muvhulawa Mugwena also facilitated efficient communication and work relationship between Siloam Royal House and University of Venda. The Station Commissioner of Siloam Police Station and his staff are acknowledged for allowing installation of the weather station and housing its data logger.

Mr. Tinyiko Rivers Nkuna from the Department of Hydrology and Water Resources at the University of Venda is acknowledged for assisting in field work, which included monitoring and maintenance of field equipment. He has also been a supportive and understanding colleague. Mr. Singo Tendani Elphus who assisted with field monitoring of groundwater abstractions is also acknowledged. Ms. Elelwani Malima is acknowledged for taking over the responsibilities of monitoring and maintenance of field equipment from Mr. T.R. Nkuna to ensure sustenance of continuous data monitoring beyond the PhD study, and for being a reliable friend.

Miss Shudufhadzo Mukwevho is appreciated for faithfully playing the role of a research assistant while she was also studying full time for her Honours Degree. Her support lessened my work load in the Department of Hydrology and Water Resources throughout the last two years of my PhD study. Dr. Ikudayisi and Dr. Banda who were postdocs at the School of Environmental Sciences, University of Venda are acknowledged for assisting with the lecturing load during the critical and final stages of this thesis. Their support is highly appreciated as it facilitated speedy progress of this work.

I also wish to acknowledge my 2017 Honours students (Shudufhadzo Mukwevho, Collen Maphaha, Dave Musitha, Sara Mammburu, Ndaedzo Nthulani, Nikiwe Makhubele, Mbaimbai Vhahangwele and Ndou Innocent) and Deseree Mashinye (Masters student) for their support and understanding that I was also under pressure when I could not give feedback on their Honours and Masters dissertations in time. Their hard work and resilience that contributed to them still graduating in time is highly appreciated.

Ms. Muvhulawa Mugwena is acknowledged for playing a huge role of being second mother to my children and for treating them as if they are her own. It would have been a very difficult and unmanageable task to juggle motherhood, work and studying without her support. I also acknowledge my mother and my siblings, particularly my brothers, Meshack, Maanda and Gundo for their support.

ABSTRACT

This study developed operating rules for groundwater supply from a probabilistic (risk-based) approach. Groundwater supply systems are often operated without relating groundwater yield/availability to demand which makes groundwater resource planning and management challenging and unpredictable. Risk-based approaches for developing groundwater operating rules comprehensively incorporate assurance of supply and also account for uncertainty due to model inputs, model structure and climate variability. A groundwater resource unit (GRU) was delineated and its hydrogeological conceptual model developed. Automatic curve matching was used to identify appropriate aquifer models and test solutions for estimating hydraulic characteristics (storativity, transmissivity and hydraulic conductivity) based on Aquifer Test Solver (AQTESOLV) Pro version 4.5. Limited groundwater levels and rainfall data were infilled and/or extended using Output Error-Non-linear Hammerstein Weiner (OE-NLHW) and non-parametric regression (NPR), respectively. Performances of these models were based on relative error (RE), correlation coefficient (COR), root mean square error (RMSE), coefficient of determination (R^2) and Nash Sutcliffe coefficient of efficiency (NSE). A program for generation of monthly groundwater levels for the GRU was coded in FORTRAN based on the revised version of the Pitman model (referred to as GW-PITMAN model). The model was calibrated using groundwater levels from a neighbouring borehole due to lack of observed representative data for the GRU. Validation was done by establishing the realistic nature of simulated runoff, recharge and groundwater levels. A Variable Length Block (VLB) bootstrapping model was used for simultaneous generation of stochastic inputs (rainfall, evaporation and groundwater levels) of the groundwater operating rules model. Operating rules were developed from statistical analysis of 100 base yields for the GRU simulated from 5-year long stochastically generated sequences (with length of 34 years) of rainfall, evaporation and groundwater levels. The hydrogeological conceptual model indicated presence of faults and diabase dykes which influence preferential flow paths and storage of water in the aquifer. Identified aquifer test solutions were found to be suitable for estimation of hydraulic characteristics, since they had generally good model fits and low mean residual errors. Heterogeneous aquifer types were identified though leaky aquifer dominated. Storativity, transmissivity and hydraulic conductivity values ranged from 0.0003-0.060, 0.78-12.3 m^2/day and 0.074-0.460 m/day, respectively, indicating limited storage with potential for local groundwater supply for private consumption. Graphical fits for observed and estimated rainfall and groundwater levels were mostly comparable, though scatter plots indicated cases of underestimation and overestimation of observed values. R^2, COR, NSE, RMSE and RE values were 0.76 and 0.7, 0.87 and 0.84, 0.75 and 0.68, 3.67 and 3.03 mm and 30 and 29% for both calibration and validation runs, respectively, for NPR model. R^2, COR, NSE, RMSE and RE were 0.99 and 0.86, 0.97 and 0.93, 0.99 and

0.84, 0.03 and 0.01 m and 0.08 and 0.11% for both calibration and validation runs, respectively, for OE-NLHW model. The models were therefore found to have efficient calibration and validation, and were thus, suitable for data extension. Estimated groundwater levels, streamflow and groundwater recharge for both calibration and validation runs of the GW-PITMAN model, generally fluctuated with changes in rainfall, indicating that they are realistic. Majority (9 out of 10) of the historical statistics were mostly well preserved by VLB, except for skewness. Historic highest groundwater levels were also not well preserved. Superimposing the cumulative demands on the base yield curves and analysis of percentages of water demands that can be supplied indicated that the groundwater system could not meet the water demands at all times. To promote sustainable multipurpose use of water that can enhance rural livelihoods, allocating water using priority classification was found to be essential. Operating rule curves for groundwater supply were derived using a risk-based approach. The operating rule curves indicated that if priority classification is used all water demands are met up to maximum groundwater level of 25 m. The developed operating rule curves are therefore expected to improve water supply to both domestic and productive water uses, if they are adequately implemented and hence improve livelihoods. The procedures followed in developing risk-based groundwater operating rules for Siloam Village were summarised to assist in their application in any delineated groundwater resource unit. Though minimal infrastructure is available to support implementation of the operating rules, additional monitoring boreholes are required to aid in estimation of average groundwater levels for further calibration and validation of the GW-PITMAN model. Detailed geological and geophysical investigation are required to improve on characterisation of the GRU and its hydrogeological conceptual model. Undertaking a study of this nature in other areas including those which are data-scarce could promote wide implementation of risk-based groundwater operating rules.

CONTRIBUTIONS OF THE STUDY

This section indicates the publications and presentations emanating from the study and contributions to knowledge.

Peer reviewed publications

1. Makungo, R. and Odiyo, J.O. (2017) Estimating groundwater levels using system identification models in Nzhelele and Luvuvhu areas, Limpopo Province, South Africa, *Physics and Chemistry of the Earth*, vol. 100, pp. 44-50
2. Makungo, R. and Odiyo, J.O. (In Press) Application of non-parametric regression in estimating missing daily rainfall data, International Journal of Hydrology Science and Technology, Manuscript no. IJHST-153438.

Conference presentations

1. Makungo R. and Odiyo J.O. (2014) Application of non-parametric regression in estimating missing rainfall data: case study of Siloam, South Africa, 8th Egerton International Conference, 26-28 March 2014, Kenya.
2. Makungo R. and Odiyo J.O. (2015) An approach for groundwater storage-reliability analysis and operating rules for rural areas: Siloam Village case study, 14th Biennial Groundwater Conference, 21-23 September 2015, Muldersdrift, Johannesburg, South Africa.
3. Makungo, R. and Odiyo, J.O. (2015) Estimating groundwater levels using system identification models in Nzhelele and Luvuvhu areas, Limpopo Province, South Africa, 16th WaterNet/WARFSA/GWPSA Symposium, 28 - 30 October 2015, Le Meridien lle Maurice Hotel, Mauritius.
4. Odiyo J.O. and Makungo R. (2016) Comparison of multiple imputation and non-parametric regression in estimating missing rainfall, 17th WaterNet/WARFSA/GWPSA Symposium, 26-28th of October 2016, Gaborone, Botswana.
5. Makungo R., Ndiritu, J.G., Odiyo J.O and Mwaka B (2018) Hybrid manual-automatic calibration of the revised Pitman model using groundwater levels, SANCIAHS Symposium
6. Makungo R., Ndiritu, J.G., Odiyo J.O and Mwaka B (2018) Generating stochastic hydrological inputs for groundwater resource assessment using Variable Length Block method, First Pan African International Research Congress on Knowledge Generation and Dissemination (1st PAIRC-2018), 18-21 June 2019, Kisumu, Kenya.

The study's contributions to knowledge include:

1. A first attempt to calibrate GW-PITMAN model using groundwater levels following a hybrid approach which constituted setting of model parameters based on available physical and hydrogeological data (manual component) and their optimisation using SCE-UA constituted (automatic component).
2. Further knowledge on hydraulic characteristics of fractured crystalline basement aquifers in semi-arid areas
3. Knowledge and application of alternative approaches to estimate missing rainfall and groundwater levels data in data scarce areas.
4. Application and assessment of the VLB generator in generating stochastic rainfall, evaporation and groundwater levels (multivariate stochastics) at multiple sites.
5. Incorporation of risks of failure/levels of assurance of supply in groundwater resource assessments
6. Development of operating rules for groundwater supply in Siloam Village using a risk-based approach, which is typically used for surface water systems in South Africa.
7. Generalised procedure for development of groundwater operating rules for application in any groundwater resource unit

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LIST OF ABBREVIATIONS

AAY	Aquifer Assurance Yield
AAYM	Aquifer Assurance Yield Model
$Abst_{actl}$	Volume of abstractions on lower slope per week per m strip of the groundwater resource unit
$Abst_{actu}$	Volume of abstractions on upper slope per week per m strip of the groundwater resource unit
$Abst_P$	Proportion of groundwater demand actually abstracted
AFYM	Aquifer Firm Yield Model
A_{imp}	Proportion of the groundwater resource unit area which is impervious
ANFIS	Adaptive neural fuzzy inference system
ANN	Artificial neural network
AQTESOLV	Aquifer Test Solver
ARIMA	Autoregressive integrated moving average
ARMA	Auto-regressive moving average
ARX	Autoregressive exogenous
ASTER GDEM	Advanced Spaceborne Thermal Emission and Reflection Radiometer Global Digital Elevation Map
AWP	Available weighted precipitation
b	Aquifer thickness
BP	Backpropagation
dBH	Distance from the borehole distance to river.
COR	Correlation coefficient
DEM	Digital elevation model
DGW	Depth to groundwater
dVL	Groundwater volume in the lower slope
dVU	Groundwater volume in the upper slope
DWA	Department of Water Affairs
DWAF	Department of Water Affairs and Forestry
DWS	Department of Water and Sanitation
e(t)	Model error
EMMCMC	Expectation-maximization algorithm based on Monte Carlo Markov Chain
E_p	Evaporation
ESO	Explicit stochastic optimisation
ET_0	Evapotranspiration
EvapS	Evaporation from the soil
FAO	Food and Agricultural Organisation
FNN	Feedforward neural network
FORTRAN	Formula Translation
FT	Runoff at soil moisture equal to ST
g(x)	Wavelet network
GP	Genetic programming
GPOW	Power of the moisture storage-recharge equation.
GRIP	Groundwater Resources Information Project
GW	Maximum recharge
GWF_l	Volume of flow from lower slope

GWF_{lnew}	New groundwater slope of lower slope
GWF_u	Volume of flow from upper slope
GWF_{unew}	New groundwater slope of lower slope
GWH	Groundwater height
GWL_{est}	Estimated groundwater levels
GWOMP	Groundwater Operational Management Package
GWS_l	Groundwater table from lower slope
GWS_u	Groundwater table from upper slopes
H_c	Height of water table at the connection between lower and upper slope
h_i	Half the window width of the local polynomial regression centered at the focal
HONNs	Higher-order neural networks
H_u	Height of water table at end of the upper slope
IDW	Inverse distance weighting
IhsMax	Maximum upper groundwater gradient
IlsMax	Minimum lower groundwater gradient
ISO	Implicit stochastic optimisation
K	Tricube kernel weighting function
KC	Kaapvaal craton
k_c	Pan coefficient
K_h	Hydraulic conductivity
KNN	k-nearest neighbour
L_{GRU}	Length of the groundwater resource unit.
LM	Levenberg-Marquardt
LMB	Limpopo Mobile Belt
LOWESS	Locally-weighted-regression scatter-plot smoothing
LP	Linear programming
LR	Linear regression
MCDA	Multiple criteria decision analysis
MLP	Multilayer perceptron
MLR	Multiple linear regression
nb	Order of the polynomial *B*
nf	Order of the polynomial *F*
NGS	National Groundwater Strategy
n_k	*D*elay from input to output in terms of number of samples
NN-ARX	Neural network-autoregressive extra input
NPR	Non-pararametric regression
NR	Normal ratio
NSE	Nash Sutcliffe coefficient of efficiency
NWA	National Water Act
OBF	Objective function
OE-NLHW	Output error-non-linear hammerstein-wiener
PE	Potential evapotranspiration
PE_{max}	Maximum potential evapotranspiration
PIRFICT	Predefined impulse response function in continuous time
pls	Proportion of the catchment with lower side groundwater table
POW	Power of runoff soil moisture storage curve.

pRechl	Recharge from upper slope
pRechu	Recharge from lower slope
PSO-LSSVM	Least square support vector machine coupling particle swarm optimization algorithm
R	Evaporation-moisture storage relationship parameter
R^2	Coefficient of determination
RBF	Radial basis function
RE	Groundwater recharge
RE	Relative error
RE_l	Volume of recharge per 1 m strip of the groundwater resource unit on lower slope
RE_u	Volume of recharge per 1 m strip of the groundwater resource unit on upper slope
RMSE	Root mean square error
RNN	Recurrent neural network
ROR	Run-of-river
RSW	Width of the riparian strip
RWL	Rest water level
S	Actual soil moisture
SCE-UA	Shuffled Complex Evolution developed at the University of Arizona
SL_1	Soil moisture below which no runoff occurs
SL_2	Minimum moisture storage below which no groundwater recharge
SMAR	Soil moisture analytical relationship
SNN	Static neural network
SR_{imp}	Surface runoff from impervious areas
SR_{per}	surface runoff from pervious areas
SR_{per}	Surface runoff from pervious areas
ST	Total soil moisture capacity
STOMSA	Stochastic Model of South Africa
STOR	Storativity
SVM	Support vector machine
TFN	Time transfer function
TLF	Time-lagged feedforward
TMG	Table Mountain Group
T_r	Transmissivity,
TRE	Total groundwater recharge
USA	United States of America
VLB	Variable length bootstrap
W	*M*aximum deviation of cumulative rainfall
W_{GRU}	Width of the groundwater resource unit
w_i	Weighting function
WI	Weighted interception
WP	Weighted rainfall
x_0	Focal (target value)
Z_1	Minimum catchment absorption rate
Z_2	Mean catchment absorption rate
Z_3	Maximum catchment absorption rate

ZC	Zimbabwe craton
z_i	Scaled distance between the *x* value for i^{th} observation and the focal (target value)

LIST OF SYMBOLS

β_0, β_1 and β_p	Regression coefficients at time steps *0, 1* and *p*
β_k	*R*ow vector
$\beta_k (x-\gamma_k)$	Scalar
$\bar{e}$	Residual mean
e_i	Residual
σ_e	Residual standard deviation
σ^2_e	Residual variance
q^{-1}	Time-shift operator

CHAPTER 1: INTRODUCTION

1.1 Background

More than 60% of the South African population is supplied with groundwater and this number increases to 90% in some provinces (Braune and Xu, 2008). In Limpopo Province, groundwater accounts for almost 70% of rural domestic water supply (du Toit *et al.*, 2011). Rural communities and irrigation farming make extensive use of groundwater, extracting a total of about 850 million m^3/year (FAO, 2004). The rural groundwater domestic use in South Africa increased from 120 x 10^6 m^3/annum in the year 1986 to 310 x 10^6 m^3/a in the year 2000 (Braune and Xu, 2008). This shows that groundwater plays a vital role of supplying water demands and supporting economic growth in South Africa. However, in contrast to its strategic role as an essential resource to help achieve community development and poverty alleviation, groundwater has remained a poorly understood and managed resource (FAO, 2003).

According to DWA (2010), available groundwater resource estimated for drought conditions is 7 500 x 10^6 m^3/annum and the present groundwater use of between 2 000 and 4 000 x 10^6 m^3/annum, means that there is potential to considerably increase groundwater supplies in South Africa. Despite this, overall potential yields of boreholes in the Limpopo River Basin, for example, are relatively low, limiting the extent to which groundwater can be used for large scale water supply (FAO, 2004). There are also variations in local groundwater yields. For example, in the Nzhelele River Catchment, which is dominated by intergranular and fractured aquifers, the groundwater yield varies from as low as 0.5 l/s to 11.4 l/s (du Toit *et al.*, 2002). The variations in groundwater availability and yields at a local scale require a comprehensive assessment and development of probabilistic (risk-based) operating rules to ensure its sustainable use. Probabilistic or risk-based methods aim at incorporating the real-world uncertainties of not knowing future inflows into the operating rules (Ramirez, 2004).

1.2 Statement of the research problem

South Africa resort to the use of groundwater to supplement surface water supply during drought periods. For example, a number of emergency boreholes for groundwater supply were drilled as part of drought relief programme during the 1992/93 drought. Rehabilitation and refurbishment of existing boreholes, and drilling of new boreholes were done to augment existing stressed schemes or for use of groundwater as a sole source of supply in needy communities during the 2015/16 drought (DWS, 2016). Although groundwater is also vulnerable to drought, this fact is often ignored and water supply systems or drought relief programmes are planned without assessing the aquifer systems accordingly (MacDonald and Calow, 1996). Calow *et al.* (2002) and DWS (2016) emphasised the need for comprehensive assessment, planning and prioritisation of groundwater resources. This would further help in taking prompt decisions in times of extreme events even on a local scale, and for information purposes (DWS, 2016).

In South Africa, comprehensive yield–reliability analysis and development of operating rules for large surface water resource systems with storage dams has become the norm and has been advanced to cater for environmental flow requirements (Odiyo *et al.*, 2015). In addition, reliability based operating rules have been developed for run-of-river (ROR) abstraction schemes (Odiyo *et al*., 2015) and combined run-of-river and harvested rainwater (Ndiritu *et al*., 2011a). This has not been done for groundwater supply. DWAF (2004) indicated that the local and international approaches for quantifying groundwater resources do not incorporate the assessment of risk of resource failure or assurance of supply. Stochastic (probabilistic) based approaches are required in quantification of groundwater resources and development of their operating rules at both local and international levels. Approaches that do not incorporate stochastics do not account for uncertainty that results from climate variability. Thus, operating rules developed from such approaches may not be realistic.

Within the context of groundwater use, operating rules show the demand and period to be supplied for a specified level or storage and level of reliability. In the absence of this information over-allocation of groundwater may lead to declining groundwater levels resulting to aquifer depletion, subsidence and salt water intrusion. For example, during the 2015/16 drought, drop in groundwater levels was recorded in most parts of South Africa including Limpopo, KwaZulu Natal, Eastern Cape and Free State Provinces (DWS, 2016). Piesse (2016) reported that in recent years, farmers in the Free State Province drilled boreholes up to 500 m in depth and found no water as a result of drought. This shows that though groundwater has been regarded as an alternative source of water supply during dry periods, it is also susceptible to drought. More than 60% of the boreholes in the Western Cape have shown decline in groundwater levels since January 2015 as a result of drought (Western Cape Government, 2017). However, there are plans to further develop groundwater for long-term security of municipal supplies. This calls for the development of risk-based operating rules which would aid in sustainable use of groundwater during drought conditions.

Lack of risk-based groundwater operating rules leads to highly subjective groundwater allocation that is mainly based on rule of thumb. In most instances, when boreholes dry up, blame is placed on the groundwater system instead of the approaches used in operating the system. Groundwater systems are often operated without relating groundwater yield/availability to demand. This is exacerbated by poor groundwater monitoring networks and discontinuous monitoring and measurement of aquifer data. For example, most boreholes that are drilled in rural areas of South Africa, including Siloam Village, are typically production boreholes aimed at domestic water supply while there are limited groundwater monitoring boreholes. This makes groundwater resource planning and allocation challenging and unpredictable as stated in Knüppe (2011).

In Siloam Village, ROR abstractions can only supply the domestic water demand up to 90% reliability throughout the year (Makungo, 2009) while harvested rainwater can only supply the domestic water demand up to 96% reliability for a period of 4 months (Ndiritu *et al.*, 2011a). The latter study showed that integrating the utilisation of ROR and harvested rainwater can substantially improve the level of domestic supply throughout the year at 96% reliability. Thus, to achieve a reliability of 98% required for domestic water use in South Africa, groundwater is required to supplement run-of-river abstractions and harvested rainwater.

1.3 Motivation

Most groundwater studies estimate sustainable yields of aquifers but do not incorporate risk of failure of water supply. Examples of such studies include Van Tonder *et al.* (2000), Misstear and Beeson (2000), Monirul and Kanungoe (2005), Uddameri and Honnungar (2007) and McDowell (2010). Analysis of risk of failure is essential for ensuring the assurance of supply from specific yields. A study that has incorporated the reliability of the estimated yield is that of Khan and Mawdsley (1988), though it focused on an unconfined aquifer environment and did not follow stochastic (risk-based) approaches.

Developing operating strategies/rules for water supply schemes with multiple sources of supply can be done by developing separate components of the operating strategy for each source of supply (Makungo, 2009). These strategies can then be combined to derive an optimum and integral operating strategy. For example, Makungo (2009) developed a ROR abstractions operating strategy for rural water supply while Ndiritu *et al.* (2011a, b) developed integrated operating rules combining harvested rainwater and ROR abstractions for rural water supply. Groundwater operating rules for such rural areas have not yet been developed. This justifies the need to develop groundwater operating rules which would support conjunctive use of run-of-river, harvested rainwater and groundwater to improve the reliability of water supply in such areas. Development of groundwater operating rules would also aid in allocation and sustainable use of groundwater resources even during drought periods.

The study conducted groundwater yield-reliability analysis based on stochastic methods, which to the knowledge of the author has not been done in South Africa and done to a limited extent globally. Examples of limited global studies include those that focused on operating rules for conjunctive use of surface and groundwater including Knapp and Olson (1995), Philbrick and Kitanidis (1998), Marques et al. (2010), and Dracup and Dale (2011). These studies only considered surface water inflow as a stochastic variable and ignored inflow to the groundwater reservoir. This creates the need for a study that would use a stochastic-based approach to develop groundwater operating rules that incorporate reliability of supply and assess uncertainty more realistically. This will enable the incorporation of risk of failure in the integration of groundwater, ROR abstractions and harvested rainwater operating rules.

The study also tested data-driven methods for infilling and/or extending data required for groundwater modelling. This was motivated by limited data in the study area, which required extension. Testing and validation of such data extension approaches provide alternative approaches that can be applied in other data scarce areas.

1.4 Objectives of the study

1.4.1 The main objective

The main objective is to develop risk-based operating rules for groundwater supply using Siloam Village as a case study.

1.4.2 The specific objectives

The specific objectives include:

- To delineate and characterise a groundwater resource unit for Siloam Village
- To infill and/or extend data required for generating groundwater levels for the groundwater resource unit and stochastic analysis.
- To generate stochastic inputs for groundwater base yield-recurrence interval analysis
- To derive risk-based groundwater supply operating rules and formulate an approach for their implementation.

1.5 Research questions

- What are the characteristics of the groundwater resource unit for Siloam Village?
- What methods can be used for generating groundwater levels for the groundwater resource unit and stochastic analysis?
- How are stochastic inputs for groundwater base yield-recurrence interval analysis generated?
- How are risk-based groundwater supply operating rules derived and an approach for their implementation formulated?

1.6 Hypotheses

- Groundwater yield is a stochastic variable which can be predicted based on probabilistic relationships.
- The approaches used for deriving probabilistic operating rules for surface water reservoirs can be applied to derive risk-based groundwater operating rules.

1.7 Structure of the thesis

This thesis consists of 10 chapters. Chapter 1 provides the "Introduction" and it covers the background, statement of problem, objectives and motivation for the study. The literature which covers principles on groundwater resource unit delineation, hydrogeological conceptual models, and reservoir operating rules is in Chapter 2. A review of methods relevant in the study is also covered in Chapter 2. Chapter 3 describes the characteristics of the selected case study area as well as data used in the study. The methodological procedures followed in the study are described in Chapter 4. Chapter 5 describes the results on delineation of groundwater resource unit, its hydrogeological conceptual model and hydraulic characterisation. Chapters 6, 7 and 8 provide the results of extension of rainfall and groundwater levels data, GW-PITMAN modelling and generation of groundwater levels, and generation of stochastic rainfall, evaporation and groundwater levels, respectively. Results on stochastic groundwater base yield analysis and development of risk-based operating rules are provided in Chapter 9. The conclusions and recommendations are in Chapter 10.

CHAPTER 2: LITERATURE REVIEW

2.1 Preamble

This chapter reviewed literature on operating rules including related studies and stochastic based approaches. This aided in identifying gaps and current applications of stochastic based approaches for deriving groundwater reservoir operating rules. In addition, literature on methods for stochastic generation of hydrological/weather variables was reviewed to identify methods that can be applied in this study. Literature on groundwater resource assessment was essential to establish commonly used procedures and their applicability in data scarce areas. Literature review was also focused on appropriate methods for infilling and/or extending input data required when developing stochastic based operating rules for groundwater supply. This was justified by the fact that data scarce areas lack adequate, reliable and continuous data. This review only focused on methods/models that are used to infill and/or extend the input data required for this study to limit the extent of literature review since there is a wide spectrum of literature on methods for data infilling and/or extension of hydrological data.

Literature on groundwater resource unit delineation and hydrogeological conceptual models was also reviewed. The groundwater resource unit provides the basis for groundwater balance quantifications. Models for determining aquifer type and flow behaviour in fractured aquifers using diagnostic plots were also reviewed.

2.2 Review on groundwater resource unit delineation and hydrogeological conceptual models

Groundwater development under the National Water Act (Act no 36 of 1998) requires establishment of groundwater resource units and calculation of the amount of allocable groundwater taking into account the basic human need and ecological reserve (Levy, 2011). A groundwater resource unit is defined as a system that has been delineated or grouped into a single significant water resource based on one or more characteristics that are similar across that unit (Riemann and Blake, 2010). It can also be referred to as a groundwater response unit. A groundwater response unit is the smallest groundwater unit considered and is demarcated on the basis of homogeneity and geohydrological region type (GEOSS, 2006).

Groundwater response units are delineated in order to identify areas which are hydrogeologically similar (for monitoring and reporting purposes) and areas where there is a boundary between an aquifer and an aquitard or aquicludes (GEOSS, 2006). The delineation is based on geological formations, geologic features such as faults, dykes and lineaments. Xu *et al.* (2009) considered contacts between different geological formations, faults, main rivers, primary catchments, discharge boundaries and extent towards where the Table Mountain Group (TMG) aquifers die out as boundaries when delineating the TMG into 15 hydrogeological/groundwater units. Riemann and Blake (2010) delineated 5 model sub-domains of the confined Peninsula Formation Aquifer based on geological and structural features. Blake *et al.* (2010) delineated separate model domains for the Peninsula and Skurweberg formation aquifers in Cape Town, South Africa using faults, lithological contacts and dykes.

A groundwater divide separates areas where water flows in one direction from areas where it flows in another. The concept of groundwater divide has been used for delineation of groundwater units in studies such as Sheets and Simonson (2006). Groundwater divides are frequently simulated as no-flow boundaries in groundwater flow models to limit the areal extent of the system being analysed (Reilly, 2001). Once a groundwater resources unit has been delineated it can serve as a groundwater reservoir where which provides the basis for groundwater assessments and development of operating rules.

The development of a conceptual model is the first step towards understanding a groundwater system (Wilson and Davidson, 2011). A hydrogeological conceptual model is a pictorial presentation of the groundwater flow system incorporating all available geological and hydrogeological data into a block diagram or geological cross-section (Anderson and Woessner, 1992). It is developed to describe the inflows and outflows within a groundwater resource unit. The purpose of developing a conceptual model is to arrive at a sufficient understanding of the relationships between the principal characteristics of a system so that deductive and/or mathematical methods can be used to evaluate possible outcomes of changes within the system for a range of feasible situations (Brassington and Younger, 2010).

Construction of a hydrogeological conceptual model involves defining geological and hydrological frameworks of the study area. Data for the geological framework is typically obtained from geological maps, borehole logs, geophysics and additional field mapping (Wilson, 2005). Construction of the geological framework then allows the hydrological framework to be defined involving the following: (a) identifying the boundaries of the hydrological system, (b) defining hydrostratigraphic units, (c) preparing a water budget, and (d) defining the flow system (Sefelnasr, 2007).

A mass/water balance for an aquifer describes all the inputs to and outputs from a system (Wilson and Davidson, 2011). Preparation of a water budget involves the identification and quantification of all flow magnitudes and directions of the source of water to the groundwater system as well as the outflow from the system (Sefelnasr, 2007). In some cases, the development of a conceptual model of a groundwater system can be an end in itself, as it forms the basis for the majority of hydrogeological projects where the understanding of the system provided by the conceptual model allows for decisions to be made and the risks associated with new developments to be evaluated to a satisfactory level of accuracy (Brassington and Younger, 2010).

2.3 Reservoir operating rules

Basson *et al.* (1994) noted that it is generally not economically feasible to develop and operate water resources to meet demand all the times especially in arid and semi-arid regions where the water resources are scarce and limited. Limited water resources coupled with changing climatic conditions, increasing population, economic development and living standards cause increase in water demands as stated in Macian-Sorribes (2017) making it crucial to improve water resource systems efficiency. Efficient operating rules are therefore required to improve water supply reliability and sustainable use of available water resources. Operating rules are statements on how to schedule water releases from a given source at a given time (season) (Johnson, 1993). Operating rules can assist in judging when the storage reservoirs can supply more than their minimum yield for a given risk (Ratnayka *et al.*, 2009).

Reservoir operating procedures include a set of instructions, equations, tables or simply judgment decisions by which reservoir releases and diversions are determined based on current or forecasted state of the system (Guggino *et al*., 2012). The purpose of operating rules is to distribute any necessary deviations from ideal/target conditions in a manner that satisfies mandated laws or regulations and/or that minimises the discomfort to all users in the system (Johnson, 1993; McMahon and Adeloye, 2005).

Reservoirs are often operated considering a number of conflicting operational objectives related to environmental, economic and public services (Chu *et al.*, 2015). Operational objectives provide specific details of the purpose of the reservoir. For example, a primary operational objective of a water supply reservoir may be to improve the assurance of water availability at particular times and places while for a flood control reservoir it may be to assure the required flood storage in order to avoid or reduce downstream damages (Guggino *et al.*, 2012). These objectives are often conflicting and unequal and therefore require optimisation of reservoir operation to determine balanced solutions between them (Ngo, 2007). To meet the objectives for which the reservoir was planned, it is therefore vital to formulate guidelines for its operation (Kerachian and Karamouz, 2006).

Reservoir operation is inherently stochastic given the uncertain nature of reservoir inflows (Draper, 2001). The uncertainty arises when inflows into the reservoir cannot appropriately represent highly variable hydrologic conditions or when they cannot be reliably forecasted for a long period (Celeste and Billib, 2009). In addition, the climate is known to exhibit large inter-annual and inter-decadal variability (Ndiritu *et al.*, 2017), which also leads to uncertainty. Natural variations of a climatic system, as well as the potential influence of human activity on global warming, have changed the hydrologic cycle and threaten current water resources management; hence, the conflicts between different objectives in reservoir operation may become more challenging (Yang *et al.*, 2016).

2.3.1 Procedures for development of reservoir operating rules

Generally, system analysis models used to optimise reservoir operation may be categorised as simulation, optimisation and combination of simulation and optimisation models (Ngo, 2007). A simulation model basically provides solutions that obey the equations governing the relevant processes in the system while optimisation models identify an optimal management strategy from a set of feasible alternative strategies. Simulation models require numerous runs of a model with alternative policies to detect on near optimal solution (Simonovic, 1992). They can therefore not directly derive operating rules. Optimization models automatically search for an optimum set of decision variable values (Wurb, 2005). Optimisation tools are utilised to facilitate optimal decision making in the planning, design and operation of especially large scale water resources systems (Datta and Hakrishna, 2005). Optimisation methods may be limited due to the complexity of the systems including several components like reservoirs, aquifers, pumping systems, hydroelectric power plants, demand sites, amongst others (Shourian *et al.*, 2008). The combination of simulation and optimisation produces an engineering design tool that can aid in the formulation of design criteria and assist decision makers in assessing the impacts of trade-offs (Haddad and Mariño, 2010).

Operating rules are derived from three approaches which include direct optimisation of the system's operation, using *a priori* operating rule forms (predefined operating rules) and inferring rules from optimisation results (Macian-Sorribes, 2017). Direct optimisation approach involves applying an optimisation algorithm to long-term data (monthly time steps with planning horizons of more than a decade) in order to extract and analyse the ideal operation of the system and its associated performance. Optimisation models offer an expanded capability to systematically select optimal solutions, or families of solutions, under agreed upon objectives and constraints (Labadie, 2004). Common reservoir operating rules developed from optimisation results are storage allocation rules, storage target rules, and release rules (Nelson *et al.*, 2016).

Predefined operating rules are defined before they are evaluated using simulation models (Oliviera and Loucks, 1997). In practice, predefined operating rules are usually defined by optimization models and evaluated using simulation models while in another cases, the rules can are initially assumed and tested with historical or synthetic inflow records to determine their effectiveness (Tospornsampan *et al.*, 2004). In many practical situations, predefined operating rules remain a basis for reservoir operation, providing guidelines for reservoir releases to meet planned demands.

Inferring operating rules for reservoir operations involves obtaining general rules by which reservoir operations can be controlled while satisfying the objectives of the system operations (Mousavi *et al.*, 2007). Fuzzy rule-based modelling can be effectively used for inferring operating rules by simulating historical operations (Umadevi *et al.*, 2014). Other inferring procedures that can be used include regression and interpolation equations, simple statistics, diagrams and tables, data mining, artificial neural networks and reinforcement learning (Mousavi *et al.*, 2007; Macian-Sorribes, 2017).

Macian-Sorribes (2017) categorised models used for optimisation of reservoir systems based on purpose of the algorithm and time horizon, as long-term optimisation (based on historic records) and real-time optimal control with forecasting (based on short time horizons (hourly or daily time steps and time spans of weeks or months)). Figure 2.1 provides a categorisation of long-term optimisation models which are classified as implicit stochastic optimisation (ISO), explicit stochastic optimisation (ESO) and heuristic programming. Labadie (2004) provided a detailed review of ISO, ESO, real-time optimal control with forecasting and heuristic programming methods and expanded the review to include network flow optimisation and discrete-time optimal control theory, and multiobjective optimisation and stochastic optimal control models which are ISO and ESO methods, respectively, not covered in Macian-Sorribes (2017).

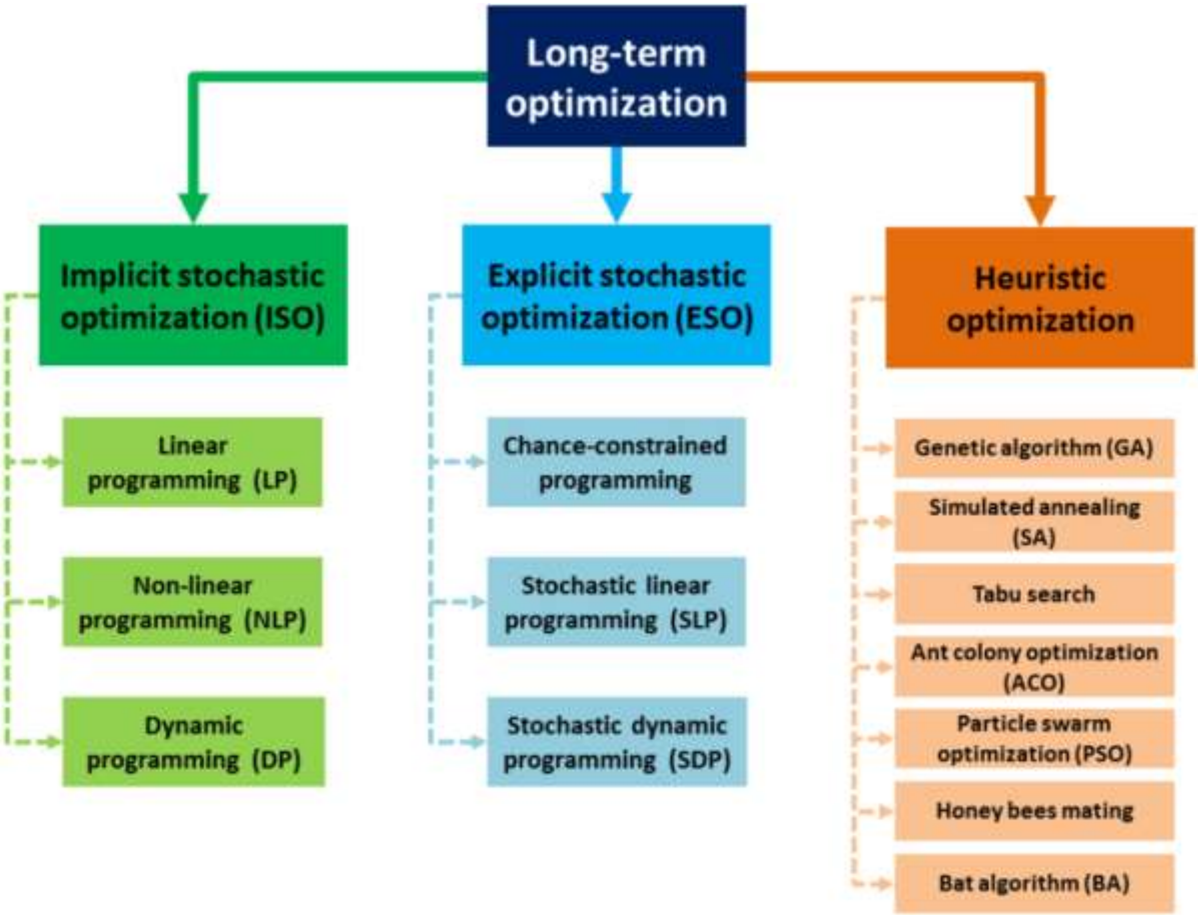


Figure 2.1: Long-term optimisation algorithm classification (Macian-Sorribes, 2017)

ISO involves performing deterministic optimisation on long historical or stochastically generated inflow sequences (Lee and Labadie, 2007). In ESO, the inflow process is directly described using its probability distributions, and then traditional optimisation methods can be applied to solve the problem (Zhou *et al.*, 2016). Celeste and Billib (2009) showed that ISO gives better results as compared to ESO. This may be due to that ISO uses observed or synthetic inflow scenarios which gives it a computational advantage as compared to ESO which utilises the probability density functions of the inflows. Lund and Ferreira (2006) noted that both ISO and ESO are not perfect in establishing optimal operating rules. ISO is more detailed and can be solved more quickly than ESO, though it can be resource (time and money) consuming when applied to large reservoir systems (Sulis, 2014). Lund and Ferreira (2006) indicated that ESO suffers from great computational inconvenience, limited computational feasibility and naturally require explicit presentation of probabilistic streamflows or other uncertain aspects of the problem which is a difficult task. Despite this, both ISO and ESO have widely been used in developing reservoir operating rules.

Real-time optimal control with forecasting is only possible at short time horizons and in water resource systems in which the objective is unique and clearly defined, such as maximizing hydropower production or minimising pumping costs (Macian-Sorribes, 2017). ISO and ESO can be applied for determining long-range guide curves and policies but real-time optimal control models are then designed to track these long-term guidelines over shorter time horizons in hourly or less or daily time increments (Labadie, 2004).

Probabilistic or risk-based methods aim at incorporating the real-world uncertainties of not knowing future inflows into the operating rules (Ramirez, 2004). Applications of risk-based principles and stochastic approaches to water planning uses statistics of the historical record to estimate flow frequencies and probabilities of system failure (Borgomeo *et al.*, 2014). Risk of failure is the probability of not being able to supply base yield associated with a specified target draft (demand) at least once over a specified period of time (Blersch, 2014). Stochastic methods also account for uncertainties associated with reservoir operation. Stochastic programming is a framework for modelling optimization problems that involve uncertainty (Liu *et al.*, 2012).

Stochastic programming has been widely applied in deriving operating rules for surface water reservoirs. Harboe and Ratnayake (1993) applied stochastic programming to arrive at a target which considers the effects of all possible critical periods. Celeste *et al.* (2009) applied stochastic programming in deriving operating rules for the Coremas-Mãe d'Água system taking into account the uncertainties of hydrologic variables. In South Africa, an approach that combines simulation with network flow programming and a detailed evaluation of supply reliabilities to multiple users has been applied since the late 1980s for developing reservoir operating rules (Ndiritu *et al.*, 2017). The methodology is applied to all major water resource systems in South Africa and has also found application in some of the neighbouring states (Basson and Van Rooyen, 2001). Ndiritu *et al.* (2017) followed this basic approach but used Shuffled Complex Evolution optimiser developed at the University of Arizona (SCE-UA) that allows for optimisation of non-linear functions in place of the network flow programming when deriving reservoir operating rules for Hluhluwe Dam in South Africa. Network flow programming is a computationally efficient form of linear programming which can be applied to problems that can be formulated in a specified format representing a system as a network of nodes and arcs (Wurbs, 2005). The optimisation aims at minimising the total cost of flow in the network and the operating rules are derived from the optimal solution (Ndiritu, 2003).

2.3.2 Examples of studies on surface reservoir operating rules

Kangrang *et al.* (2018) applied conditional genetic algorithm and conditional tabu search algorithm to develop optimal operating rules for the Ubolrat Reservoir located in northeastern Thailand. The optimal future operating rule curves were more suitable to mitigate drought and floods in the study area. Sasireka and Neelakantan (2017) used a two-point linear hedging method to develop operating rules for Bargi reservoir in India. The application of hedging rule was found to significantly improve the reliability of water supply and irrigation release and firm power production. The municipal and irrigation supplies were satisfied with 100% reliability while that of firm power increased from 10.95 to 12.84%. Thankachan and Anitha (2015) applied a system approach to develop optimal operation plans for reservoirs in northern Kerala, India. Irrigation and water supply reliabilities of 86 and 77%, respectively, were obtained for Peruvanannamuzhi reservoir while power generation could be met with 90% dependability.

Zhou *et al.* (2018) applied a two-stage optimal reservoir operation model with a dynamic programming-progressive optimality algorithm (DP-POA) for optimal co-operation of Xiluodu, Xiangjiaba and Three Gorges Dam cascade reservoirs in the upper Yantze River, China. This was aimed at flood control at multiple points downstream during the flood season. The results indicated that optimal operation of Xiluodu and Xiangjiaba reservoirs can reduce the inflow of Three Gorges Dam for effective control of floods. Nikoo *et al.* (2013) used M5P and Support Vector Regression (SVR) models for the derivation of rules for optimum reservoir-river-groundwater monthly operation in the Zayandehrood hydro-system in the central part of Iran. Model results indicated the capability of integrated water quantity and quality management in controlling and reducing the TDS concentration up to 43 % in the river.

Paredes and Lund (2005) derived operating rules for the refill and drawdown seasons for water supply reservoirs in parallel considering water quality based on linear programming technique. Shasta and Whiskeytown reservoirs in California were used as case studies to illustrate the technique with temperature as a water quality variable. The derived operating rules were found to be potentially useful for ensuring release of water to reduce downstream temperatures. Liu *et al.* (2018) derived emergency operation rules for water supply in dry seasons through the trial and error method in the Danjiangkou Reservoir in Han River, China. The proposed emergency operation rules which considered forecast uncertainties and risks resulting from potential droughts and sudden water pollution are expected to provide important insights into reservoir water supply in dry seasons. Karamouz and Kerachian (2004) developed an algorithm combining an ANFIS-based water quality simulation model and a GA-based optimization technique for determining optimal operating rules for 15-Khordad Reservoir in the central part of Iran. The developed operating rules were capable of reducing salinity of water allocated to different water users as well as the salinity build-up in the reservoir.

Yang *et al.* (2016) combined the reservoir operation function and operating rule curves to develop an adaptive multi-objective operation model that adapts to climate change. The multi-objective operation model was applied to Danjiangkou reservoir located at the middle reach of Hanjiang River in China and was found to adapt climate change and to maximize the annual power generation and reservoir water supply yield by 18.7%. The study followed the *priori* operating rule forms approach since operating rules were defined in advance and were then optimised.

2.3.3 Examples of studies on groundwater operating rules/strategies

Water supply systems that obtain water from groundwater require operating rules/strategies to regulate competing water uses, ensure the beneficial use of water and also account for the groundwater reserve. Planning and operation of groundwater reservoirs require good knowledge of their characteristics and limitations of the aquifer, an estimate of their natural replenishment and outflows, as well as the determination of a programme for pumping (Harpaz and Schwrz, 1967). Management strategies are needed to address the unique characteristics and roles of groundwater (Pietersen, 2006).

In most groundwater studies the terms operating rules, operating policies, management strategies or policies have been used synonymously and will be treated as such in this review. For example, most of the studies that have been reviewed in the current study refer to operating rules as operating policies (for example, Shamir and Bear (1984)), operating management strategies (for example, Ökten and Yacigil (2005)) and management policies (for example, McPhee and Yeh (2004)). Studies such as Gallagher and Leach (2010) clearly referred to operating rules. The current review included studies on groundwater management strategies whose focus was on operating rules to provide adequate background information and assist in identifying if there are any approaches that can be applied in this study.

Das and Datta (2001) presented a review on application of optimisation techniques in groundwater quality and quantity management. The study demonstrated the combined use of simulation and optimisation techniques in determining planning and management strategies for optimal development and operation of groundwater systems. This study has reviewed examples of studies that have applied the latter techniques for deriving groundwater operating rules. Examples of studies reviewed by Das and Datta (2001) that used such techniques to derive groundwater operating rules include Willis (1983), Willis and Liu (1984) and Hallaji and Yazicigil (1996).

Willis (1983) used linear programming (LP) to determine the optimal pumping scheme for three consecutive periods in order to meet agricultural water demands for an unconfined aquifer in the Yun Lin basin in Taiwan. The objectives were to maximise the sum of hydraulic heads and minimise the total deficit. Willis and Liu (1984) applied an optimisation model to the Yun Lin groundwater basin in southwestern Taiwan to generate optimal planning policies and a set of non-inferior solutions. The optimal operating policy was aimed at maximising the sum of the hydraulic heads and minimising the total water deficit for the entire basin. Hallaji and Yazicigil (1996) proposed six LP models for steady and transient states, and one quadratic optimisation model for steady state management of the coastal aquifer in southern Turkey. The optimisation achieved drawdowns that would meet the demand without causing salt water intrusion.

Harpaz and Schwrz (1967) performed an optimisation analysis on a simplified single cell model representing a limestone aquifer system, in the central part of Israel. This was aimed at establishing optimal operation the aquifer as a water supply reservoir. The study presented four combinations of alternative plans representing two extreme climatic conditions (dry and wet years). A historic sequence of 12 years which included successive dry years with approximated recurrence probability of 10% was used. The procedure for determining optimal operation policy developed by Bear and Levin (1966) based on the sequential decision process algorithm was used. Two alternative plans aimed at satisfying the local demand only, and supplying both local and external demands and artificial recharge were tested for both the dry and wet periods. All the plans were not able to reach optimal solutions as demands were not always met without violating the constraints (water levels failing below permissible level and low spring discharges).

Yazdanian and Peralta (1986) developed a method for designing a regional groundwater withdrawal strategy that maintains a set of optimal potentiometric surface elevation using the goal programming approach. Goal programming approach is a multi-objective programming process that tries to find the best compromise between individual objectives. The method was applied to the Grand Prairie region of Arkansas. The study concluded that the method was well suited for designing sustained yield strategies since it was able to maintain optimal groundwater elevations.

Das Gupta *et al.* (1996) noted that most of the earlier groundwater quantity management models (Aguado *et al.,* 1974; Dreizin and Haimes, 1977; Wanakule *et al*., (1986); Lindner *et al.*, (1988); Peralta *et al.,* 1991) were applied to hypothetical conditions while their practical applications were limited to single-aquifer systems for short periods of time. Das Gupta *et al.* (1996) developed and applied an operational groundwater management model for pumping and recharge policy by maximising net relative benefit or minimising operation cost, subject to a specified allowable drawdown, minimum pumping requirement and maximum allowable recharge. The model was developed for the Bangkok, Phra, Pradaeng and Nakhon Luang aquifers in Bangkok. The study simulated the hydraulic response of a multi-aquifer system using the finite-difference alternating direction implicit scheme. Optimum pumping distributions satisfying defined minimum potentiometric heads as well as economic criteria (minimising operation cost) were achieved.

McPhee and Yeh (2004) used groundwater simulation and optimisation approach to construct a decision support system for solving a groundwater management problem for the Upper San Pedro River Basin, located in southeastern Arizona. The approach used was such that, once the algorithm identified a set of efficient solutions (alternatives), concepts borrowed from fuzzy set theory were applied to rank the alternatives and to assist decision makers in selecting a suitable policy. The tested policy analysis options include assuming that pumping rates are constant over the entire planning horizon and that pumping rates are allowed to vary within the planning horizon (20 years).

Ökten and Yacigizil (2005) developed a numerical groundwater flow model for the Sandy Complex aquifer in the Ergene River Basin, Thrace Region, Turkey. Groundwater pumping scenarios were developed to determine the safe and sustainable yields, and the limits of utilisation for the Sandy Complex aquifer for a planning period of 30 years. Safe yield was considered to be the annual amount of groundwater pumped from an aquifer without exceeding the annual recharged through precipitation, surface water and subsurface inflow while sustainable yield allows adequate provision of water to sustain streams, springs, wetlands, and groundwater dependent ecosystems. Analysis of eight scenarios in which the annual pumping rates were decreased in order to be equal to 100, 90, 80, 70, 60, 45 and 35% of the annual recharge indicated that as annual pumpage decreased, the declines in groundwater levels and reserves also decreased. It was suggested that efficient water-management policies and plans to prevent the eventual depletion of the aquifer system should be developed since the existing pumping rates were greater than both the sustainable and safe yields of the system The policies and plans should incorporate controls on new development, water metering on all wells, annual water-use reporting, water conservation measures, artificial recharge structures and efficient irrigation schemes should be considered.

Gallagher and Leach (2010) developed a module of MODFLOW package called the Groundwater Operational Management Package (GWOMP) to improve the link between water resource planning objectives and the simulation of future groundwater system behaviour under different management schemes. The programme produces a detailed account of the extractive deficits recorded within management areas over the simulation period, as well as the history of trigger activation and operational decisions, which, when examined in association with the model simulated head and flow response under the operating rules, allows a robust statistical assessment to be made of the potential impacts on groundwater-dependent ecosystems and the reliability of water supply. The study further reported successful application of GWOMP in the water resource planning and operating plans for Pioneer Valley and Burnett basins in Queensland.

Pietersen (2006) used a multiple criteria decision analysis (MCDA) approach to identify critical alternative courses of action and to develop a decision making framework for sustainable groundwater management. Strategies which included involvement of appropriate users in technology selection, facilitating democratic decision models, developing tariff structures with the involvement of appropriate users, developing a regional conceptual ground-water flow model, select the most favourable target(s) for development (target selection should be based on a combination of favourable factors), delineation of protection zones, implementation of artificial recharge and water harvesting systems and drought prediction were proposed for sustainable groundwater management in Namaqualand, South Africa. The study noted that the application of the tool in a participatory environment will require further refinement and adaptation (including further work related to sensitivity analysis). Pietersen (2006) study followed the methodology used in developing the decision model for groundwater in Namaqualand by Pietersen (2004).

The reviewed studies considered groundwater as sole source of water supply and have not incorporated the reliability of groundwater supply in their analyses. The fact that groundwater is also vulnerable to drought as explained in section 1.2, further calls for the development of risk-based groundwater operating rules which will aid in management of groundwater during drought conditions. A groundwater system also requires stochastic based operating rules due to uncertainties attributed to inherent temporal and spatial variability of its variables. Risk-based operating rules have comprehensively been developed for surface water systems in studies such as Maré *et al.* (2007), Mallory *et al.* (2017) and Ndiritu *et al.*

(2017). Rule curves developed in Maré *et al*. (2007) and Mallory *et al.* (2017) where capable of establishing whether there is water surplus or deficit in the systems. When there is a deficit curtailments/restrictions are introduced to avoid depletion of water. Ndiritu et al. (2017) indicated after implementation of the derived risk-based operating rules, Hluhluwe Dam storages increased after being generally lower than those of dams within the same region due to poor operation. The approaches by Maré *et al.* (2007), Mallory *et al.* (2017) and Ndiritu *et al.* (2017) are therefore applicable for development of risk-based groundwater operating rules as they account for uncertainty and assist in sustainable use of the resource.

Examples of studies that followed stochastic approach in deriving groundwater operating rules include Philbrick and Kitanidis (1998), Knapp and Olson (1995) , Marques *et al.* (2010) and Dracup and Dale (2011). A study by Philbrick and Kitanidis (1998), based on a hypothetical system containing surface and aquifer storages, only considered inflow from streams as stochastic variable when developing optimal policies for conjuctive use system. The policies developed were in graphical form indicating controlled pumping and recharge, releases to water supply and downstream users as a function of water available in the current year. Knapp and Olson (1995) extended a basic groundwater model for Kern County in California to include stochastic surface supplies and artificial recharge when deriving optimal decision rules. Optimal decision rules based on theoretical analysis generally increased hydraulic heads while decreasing surface flows. Marques *et al.* (2010) used stochastic quadratic programming to optimise conjunctive use operations of groundwater pumping and artificial recharge with farmer's expected revenue and cropping decisions. However, the study only considered the stochastic nature of surface water availability. Results indicated potential gains in expected net benefits and reduction in income variability from conjunctive use, with increase in high value permanent crops along with more efficient irrigation technology (Marques *et al.,* 2010).

Dracup and Dale (2011) developed an inter-annual stochastic model of the operations of a conjunctive use system that includes a reservoir as a source of surface water and an aquifer. Monthly inflows into the reservoir were considered to be a stochastic variable at the annual time scale. In the dry scenario conjunctive use lead to a decrease in groundwater pumping costs and an increase in hydropower benefits, compared to the base case. Joodavi *et al.* (2015) developed an explicit stochastic optimization model for finding optimal crop patterns and amount of groundwater extraction in Firouzabad aquifer located in south-central part of Iran. The model determined optimal allocation of the agricultural area to four selected crops (wheat, barley, corn, and rice) to ensure sustainable management of aquifer. The study considered that aquifer recharge might stochastically fluctuate in time because recharge from precipitation is subject to stochastic rainfall. Though the study incorporated stochastic analysis of groundwater, it was also focused on conjunctive use of surface and groundwater for winter crops.

This review shows that studies on operating rules for conjunctive use of surface and groundwater are focused on optimising irrigation water supply and hence there is limited focus on domestic water supply. The current review did not find documented studies on stand-alone stochastic based operating rules for groundwater supply. Thus, there is lack of operating rules that adequately specify the reliabilities associated with water allocation decisions for groundwater reservoirs. Stochastic based groundwater operating rules are critical particularly in areas that are dependent on groundwater as they could aid in its management during drought conditions.

2.3.4 Comparison of operating rules for surface and groundwater reservoirs and implementation

Groundwater reservoirs are typically aquifers that store and supply usable quantity of water for various purposes. They are comparable to surface water reservoirs as they both store water that can be supplied for different uses. This therefore means that optimisation approaches that are applied in development of surface water reservoirs can also be applied for groundwater reservoirs. This has been confirmed from the examples of the studies reviewed in sub-sections 2.3.2 and 2.3.3 which indicated that optimisation techniques are used for developing operating rules for both surface and groundwater reservoirs. However, the reviewed studies lack application of stochastic (risk-based) approaches that incorporates risk of failure or assurance of supply in groundwater operating rules.

The difference in surface and groundwater reservoirs is that measurements of water levels in a surface water reservoir start from the bottom of the reservoir to its surface while in groundwater reservoir measurements start from the ground surface to the groundwater table. The procedures followed in developing risk-based operating rules for water supply in South Africa including Basson and Van Rooyen (2001) and Ndiritu *et al.* (2017) consider reservoir storage levels in terms of percentage of full supply capacity. If such an approach is to be followed, development of operating rules for groundwater reservoirs should be in terms of groundwater heads. Groundwater head is the height of water in the reservoir from a specified datum and it is equivalent to water level in a surface water reservoir. This means that for practical application, operating rules for groundwater reservoirs will require conversion of groundwater head to groundwater levels. This can be done by relating the groundwater head, groundwater level and borehole depth. Operating rules derived through this approach are expected to indicate the volume of water that can be supplied for different groundwater levels.

In practice, operating rules are implemented by reservoir operators who usually follow the rule curves, which stipulate the actions that should be taken conditioned on the current state of the system (Celeste and Billib, 2009). Operating rules therefore guide reservoir operators on the actual operation of the reservoir. Developed operating rules therefore need to be acceptable and simplified for implementation by reservoir operators. Approaches such as fuzzy logic, are flexible and allow incorporation of expert opinions, which makes them acceptable to reservoir operators (Panigrahi and Mujumdar, 2000). The limitation of fuzzy logic is that it suffers from the curse of dimensionality (computational effort increases exponentially with the complexity of the considered system) (Russel and Campbell, 1996), limiting its application to single reservoir systems (Panigrahi and Mujumdar, 2000).

In South Africa, reservoir operators are included as stakeholders during the development of operating rules for surface water reservoirs (see DWAF, 2008b; DWA, 2010). This ensures that developed operating rules are acceptable and simplified for ease of implementation by operators. Developed operating rules are also simplified into graphical format to simplify their interpretation and implementation.

2.4 Generation of stochastic hydrological/weather variables

Generated stochastic hydrological inputs are typically used in derivation of reservoir operating rules. Use of stochastic sequences improves the precision with which water resources system performance indices can be estimated (Louck and van Beeck, 2017). Efstratiadis *et al.* (2014) noted that probabilistic assessment through stochastic simulation is of high importance since synthetic time series provide large samples or ensembles of different time series to evaluate a wide range of possible outcomes. There is vast literature on methods/models that have been developed and tested for stochastic generation of hydrological/weather variables. This review focused on multi-site and multi-variate methods for stochastic generation of hydrological/weather variables. This was aimed at enabling selection of a method/model that is applicable and suitable for the current study. Multisite generators make it possible to reproduce the space–time variation of variables at several sites (Breinl *et al.*, 2013). Multi-site stochastic weather generators reproduce the interstation correlations and self-consistency in weather series (Qian et *al.*, 2002), while multivariate schemes enable the preservation of cross-correlations between variables (Efstratiadis *et al.*, 2014). In some applications, it is important to preserve the spatial correlations when simulated series are reused as input to process models (Apipattanavis *et al.*, 2007).

Parametric, semi-parametric and non-parametric methods are used in stochastic hydrology. Parametric approaches use pre-specified functions to approximate the observed precipitation distribution (Rayner *et al.*, 2016). Parametric approaches sample the variable from a probability distribution function and assumes the transformed weather data to be normally distributed (Al-alawi *et al.*, 2017). Most of the parametric approaches are linear in their form and they can only capture linear relationships between the variables (Rajagopalan *et al.*, 2010). Parametric stochastic methods typically use large numbers of parameters, and the methods applied to preserve cross-correlations and to disaggregate annual variables are usually complex (Ndiritu, and Nyaga, 2014).

Non-parametric approaches are based on resampling methods such as bootstrapping (Breinl *et al.*, 2013). They often use resampling and simulation methods that do not need to meet any inherent data assumption (Herrera *et al.*, 2017). They allow generation of sequences that match the observed distribution with arbitrarily-high precision, at the cost of introducing arbitrarily many parameters (Rayner *et al.*, 2016). Non-parametric methods avoid the difficult model specification issues and can circumvent many other problems associated with the parametric methods (Sharif, 2006). One of the difficulties with the non-parametric methods for some applications is that future climate regimes cannot be easily constructed through simple parameter adjustments (Wilks and Wilby, 1999). Non-parametric methods are gaining wide prominence and are being applied to a variety of hydrologic and climatologic applications (Rajagopalan *et al.*, 2010) after decades of domination by parametric approaches (Ndiritu and Nyaga, 2014). Semi-parametric models are used to combine advantages of parametric and non-parametric stochastic methods. Similar to parametric, semi-parametric models require statistical assumptions regarding the probability distributions of climate variables and spatial correlations are assumed for multi-site applications (King, 2012).

Mehrotra and Sharma (2006) developed a semi-parametric stochastic model for simultaneous generation of daily precipitation multi-site in Sydney, Australia. The generator preserved realistic spatial correlations, accommodated seasonality, and reproduced a number of key aspects of the distributional and dependence properties of observed rainfall. A semi-parametric copula-based generator was developed to simulate precipitation, maximum temperature and minimum temperature in Mexico by Juárez-Torres *et al.* (2013). The approach captured the nonlinear dependence structure and the occurrence of extreme events more accurately and had acceptable replication of observed weather patterns.

Lee *et al.* (2010) improved a parametric multisite weather generator and applied it using historical data from South Korea. The weather generator has procedures that obtain a symmetric positive definite estimate for the covariance matrix, automatically selects a distribution that represents precipitation amounts well and minimises the computational burden. The results showed promising performance in terms of spatial correlation and long term variation of precipitation. Apipattanavis *et al.* (2007) used a modified semi-parametric multivariate and multisite weather generator which combines Markov Chain for generating the precipitation state and KNN bootstrap resampler for generating the multivariate weather variables. This was applied in generation of daily precipitation, maximum temperature, and minimum temperature at Pergamino in Argentina. Comparison of the results from a traditional k-NN weather generator, showed that the spell statistics are captured better in the modified method.

Kenabatho *et al.* (2012) investigated performance of multi-site stochastic rainfall models based on generalised linear models under semi-arid conditions in Botswana. The results showed consistent characteristics of observed and simulated rainfall while spatial and temporal validation tests showed adequate simulation of rainfall for the periods and gauges not used during model fitting. Kigobe *et al.* (2012) developed multi-site stochastic daily rainfall models that have the capability for extending and infilling historic data sets in Uganda. The models were able to reproduce inter-site and temporal patterns of precipitation, regional daily, monthly and annual statistics and joint probability of daily occurrence between rainfall zones. Ndiritu (2011) developed a variable length bootstrap (VLB) for synthetic generation of streamflow at multi-sites. The method was tested using data from 5 reservoirs in South Africa, and was compared to Stochastic Model of South Africa (STOMSA) which is a parametric generator widely used in South Africa. VLB adequately replicated historical annual statistics and reproduced the annual serial and cross-correlations better than STOMSA. Ndiritu and Nyaga (2014) adapted VLB to stochastically generate multisite rainfall. This was tested using rainfall data from widely spread out stations in South Africa. VLB generator replicated all the statistical measures reasonably well at the annual and monthly time scales. VLB model performed better than PEGRAIM-W at the annual and monthly time scales although both models were found to perform reasonably well for practical application.

Abraha and Savage (2006) used climate data generator to generate stochastic time series of precipitation, minimum and maximum air temperatures, and solar radiant density in KwaZulu-Natal, South Africa. The generated baseline weather data was similar to the observed for its distributions of daily rainfall and wet and dry day series, monthly total rainfall and its variances, daily and monthly mean and variance of precipitation, minimum and maximum air temperatures, and solar radiant density. Efstratiadis *et al.* (2014) improved Castalia stochastic generator for generating synthetic time series of hydrometeorological variables (wind speed, sunshine duration, rainfall and streamflow) at multiple locations and at daily, monthly, and annual time scales. Testing of Castalia software based on case studies in Athens and Eastern Greece showed that the software preserved the mean, standard deviation, skewness as well as the joint second order statistics. Greene *et al.* (2012) generated multivariate stochastic climate sequences for the Berg and Breede Water Management Areas in the Western Cape Province, South Africa. The methodology incorporated a first-order vector autoregressive model and a modified KNN resampling algorithm. The generated sequences preserved both spatial coherence and the temporal characteristics and linked sub-annual statistics (spell-related behaviour and precipitation extremes) to climatic trends.

2.5 Groundwater resource assessment

Groundwater resource assessment is aimed at quantifying the volume of groundwater that can be allocated for use. It is important to know the volume of available groundwater resources when developing operating rules. Approaches to quantifying groundwater hinge on the water balance equation or some components of this equation (DWAF, 2004). The hydrologic equation for groundwater regime (groundwater balance equation) is a specialized form of water balance equation that requires quantification of the components of inflow to and outflow from a groundwater reservoir as well as changes in storage (Kumar, 2004). The basic concept of water balance is:

$$I - O = \pm \Delta s \qquad (2.1)$$

where *I*= Input to the system, *O*=outflow from the system and Δs = change in storage in the system. Methods of solution to the water balance equation vary from simple analytical to complex numerical approaches (DWAF, 2004). With water balance approach, it is possible to evaluate quantitatively individual contributions of sources of water in the system, over different time periods, and to establish the degree of variation in water regime due to changes in components of the system (Kumar, 2004). Figures 2.2 (a) and (b) indicate the groundwater balance components of a natural system before and after its modification due to pumping, respectively. This shows that the groundwater balance of an area may require updating overtime to reflect changes in its components. The confidence in the results is a function of the number of components used in the water balance equation and the accuracy of the data used (DWAF, 2004).

Introduction of the National Water Act (NWA) in 1998 and recognition that South Africa is a water-scarce country have placed a new emphasis on groundwater and its associated integrated management (Dennis, 2007). In response to this, a number of studies carried out after introduction of the NWA focused on developing and/or applying approaches for determination of sustainable quantity of groundwater (yield) and/or their levels of assurance. Wright and Xu (2000) explored the possibilities of applying the water balance methodology to groundwater management in South Africa. The study noted that the quantity of utilisable groundwater within a region may be identified as neither entering nor leaving a geohydrological unit (*i.e.* may not be in a state of flux). Such groundwater could be considered as being held in storage. The study concluded that the robustness of applying the water balance approach to sustainable groundwater management needs to be tested in South Africa.

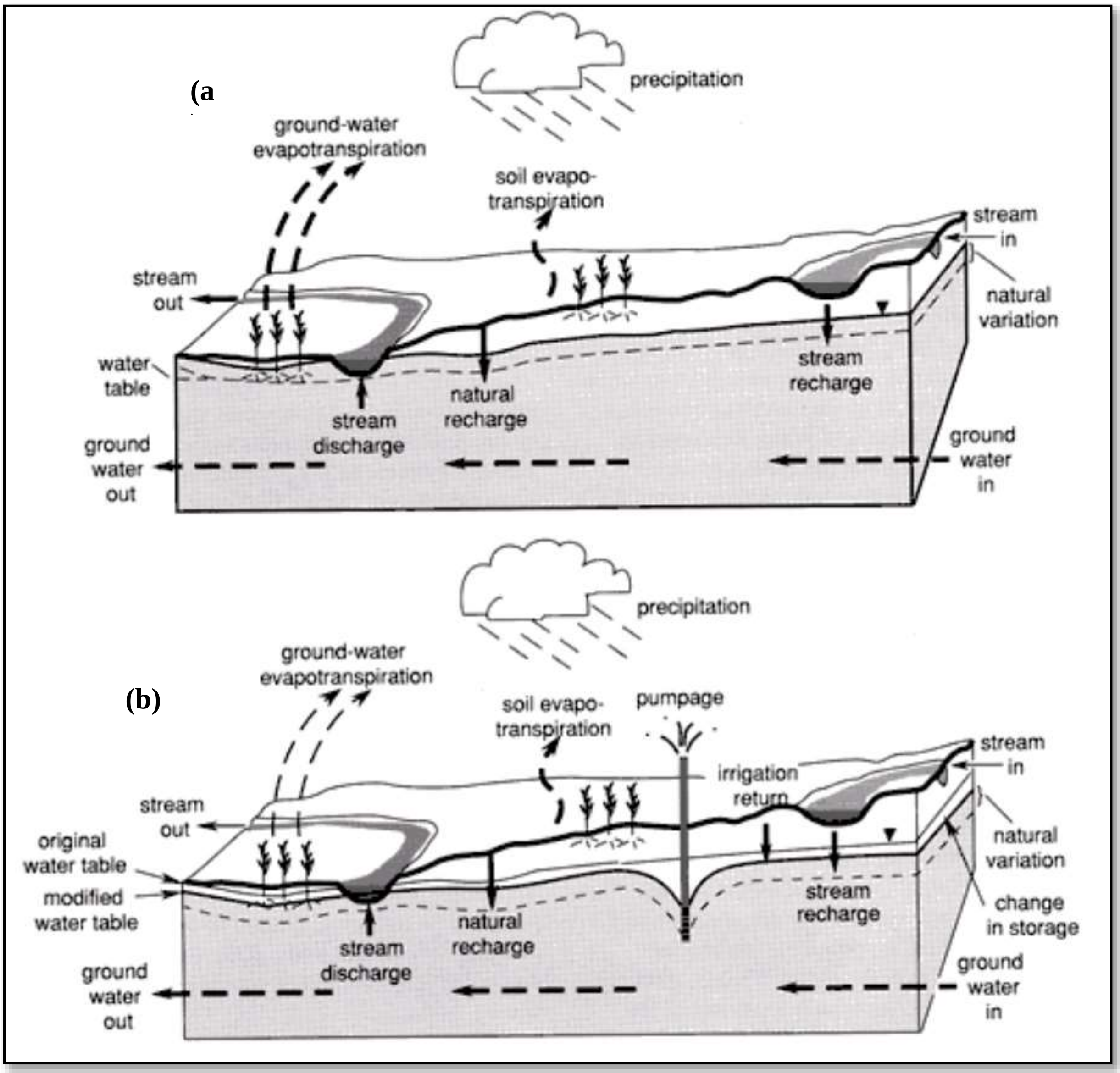


Figure 2.2: Groundwater balance for a system (a) before (natural condition) and (b) after modification (Buchanan and Buddemeier, 2005)

DWAF (2004) proposed a method, termed Aquifer Assurance Yield (AAY), as a means of including the supply assurance concept into the groundwater resource assessment. The AAY approach incorporates aspects of water balance principles as well as more detailed risk assessment, thereby allowing for reliability during drought, above average availability after major recharge events, and policy requirements. The proposed method was not tested in DWAF (2004), because it is data intensive for application on the national scale. DWAF (2006a) proposed a procedure that makes use of potential storage volumes together with parameters such as rainfall, recharge and baseflow to determine the annual volumes of groundwater available for utilisation on a sustainable basis. The developed method is applicable on a national scale or at the scale of an individual aquifer; the difference lies in the input data required (DWAF, 2006a). The method requires aquifer thickness and storage coefficient data, which are mostly available as default values at national scale from the groundwater assessment phase two (GRA II) project. The default values are therefore not useful at a local scale as they may not represent the local variations in aquifer thickness and storage coefficient. Conrad and der Voort (2000) developed a methodology to determine the sustainable utilisable potential of South African aquifers at a catchment scale taking into account the groundwater reserve (water required to maintain aquatic ecosystems and basic human needs). The methodology has been tested in areas (Atlantis, Zeerust and Beaufort West) where there is extensive groundwater data and can only be applied at catchment scale. The limitation of method is that a catchment is defined by a surface water divide and may not accurately represent a groundwater system particularly at a local scale.

Witthüser *et al.* (2009a) proposed a methodology for regional estimations of assured yields. The methodology links the data on borehole median yields and classes with data which contains assurance of supply information in order to produce a map or maps that would provide aquifer type, yield and assurance of supply information. Assurance of supply was inferred from the probability of failure which is computed by dividing the total number of times for which the aquifer is empty by the total number of time periods in the simulation. Following this approach, a 5% probability of failure represents a 95% assured yield. Witthüser *et al.* (2009a, b), DWA (2010) and Murray *et al*. (2012) indicated the importance of including level of assurance of supply (reliability) of groundwater in groundwater resource assessments. Incorporating level of assurance of supply aids in determining times when a groundwater system will be able to meet the demand and times of deficit. This can aid in making alternative water supply plans for meeting the demand during time of deficits.

In order to present groundwater yields in similar manner as in surface water supply, the same concept used in surface-water resource assessments and dam or reservoir design was adapted and applied to groundwater within the Aquifer Assurance Yield Model (AAYM) and Aquifer Firm Yield Model (AFYM) by Murray *et al.* (2012). AAYM provides assured yields similar to assurance levels given in surface water reservoir design estimated by statistical analysis of long-term time-series data of inflow against reservoir or aquifer storage. The assured yields can vary according to various design of the system and the demand (Murray *et al.*, 2012). AAYM is a simple groundwater balance model that reproduces storage dynamics based on variable volumes of inflow and outflow and provides groundwater yields with assured level of supply. The model is run in monthly time increments on a quaternary catchment scale. Inflow and outflow parameters (such as recharge as a percentage of mean annual precipitation, evapotranspiration, baseflow and threshold) have default values from GRA II, or values can alternatively be set by the user.

Murray *et al.* (2012) used the AAYM and AFYM to identify and quantify groundwater-development options for the main Karoo basin in South Africa. AAYM and AFYM are single-cell, lumped-parameter models, and make use of critical management water level below which aquifer storage levels cannot be drawn down, to provide estimates of the firm or assured yield of an aquifer. The essential components required for running the lumped-box AFYM are provided in Figure 2.3. These include recharge, evapotranspiration, baseflow and reservoir storage levels. Its limitations are related to both the assumptions on how well it simulates physical processes and the datasets from which the simulations are run (Murray *et al.*, 2012). It was suggested that users of this model should use site-specific data whenever possible to account for the latter concern. The aquifer yield models are only intended for use during the early planning stages of groundwater resource assessment studies where spatial and temporal hydrogeological information is scarce and perhaps several alternative schemes for increasing water supply are considered.

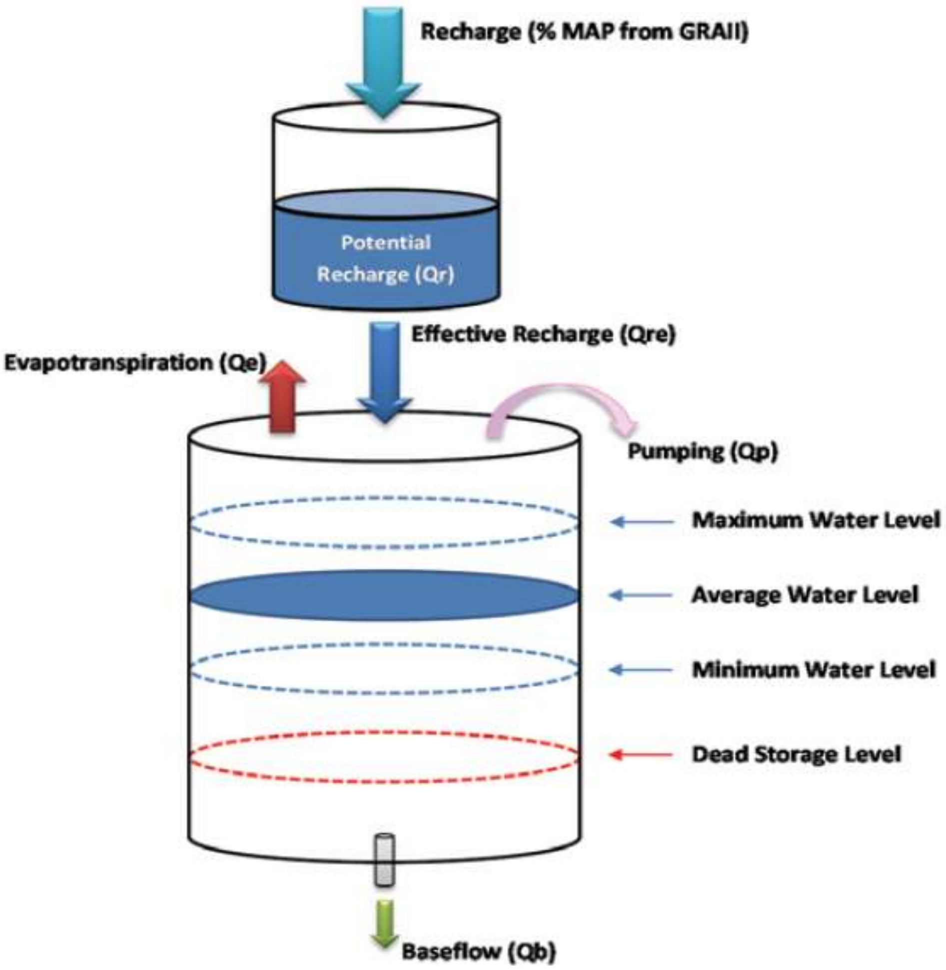


Figure 2.3: Essential components of the lumped-box AFYM (Murray *et al.*, 2012)

The Pitman model has contributed enormously to the practice of water resources assessment in South Africa and has formed the foundation of some national water resources development strategies (Hughes, 2013). Some of original design principles noted in Pitman (1973) include that:

- Only the principal components and relationships in the hydrological cycle must be selected so as to confine the model to an acceptable level of complexity.
- The model should represent the hydrologic regimes of a wide variety of catchments to an acceptable degree of accuracy.
- The model should be easily applied with existing hydrologic data to different catchments
- The model should be physically relevant so that, in addition to streamflow, estimates of other useful features, such as actual evapotranspiration or soil moisture state, can be made.

- The model should be applicable to ungauged areas

The principles boosted its wide applications mostly in other southern Africa regions including Lesotho (Khalema, 2010), Swaziland (Ndzabandzaba and Hughes, 2017), Tanzania (Tumbo and Hughes, 2015), Zambia (Mwelwa, 2004), Congo (Tshimanga and Hughes, 2012; 2014), amongst others. Hughes (2004) revised the original version of the Pitman model to simulate groundwater recharge and discharge thereby incorporating groundwater-surface water interactions. This version is herein referred to as the GW-PITMAN model following Tshimanga and Hughes (2012). The GW-PITMAN model is a conceptual type, semi distributed hydrological model, consisting of storages (interception, soil moisture, and groundwater) linked by functions designed to represent the main hydrological processes at the sub-basin scale such as infiltration, excess flow, saturation excess flow, direct overland flow, and groundwater flow (Tshimanga and Hughes, 2012). The conceptual nature of the model means that the parameters are at least 'physically-relevant' (within certain constraints associated with the time and space scales that the model typically operates over), even if they cannot be considered 'physically-based' (Hughes *et al.*, 2010). Physically relevant parameters are obtained directly using physical basin attributes and the role that they play in the rainfall-runoff process in the basin (Kapangaziwiri, 2011). The use of local information on physical catchment characteristics aids reduction of uncertainty in the estimation of model parameters (Kapangaziwiri *et al.*, 2012).

The configuration of the storage geometry of the groundwater module of GW-PITMAN allows computation of storages. The groundwater storage is represented by relatively simple geometry (Figure 2.4) based on a number of representative slope elements, determined from the catchment area and a drainage density parameter (Tanner and Hughes, 2015). Water balance components of the groundwater storage include recharge from soil storage, drainage to the channel, drainage to downstream sub-basins, evaporation losses from riparian zone, transmission losses from upstream inflows to groundwater (Hughes, 2013). Hughes (2004, 2013), Kapangaziwiri (2007) and Tanner and Hughes (2015) provided further description of the GW-PITMAN model.

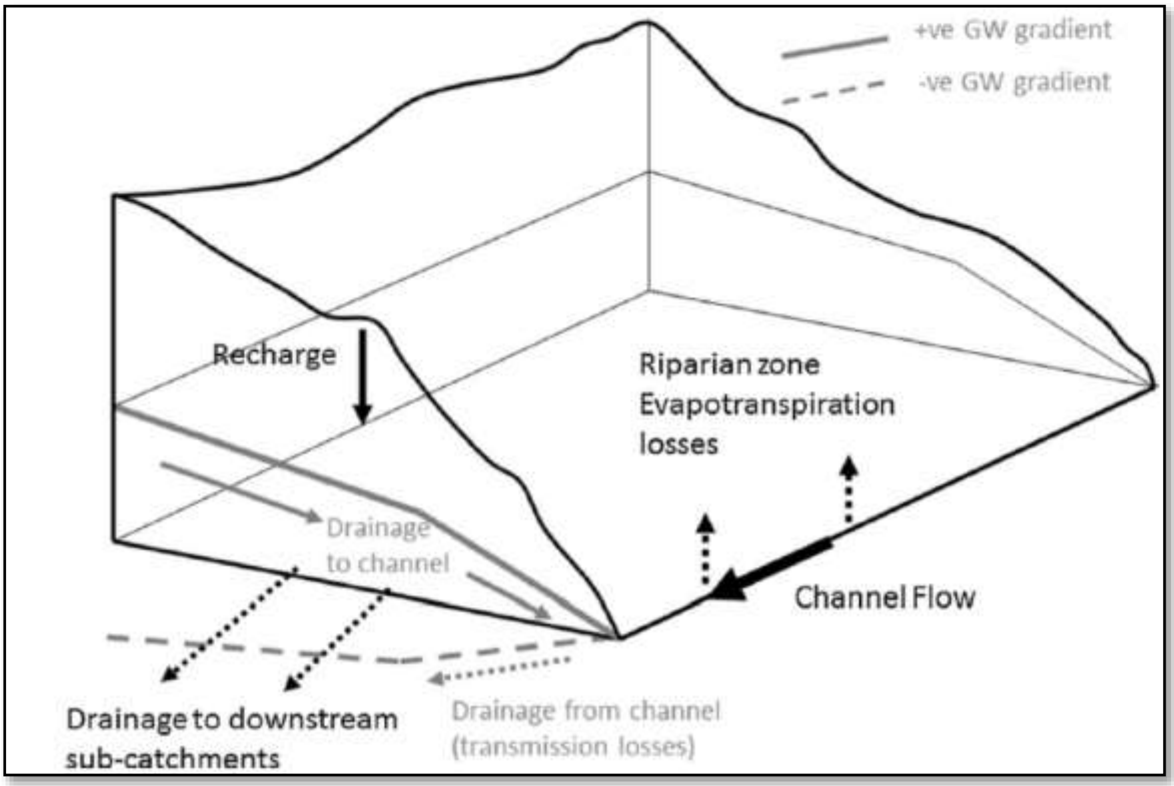


Figure 2.4: Geometry of the groundwater component of the GW-PITMAN model (Hughes, 2013).

The vertical geometry of the groundwater component of the GW-PITMAN model is defined by a simple representation of the groundwater table in each slope element (Figure 2.5A-C). Figures 2.5A-C illustrate examples where groundwater contributes to flows in the channel, groundwater level is below the channel and no contributions to the channel are possible and a situation where both gradients are negative, respectively. The groundwater storage compartment is presented as a groundwater wedge. The volume of water in the groundwater wedge is determined based on drainage width and length, hydraulic gradient and storativity (see Hughes, 2004). The model formulation consists of adding the recharge to the volume of stored groundwater, re-calculating the hydraulic gradient, calculating the outflows and updating the volume of stored groundwater for the next time step (Hughes, 2004). The GW-PITMAN model can therefore be used for groundwater assessment since it can compute groundwater storages.

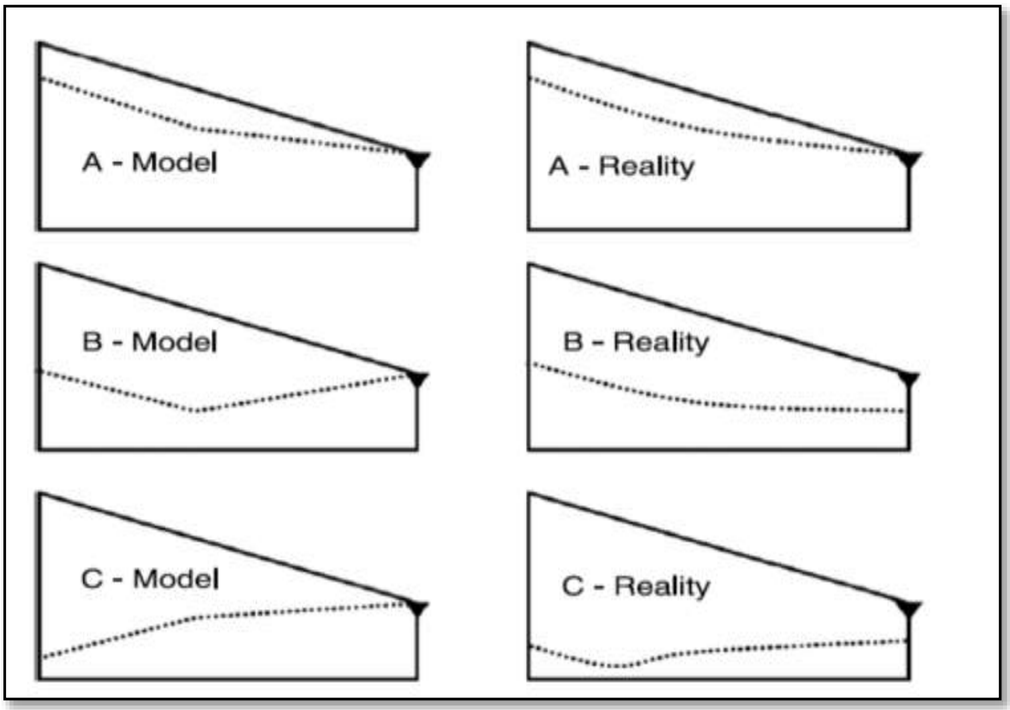


Figure 2.5: Modelled versus real groundwater conditions in a single hill slope element (Hughes *et al.*, 2010)

Legend: Dashed lines= groundwater levels, solid upper line=surface and solid triangle = river channel

2.6 Techniques for infilling missing rainfall time series data

Continuous and long term daily rainfall time series is one of the most used data in hydrological applications. However, most daily rainfall time series data are inadequate to perform reliable and meaningful analyses, and possess significant number of missing records (Hasan and Croke, 2013). This problem is more prevalent in developing countries than in developed countries (Ilunga and Stephenson, 2005). Studies such as Makhuvha *et al.* (1997b), Elshorbagy *et al.* (2000) and Simolo *et al.* (2010), amongst others, have attributed the existence of data gaps to loss of yearbooks because of wars or fire accidents, effects of extreme natural phenomena such as hurricanes or landslides, limited financial resources, poor management of data related to water resources, occasional interruptions of automatic stations, equipment failure, temporary absence of observers, cessation of measurement, no reliable hydrological networks and network reorganizations. For example, in South Africa, the overwhelming majority of gaps are caused by the temporary absence of observers, the cessation of measurement or absence of observations prior to the commencement of measurement

(Makhuvha *et al.*, 1997a). Generally traditional weighting and data-driven methods are used for estimating rainfall data (Teegavarapu and Chandramouli, 2005; Di Piazza *et al.*, 2011). Applications of these methods in estimating missing rainfall data are briefly reviewed.

2.6.1 Weighting methods

Weighting methods can be classified into deterministic interpolation (normal ratio (NR), inverse distance weighting (IDW) and non-linear interpolation as spline techniques, amongst others) and statistic interpolation methods (different varieties of kriging) (Di Piazza *et al.*, 2015). NR and IDW methods are the most commonly used traditional weighting methods for estimation of missing climatic data (Suhaila *et al.*, 2008), because of their simplicity. One of the major limitations of the NR method is that by considering all the gauges in estimating the missing data, the method could fail to take into account the redundant information because some of the gauges might have been clustered together and also may bias the estimate of missing data (McCuen, 1998). One problem of the IDW method is the arbitrary selection of time series data from neighbouring stations (Di Piazza *et al.*, 2011). In addition, Teegavarapu and Chandramouli (2005) noted that in spite of the IDW's wide success and acceptability, it suffers from major conceptual limitations, which are the arbitrariness in the choice of weighting parameter and the definition of the neighbourhood.

Teegavarapu and Chandramouli (2005) addressed some of the limitations of IDW by incorporating several conceptual improvements to the traditional inverse distance weighting method to estimate missing precipitation data in state of Kentucky, USA. The latter study also used artificial neural networks (ANNs) and Kriging approaches to illustrate the advantages of deterministic, stochastic data-driven and interpolation methods compared to traditional distance-based weighting methods in estimating the missing values. Suhaila *et al.* (2008) proposed a hybrid of modified NR, IDW and coefficient of correlation weighting methods for estimating daily missing rainfall values in Malaysia. The results of the latter study indicated that the performance of the modified methods improved estimation of missing rainfall values at the target station. De Silva (2007) compared arithmetic mean, NR and IDW and aerial precipitation ratio methods for estimating missing rainfall data at seven major Agro-ecological zones of Sri Lanka. The study concluded that IDW method was the most suitable method.

Bazgeer *et al.* (2012) compared performance of IDW averaging; regularized and tension splines, spherical, circular, exponential and Gaussian kriging for interpolating yearly precipitation in Fars Province, Iran. The study chose exponential kriging to estimate long term average precipitation because it had less errors. Kriging presents an important advantage in its ability to give unbiased predictions with minimum variance and to take into account the spatial correlation between the data recorded at different rain gauges or weather stations, its geostatistical framework is also able to accommodate secondary information in order to improve the interpolation results and it provides a measure of prediction error (kriging variance) (Ly *et al.*, 2013). Hofstra *et al.* (2008) compared global and local kriging, two versions of angular distance weighting, natural neighbour interpolation, regression, 2D and 3D thin plate splines, and conditional methods for interpolation of daily precipitation, mean, minimum and maximum temperature, and sea level pressure from station data over Europe. The study showed that no interpolation method stood out as superior to others by a large margin and several methods performed best when considering a specific criterion, climate variable or sub-domain. However, global kriging was found to be the best overall method by a small margin.

Teegavarapu (2012) used nonlinear and mixed integer nonlinear programming formulations along with binary variables for the estimation of missing precipitation data through a spatial interpolation technique at several stations in the state of Kentucky, USA. The proposed approach overcomes the limitation of spatial interpolation methods relevant to the arbitrary selection of weighting parameters, the number of control points within a neighbourhood, and the size of the neighbourhood itself. Parametric and nonparametric hypothesis tests indicate statistically insignificant differences in error and performance measures as compared to observed data.

2.6.2 Data-driven methods

Data-driven modelling is based on analysing the data about a system, in particular finding connections between the system state variables (input, internal and output variables) without explicit knowledge of the physical behaviour of the system (Solomatine *et al.*, 2008). A model can then be defined on the basis of connections between the system state variables (input, internal and output variables) with only a limited number of assumptions about the "physical" behaviour of the system (Solomatine and Ostfeld, 2008). When data are insufficient and accurate prediction is more important than conceiving the physics of a problem, black box models could be a good option (Nourani and Mano, 2007). Data-driven methods include regression, ANNs and time series analysis (Teegavarapu and Chandramouli, 2005; Di Piazza *et al.*, 2015).

Regression based methods have widely been applied in estimation of missing rainfall data. Simple linear (Daly *et al.*, 2002; Mott *et al.*, 1994; Terzi, 2012) and multiple linear (Makhuvha *et al.*, 1997b, Pegram, 1997; Terzi, 2012) regression methods are the most commonly used. Villazón *et al.* (2010) applied linear and multiple linear regression techniques for estimation of monthly precipitation in part of the Pirai River Basin located in Santa Cruz-Bolivia. Multiple linear regression technique gave a 36% reduction in the standard deviation and root mean squared error when compared to linear regression.

One of the major limitations of regression methods is the necessity to define the functional form of the relationships *a priori* (Teegavarapu and Chandramouli, 2005). In situations where the structure of the data is complex, it may be very difficult to define a function that may correctly model the relationship between two variables. Non-pararametric regression (NPR) becomes useful in solving such problems. Non-parametric method is suitable for analysis of multimodal distribution, which perhaps reflects more accurately the naturally occurring complex hydrological cycle (Adamowski, 1987). The basic idea of non-parametric approaches is to let the data determine the most suitable form of the functions (Wu and Zhang, 2006). This overcomes the limitations of other regression methods. NPR analysis traces the dependence of a response variable on one or several predictors without specifying in advance the function that relates the predictors to the response (Fox, 2002). Non-parametric function estimation refers to methods that strive to approximate a target function locally, i.e., using data from a "small" neighbourhood of the point of estimate (Lall, 1995).

A non-parametric approach based on KNN estimator has been applied in spatial interpolation of rainfall data (Ali, 1998), estimation of seasonal precipitation by disaggregating water year precipitation for 29 climate divisions in the Colorado River Basin (Kalra and Ahmad, 2011), simulation of daily rainfall spells and amounts in Sydney, Australia (Sharma and Lall, 1999), development of nonparametric seasonal wet/dry spell stochastic model for resampling daily precipitation (Lall *et al.*, 1996), stochastic generation of rainfall data (Rajagopalan and Lall, 1999, Harrold, 2002, Harrold *et al.*, 2003, Srikanthan *et al.*, 2009, amongst others). KNN has some weaknesses when the data have outliers or when a nonlinear trend exists around the missing data, due to its fundamental assumption to follow a normal distribution, which is statistically unsound (Lee and Kang, 2015). Lee and Kang (2015) compared KNN approach with five different kernel functions (epanechnikov, quartic, triweight, tricube, and cosine) to estimate missing precipitation data. The latter study showed that the kernel approaches provided higher quality interpolation of precipitation data compared to that of KNN.

Ilunga (2010) used standard backpropagation (BP) and generalised BP FNNs to infill annual total rainfall data in Orange River System, Western Cape and Eastern Cape, South Africa. The generalized BP generally performed slightly better than the standard BP technique. Nkuna and Odiyo (2011) used radial basis function (RBF) neural networks to infill missing rainfall data in Luvuvhu River Catchment, SA. RBF neural networks were found to be capable of learning complex relations using available data. Ahmad and Al-khazelah (2008) proposed a method to estimate missing rainfall data by using the filtering process based on Box-Jenkins' autoregressive integrated moving average (ARIMA) modelling technique. The best ARIMA model was used to predict monthly average rainfall for Pinang station. The results of this model using both datasets with and without missing data were compared using Naive test and comparable values were obtained. Fung (2006) found that ARIMA interpolation was the most suitable method for estimating a missing value where there is sufficient data to obtain a reliable model when compared to polynomial curve fitting, cubic spline and state space modelling. Generated time series data sets were used in the study to compare performance of these models in estimating missing data. The applicability of time series analysis methods for estimation is dependent on good correlation with past values (Daniels, 2014). Thus, ARIMA models are especially suited to short-term forecasting because most of them place heavy emphasis on the recent past rather than the distant past (Ahmad and Al-khazelah, 2008). In addition, empirical time series models such as ARIMA are not adequate when the dynamic behaviour of the hydrological system changes with time (Shiri *et al.*, 2013).

Aslan *et al.* (2010) compared performances of MI based on single arithmetic average, NR, NR weighted with correlations, multi-layer perceptron neural network and expectation-maximisation algorithm based on Monte Carlo Markov Chain (EMMCMC) methods in infilling artificially created missing values in two series of precipitation data. EMMCMC algorithm performs better than the others with respect to the normalised root mean squared error criterion.

2.7 Extension of groundwater levels time series data

Groundwater levels from observation wells provide a principal source of information regarding the hydrological stresses acting over aquifers and how those stresses influence groundwater recharge, storage and discharge (Sujay and Paresh, 2015). Groundwater levels are required for groundwater resource assessment to ensure its sustainable utilisation. However, most boreholes that are drilled in most developing countries are typically production boreholes aimed at domestic water supply. This in addition to poor groundwater monitoring networks and discontinuous monitoring or measurement of groundwater levels results in lack of continuous long term groundwater levels time series data. This creates the need to estimate and extend limited groundwater levels data.

Data required to quantify aquifer parameters are rarely available and expensive to acquire in most developing countries. In addition, approximations, assumptions and simplifications that are made in physically-based models result in errors and uncertainty in the outputs. Fitting a physical model is not possible when there is no sufficient data, and the accuracy of the numerical model to a great extent depends on how accurate the model inputs are (Sun *et al.*, 2015). To overcome the problems associated with physically-based models, data-driven methods such as ANNs, SVMs and time series analysis are used in simulating groundwater levels. These methods are easier to apply as compared to physically-based models. Fallah-Mehdipour *et al.* (2013) noted that application of simple tools to predict future groundwater levels and fill-in gaps in data sets are important issues in groundwater hydrology.

Shiri *et al.* (2013) compared performance of Gene Expression Programming (GEP), ANFIS, ANN, ARMA and SVM techniques for groundwater levels forecasting up to 7-day prediction intervals. The study found that all models performed better than the ARMA model. Venkatesan and Rajesh (2015) evaluated performance of ANN and MODFLOW in simulating groundwater levels in Sindapalli Uppodai, a sub-basin of Vaippar River basin in India. ANN model performed similarly to MODFLOW for short-horizon predictions.

System identification models or time series analysis can also overcome some of the limitations of physically-based models in cases where data on aquifer parameters, which is required for estimating groundwater levels, are not available. System identification is the art and science of building mathematical models of dynamic systems from observed input-output data (Ljung, 2010). In system identification, the groundwater system is seen as a black box that transforms a series of observations of the input or explanatory variables into a series of output variables or groundwater levels (von Asmuth and Knotters, 2004). Using a time series model, it is possible to simulate periods without observations, as long as data on explanatory variables are available (Manzione *et al.,* 2009). Different model classes that are used in system identification include linear, non-linear, hybrid, discrete, continuous, non-parametric, amongst others.

Houston (1983) explored the use of time series techniques in groundwater systems and concluded that they are applicable for forecasting and control/management of such systems. Bidwell (2005) described an approach that matches the stochastic difference equation models of time-series analysis to the physically-based, linear system, groundwater model. von Asmuth and Knotters (2004) used a method based on continuous time transfer function (TFN), which estimates the impulse response function of the system from the temporal correlation between time series of groundwater level and precipitation surplus, to describe groundwater dynamics. Bierkens *et al*. (2010) modelled the spatio-temporal variation of shallow water table depth using a regionalised version of an autoregressive exogenous (ARX) time series. von Asmuth (2012) developed Menyanthes software in which groundwater levels time series can be modelled using both the ARMA and predefined impulse response function in continuous time (PIRFICT) methods. Izady *et al.* (2013) assessed the performance of neural network-autoregressive extra input (NN-ARX) model in predicting groundwater levels of Neishaboor plain, Iran, and compared it to static neural network (SNN). ARX model is a system identification model that has been widely used in control theory for modelling various control processes. NN-ARX model's performance was significantly better than that of SNN.

2.8 Determining type of aquifer and flow behaviour in fractured aquifers from pumping tests

Aquifer test interpretation can be considered as an exercise involving identification of parameters of a system from its response to a known disturbance (pumping) (Milne-Home, 1988). An appropriate way to investigate the hydrodynamic behaviour of a fractured aquifer is to determine the flow dimension and aquifer parameters simultaneously (Chang *et al.*, 2011). In addition, all well test analysis methods require that the geometry of the system be specified (by specifying the flow dimension), and then estimates of the hydraulic properties for that given geometry can be made (Beauheim *et al.*, 2004).

Curve matching which involves fitting theoretical type curves (models) to observed drawdown data, is used for analysis and interpretation of pumping test data. The model that compares best with the real system is then selected for the calculation of hydraulic characteristics (Kruseman and de Ridder, 2000). To circumvent the difficulty in choosing an appropriate conceptual model (type curve), diagnostic plots have been used to interpret pumping test data in basement crystalline aquifers. Diagnostic plots and derivatives facilitate the selection of appropriate models for analysing pumping test (drawdown) data, and estimating appropriate hydraulic properties of the aquifer (Hammond and Field, 2014). A diagnostic plot is a plot of the drawdown and its logarithmic derivative as a function of time (Renard *et al.*, 2009). Derivative analysis of pumping test data relates the rate of drawdown change as a function of the natural logarithm of time (Hammond and Field, 2014). The most useful diagnostic plots (Baumle, 2003; Holland, 2011) include:

- drawdown (*s)* versus time (*t)* in a log-log plot (log *s* vs. log *t*)
- drawdown versus the logarithm of time (semi-log plot: *s* vs. log *t*))
- drawdown versus the square root of time (*s* vs. $t^{1/2}$)
- drawdown versus the fourth root of time (*s vs.* $t^{1/4}$)
- time derivative of the drawdown versus the time in a log-log plot.

The standard diagnostic plot for a constant-rate test is a log-log plot of elapsed time on the *x*-axis versus the pressure change and derivative of pressure change with respect to the natural log (*ln*) of time (or superposition time) on the *y*-axis (Ehlig-Economides *et al.*, 1988). However, Kruseman and de Ridder (2000) recommended that both semi-log and log-log plots of *s vs t* should be used since a semi-log plot of *s vs t* has shown more diagnostic value than a log-log plot in a number of cases. Figure 2.6 shows most typical features that are observable in semi-log and log-log diagnostic plots.

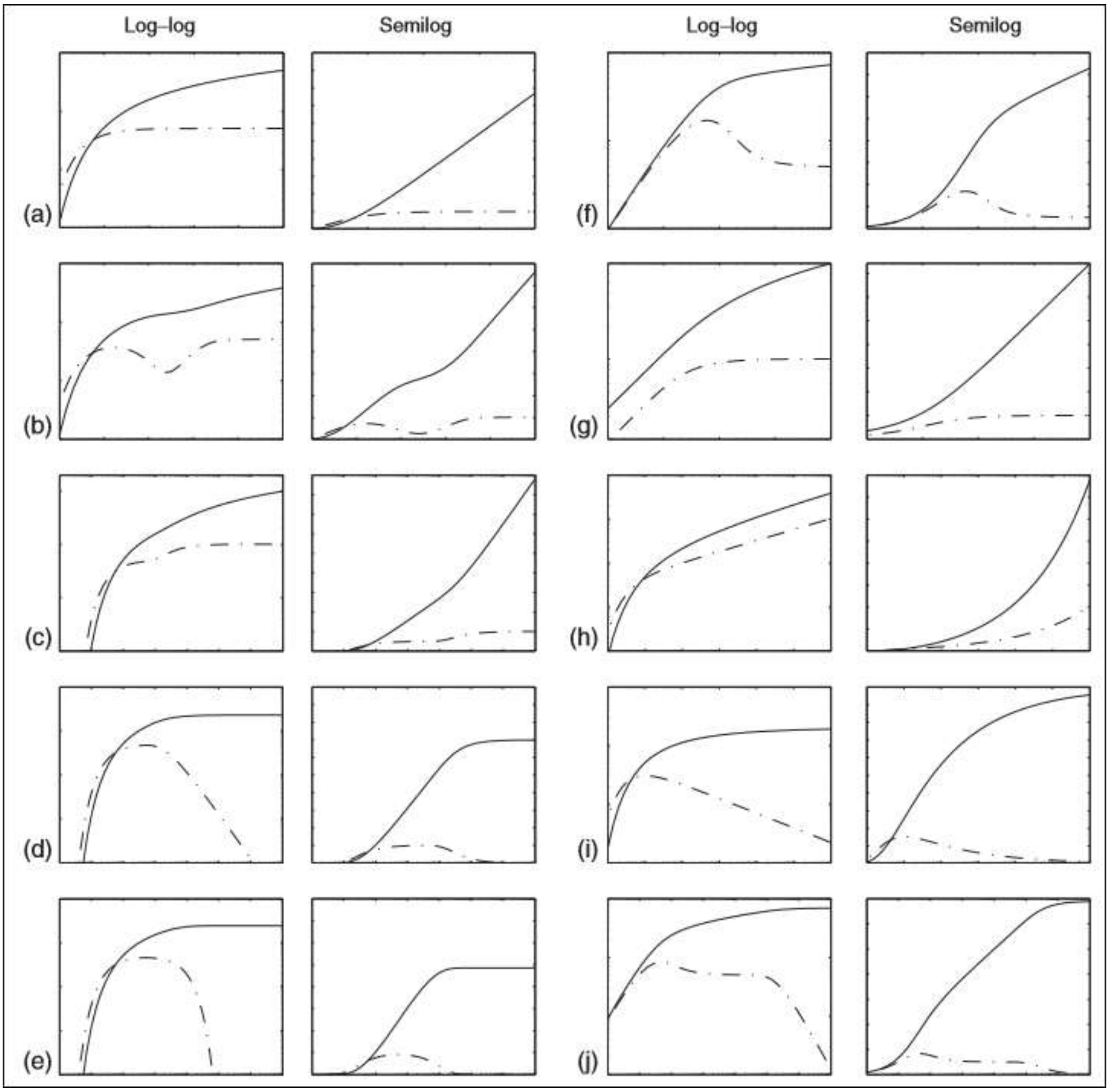


Figure 2.6: The typical diagnostic plots used in hydrogeology (Renard *et al.*, 2009)

Legend: (a) Theis model: infinite two-dimensional confined aquifer, (b) double porosity or unconfined aquifer, (c) infinite linear no-flow boundary, (d) infinite linear constant head boundary, (e) leaky aquifer; (f) well-bore storage and skin effect, (g) infinite conductivity vertical fracture, (h) general radial flow—non-integer flow dimension smaller than 2, (i) general radial flow model—non-integer flow dimension larger than 2 (j) combined effect of well bore storage and infinite linear constant head boundary

Flow dimension of a hydraulic test may reflect several characteristics of the hydrogeologic system, including heterogeneity, boundaries, and leakage (Walker and Roberts, 2003). Once the flow regime has been identified and the appropriate analytical solution chosen then simple curve matching and calculations can be carried out to determine values of transmissivity and storativity (Scottish Environment Protection Agency, 2013). A brief review of models that can be used for curve match fitting in fractured aquifers is given as follows:

2.8.1 Double porosity model

The concept of a double porosity model, as developed by Barenblatt *et al.* (1960), regards a fractured rock formation consisting of two media (matrix blocks and fractures) having different characteristic properties (Kruseman and de Ridder, 2000). In a double porosity aquifer (Figure 2.7), matrix blocks have low permeability, high (primary) porosity and high storage capacity while the fractures have high permeability and low storage capacity. As a result, the permeability and porosity of the entire formation are represented by that of the fracture network and porous block, respectively (Braester, 2003). The fractures produce flow directly into the well and matrix blocks act as a source, which feeds water into the fractures (Holland, 2011).

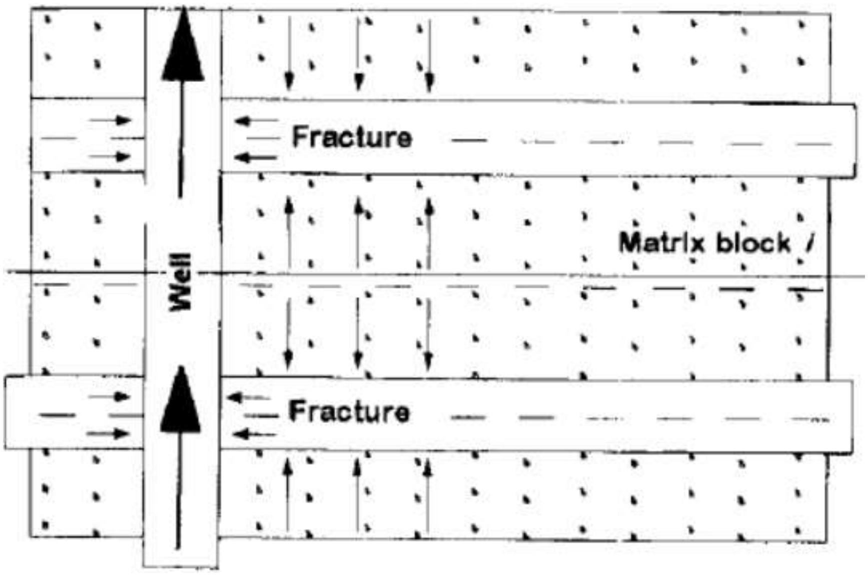


Figure 2.7: Double porosity aquifer (Gernand and Heidtman, 1997)

Geologically, the porosity of the matrix block is due to original intergranular pore space of the rock while secondary porosity is due to fractures associated with earth movements or solution channels (Milne-Home, 1988). Kruseman and de Ridder (2000) categorised flow characteristics of double porosity aquifer into three time periods, which are:

- Early pumping time, when all the flow comes from storage in the fractures;
- Medium pumping time, a transition period during which the matrix blocks feed their water at an increasing rate to the fractures, resulting in a (partly) stabilising drawdown;
- Late pumping time, when the pumped water comes from storage in both the fractures and the matrix blocks.

Renard *et al.* (2009) explained that early pumping depletes the first reservoir (fractures, for example), which is then partly compensated by a delayed flux provided by a second compartment of the aquifer (second/intermediate stage) and equilibrium is reached at the late time (last stage). During the intermediate stage, drawdown stabilises and the derivative shows a pronounced dip (Renard *et al.*, 2009).

2.8.2 Single fracture model

Gringarten and Witherspoon (1972) developed a model that relies on early time data to determine whether a vertical (Figure 2.8) or horizontal (Figure 2.9) fracture is intersecting a well. Thus, the model assumes that the pumped well is either intersected by a vertical or a horizontal single fracture. In a single fracture model, water flows along the fracture with higher permeability than that of the rock, and the fracture defines the flow pattern (Karay, 2013). The flow is one dimensional (*i.e.* it is horizontal, parallel, and perpendicular to the fracture) at early pumping times but it changes to pseudo-radial flow at late pumping times (Kruseman and de Ridder, 2000). The time required for pseudo-radial flow may be excessively long (Griffioen and Kruseman, 2004). The fracture is characterised based on drawdown from production well which typically plots a straight line on a log-log scale at early time merging with a Theis curve if the test is sufficiently long (Gernand and Heidtman, 1997). The main types of single vertical fracture models are infinite conductivity, uniform flux, finite conductivity fracture and dyke, though the dyke model is the only one developed for groundwater purposes (Verweiji and Barker, 1999). The rest have been developed for petroleum reservoirs.

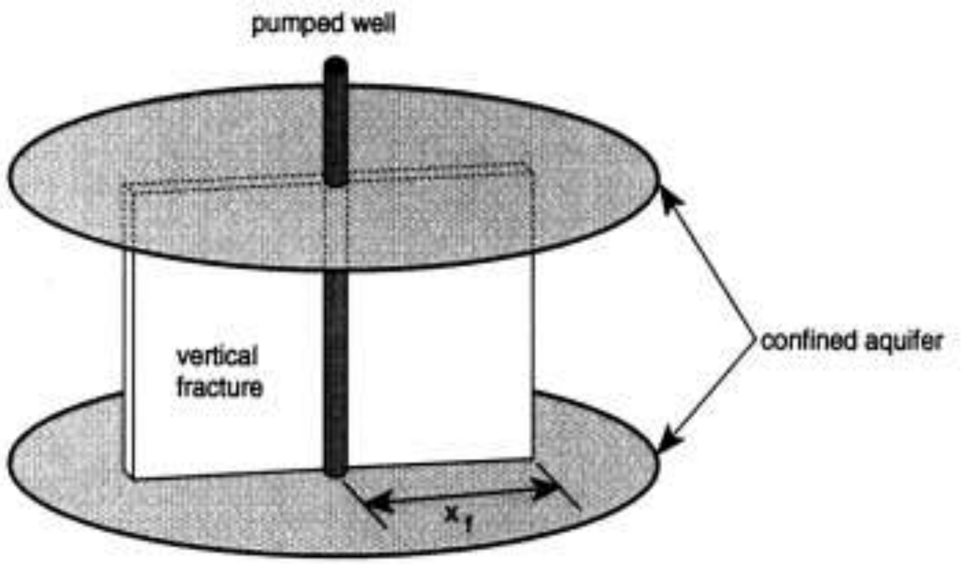


Figure 2.8: Single vertical fracture intersecting a well (Kruseman and de Ridder, 2000)
X_f is the vertical fracture half-length

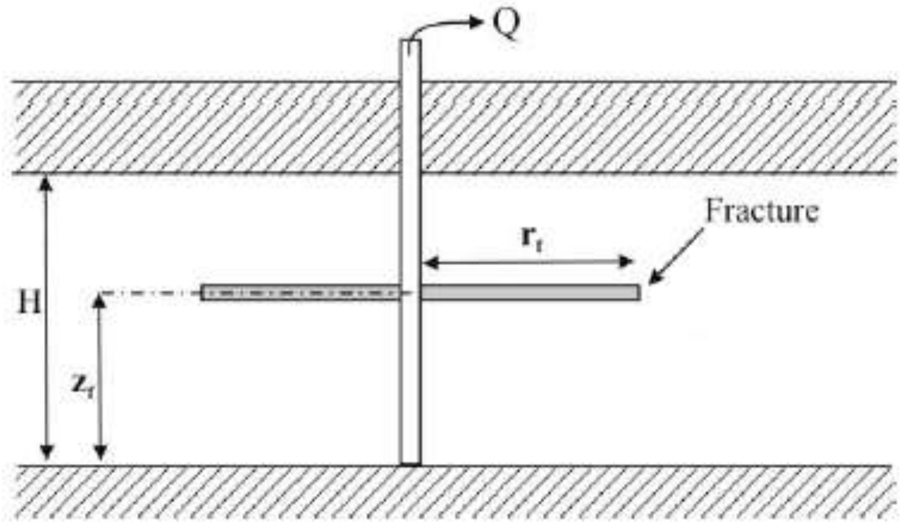


Figure 2.9: Single horizontal fracture intersecting a well (Maréchal, 2003)
Q is discharge from pumping well, r_f the radius of the horizontal fracture, z_f is the distance between the fracture and the bottom of the aquifer, H the aquifer thickness

2.8.3 Generalised radial flow (GRF) model

A GRF model generalizes the flow dimension to non-integral values, while retaining the assumptions of radial flow and homogeneity. The model was developed by Barker (1988) for hydraulic tests in fractured aquifers and regards the dimension of the flow as a model parameter. The model assumes that flow is radial and n-dimensional fractured media is homogenous and isotropic, and is described by K_h and specific storage capacity (Kuusela-Lahtinen *et al.*, 2003). Darcy's law is also assumed to be valid throughout the system. The concept of generalised flow dimension is most easily understood as an extension of the basic flow system geometries considered in classical well-test analysis (Geier *et al.*, 1996).

GRF approach to hydraulic test interpretation uses the flow dimension to describe the change in flow area versus radial distance from the borehole (Walker and Roberts, 2003). In the GRF model, the partial differential equations describing the boundary condition at the well and the flow in a homogeneous aquifer are expressed in terms of hydraulic parameters, distance, and a fractional spatial dimension (Leveinen *et al.*, 1998). An interesting feature of the model is that the flow dimension is related to the late time evolution of drawdown curves (Le Borgne *et al.*, 2004). It also generalises the basic solutions of flow to a well to fractional flow dimensions which greatly increases the range of drawdown type curves that may be fitted to observed data (Odling *et al.*, 2013).

2.8.4 Homogenous porous model

Homogenous porous model applies to fractured rock aquifer that shows behaviour similar to that of a homogenous isotropic or anisotropic porous aquifer during all or part pumping test (Verweiji and Barker, 1999). A fractured rock aquifer with many connected fractures is often assumed to fit the continuum conceptualisation, at least on a regional scale, and an equivalent of a porous medium model can be adopted (Kraemer and Haitjema, 1989). A fractured aquifer will most likely fulfil this assumption if a dense network of uniform fractures intersects the rock (Baumle, 2003). In this case, the Theis (1935) and Cooper and Jacob (1946) methods can be applied. The aquifer is assumed to be infinite in lateral extent, fully confined (no recharge or leakage), two dimensional (large extension compared to its thickness), having a homogeneous transmissivity and storativity (Holland, 2011).

2.8.5 Leaky aquifer model

Leaky or semi-confined aquifers are geologic systems in which vertical fluxes through confining overlying and/or underlying layers are not negligible. A schematic presentation of a leaky aquifer is provided in Figure 2.10. Under leaky artesian conditions, the cone of depression developed by a pumping well is influenced by the vertical permeability of the confining bed in addition to the hydraulic properties and geohydrologic boundaries of the aquifer (Walton, 1960). Hantush and Jacob (1955) developed a solution for drawdown in a pumped aquifer that has an impermeable base and a leaky confining unit above. The solution assumes that the pumped aquifer is bounded on top by a low permeability aquitard beneath a more permeable aquifer containing a standing water table (Hunt, 2012). Moench (1985) combined the Hantush theory of leaky aquifers with large-diameter well theory to produce equations that can be used in the analysis of pumped-well and observation well data for stratified formations.

The analysis of the drawdown caused by a pumping test in a leaky aquifer allows the estimation of representative hydraulic parameters of both the aquifer being tested and the aquitard through which it is recharged (Trinchero *et al.*, 2008). During the early time of pumping, water comes from storage of the pumped aquifer and the leaky confining unit

(Halford and Kuniansky, 2002). The system reaches steady state when there is equilibrium between discharge and the leakage through the confining unit from the unstressed aquifer.

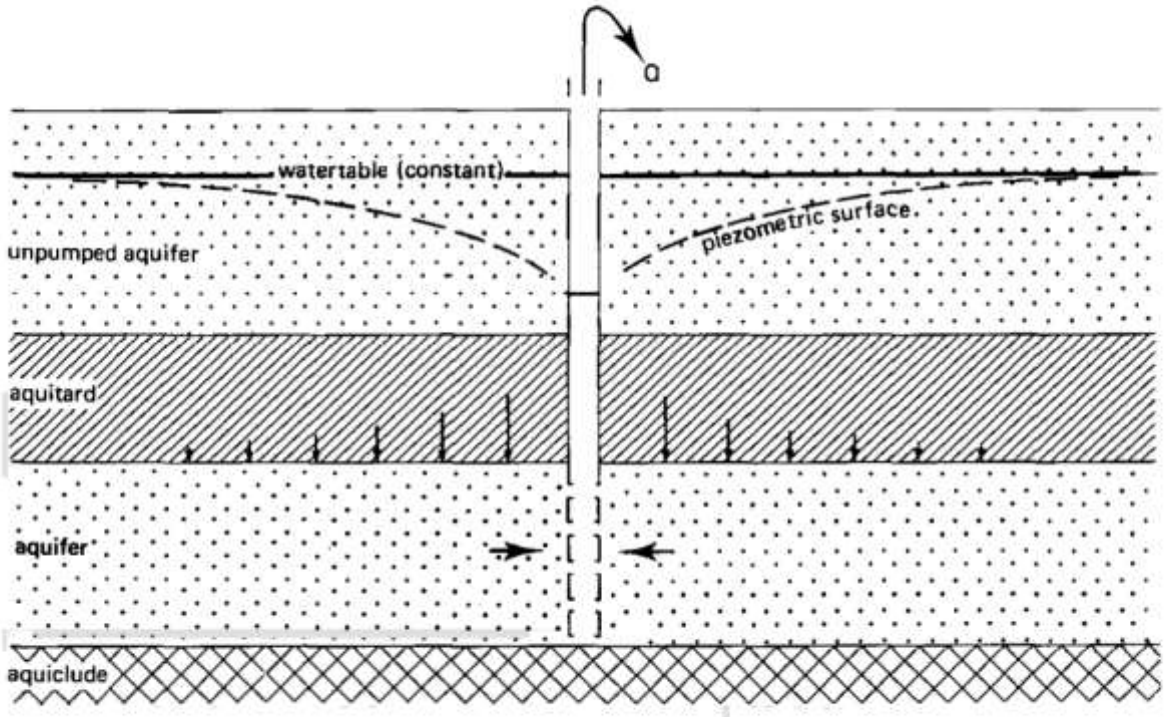


Figure 2.10: Schematic presentation of a pumped leaky aquifer (Kruseman and de Ridder, 2000)

2.8.6 Examples of studies in crystalline basement aquifers

Maréchal (2003) used Neuman (unconfined anisotropic aquifers), Gringarten (single horizontal fracture), Warren and Root (double porosity aquifer) and Barker (GRF) models to interpret pumping test data in hard crystalline rock terrain of South India. The methods allowed characterisation of the complexity of flows through fractures (Maréchal, 2003). Diagnostic plots (log *s* vs. log *t, s* vs. log *t* and derivative) were used to interpret pumping test data for boreholes located in crystalline rocks of Serre Massif, southern Italy in a study by Baiocchi *et al.* (2014). Theis, double porosity and leaky aquifer models were identified from comparison of diagnostic plots and theoretical models. Roques *et al.* (2014) used log–log plot of normalised drawdown and its derivative to interpret flow regime of crystalline rocks in

Saint-Brice en Coglès in Mancellian Domai, France. Double porosity and radial flow were identified during the 63 days pumping periods.

Nyende *et al.* (2016) used log *s* vs. log *t, s* vs. log *t* and derivative to describe flow behaviour in crystalline fractured rocks of Pallisa District within the Kyoga Basin, Uganda. Pumping test interpretation was uncertain while matching various times of the drawdown curve. The results indicated that while response at the pumping well may suggest linear single fracture flow, with availability of more data, later time showed that the aquifer responds to what would seem a radial flow (Nyende *et al.*, 2016).

Holland (2011) visually compared pumping test datasets from 2 359 boreholes to a set of typical diagnostic plots. The boreholes were from selected crystalline basement aquifers within the Limpopo Province, South Africa. Theis, double porosity, leaky aquifer, GRF models were identified. The results of the latter study showed that double porosity behaviour was displayed in 1082 boreholes within the study area. Logarithmic derivatives of the drawdown as a function of time were used to identify the flow regime, fracture intersections and boundary conditions in the crystalline basement aquifers of Namaqualand, South Africa, in a study by Pietersen *et al.* (2009). Double porosity behaviour of the aquifer became apparent in the derivative plot after initial borehole storage effects and radial flow regimes were observed at later times of the pumping test (Pietersen *et al.*, 2009).

2.9 Summary

Reviewed literature has provided essential basic concepts, background information and appropriate methods for the current study. Literature review established that it is essential to develop groundwater operating rules as they can aid in regulating competing water uses and ensure the beneficial use of water. Operating rules are also required to improve the assessment of water supply reliability. Procedures used for development of reservoir operating rules were also reviewed to aid in identifying those applicable to the current study. The review showed that in most groundwater studies, simulation models are coupled with optimisation models to derive groundwater management strategies/operating rules. It was also established that most of the reviewed studies have not incorporated stochastics and reliability of groundwater supply in their analyses thus, showing lack of information on the level of assurance of supply in most groundwater operating rules/strategies. Studies that have applied stochastics were mostly those that considered conjunctive use of surface and groundwater. This shows that a study focused on developing risk-based groundwater operating rules that incorporate reliability of water supply is of crucial importance. This literature therefore provided the basis and methods for achieving the main objective of the study, in addition to addressing the hypotheses of the study.

This review showed that most of the stochastic generators have been developed for multi-site and multi-variate generation of minimum and maximum temperature, and rainfall. The literature also showed that some of them have been adapted, improved and extended to meet specific needs. For example, Ndiritu and Nyaga (2014) extended VLB to generate both rainfall and streamflow while Efstratiadis *et al.* (2014) improved Castalia to generate multi-variate hydrometeorological variables (wind speed, sunshine duration, rainfall and streamflow). Review of stochastic generators aided in selecting a method to be used in this study to address specific objective on generating stochastic inputs for groundwater base yield-recurrence interval analysis and the research question how stochastic inputs for groundwater base yield-recurrence interval analysis are generated.

Literature on groundwater resource assessment was aimed at establishing methods that have been used to quantify the volume of groundwater that can be allocated for use, and their applicability in the current study. This assisted in addressing the main aim of the study since the volume of available groundwater resources is required when developing operating rules. The literature indicated that approaches for quantifying groundwater resources hinge around the water balance equation or some components of this equation and methods of solution to this equation vary from simple analytical to complex numerical approaches. The AAYM and AFYM were developed to aid in incorporating level of assurance of supply in groundwater assessment studies. However, the limitation of these models is that the assurance levels are based on methods that do not account for uncertainty due to natural climate variability. Thus, it is important to develop stochastic based operating rules which can account for uncertainty.

Literature review also established the procedure for delineating groundwater resource unit for application in this study. It also showed that a hydrogeological conceptual model can be developed and used to describe the inflows and outflows within a groundwater resource unit. Methods for determination of flow behaviour and hydraulic parameters in fractured aquifers were also identified. The review showed the importance of using diagnostic plots to determine aquifer type and flow behaviour in fractured aquifers. Examples of reviewed studies showed that double porosity aquifer type is mostly found in crystalline basement aquifers including those which are found in Limpopo Province. This literature supported the first specific objective and research question associated with delineation and characterisation of a groundwater resource unit for Siloam Village.

The review also showed physically based and data-driven techniques that are typically applied for estimating missing data as well as extending time series data. Data-driven methods can be used in infilling and extension of time series data in cases where physically based methods are not applicable due to model limitations or unavailability of data required by the model. For example, the arbitrariness in the choice of weighting parameters and definition of the neighbourhood are some of the limitations of IDW method for infilling missing rainfall while assumptions and simplifications that are made, and intense data requirements are limitations of physically-based groundwater models.

The review also noted that the choice of method/model will depend on data availability and nature of expected outputs. For example, Venkatesan and Rajesh (2015) reported that advantages of numerical models like MODFLOW, over black box models such as ANN include the fact that numerical models provide the total water balance of the system. Thus, if the study is focused on estimating total water balance it would be ideal to select a numerical model. However, in a case where the interest is on simulating groundwater levels in an area with limited data to describe the physical behaviour of the system, simpler techniques such as time series models can be selected for use. This is supported by Nourani and Mano (2007) who stated that when data is insufficient and accurate prediction is more important than conceiving the physics of a problem, black box models could be a good option, as reported in sub-section 2.6.2. Review of these methods aided in addressing second specific objective and research question related to infilling and/or extending data required for generating groundwater levels for the groundwater resource unit and stochastic analysis.

CHAPTER 3: CASE STUDY AREA AND DATA

3.1 Preamble

This chapter describes the basis of selection of gives an overview of the selected case study area including its characteristics and available data sets that are relevant for the study. Describing the characteristics of the study area aids in providing its environmental setting and the basis for its selection. Describing available data aids in identifying the suitable methods used for its analysis in order to achieve the main objective of the study. Description of the geology provided the background information required for developing the hydrogeological conceptual model for the study area.

3.2 Characteristics of case study area

Siloam Village, located within the Nzhelele River Catchment, is an example of a rural village that lacks adequate potable water supply for domestic use. The use of groundwater to supplement surface water and harvested rainwater supply is common in Siloam Village. In addition to one public borehole that exists in Siloam Village, some residents have drilled boreholes within their homesteads to supplement their water needs. Thus, there is an increased use of groundwater as a source of water supply. This formed the basis for selecting Siloam Village as a case study area.

3.2.1 Location of case study area

Siloam Village falls under quaternary catchment A80A of the Nzhelele River Catchment which is located in leeward side of Soutpansberg Mountain within the northern region of Limpopo Province, South Africa. The study area is found between 22°53'15.8'' S and 22°54'5'' S latitudes and 30°11'10.2'' E and 30°11'23.5'' E longitudes (Figure 3.1). It is located in Makhado Municipality in Vhembe District at 60 km North East of Makhado and approximately 45 km westward of Thohoyandou (Figure 3.2)

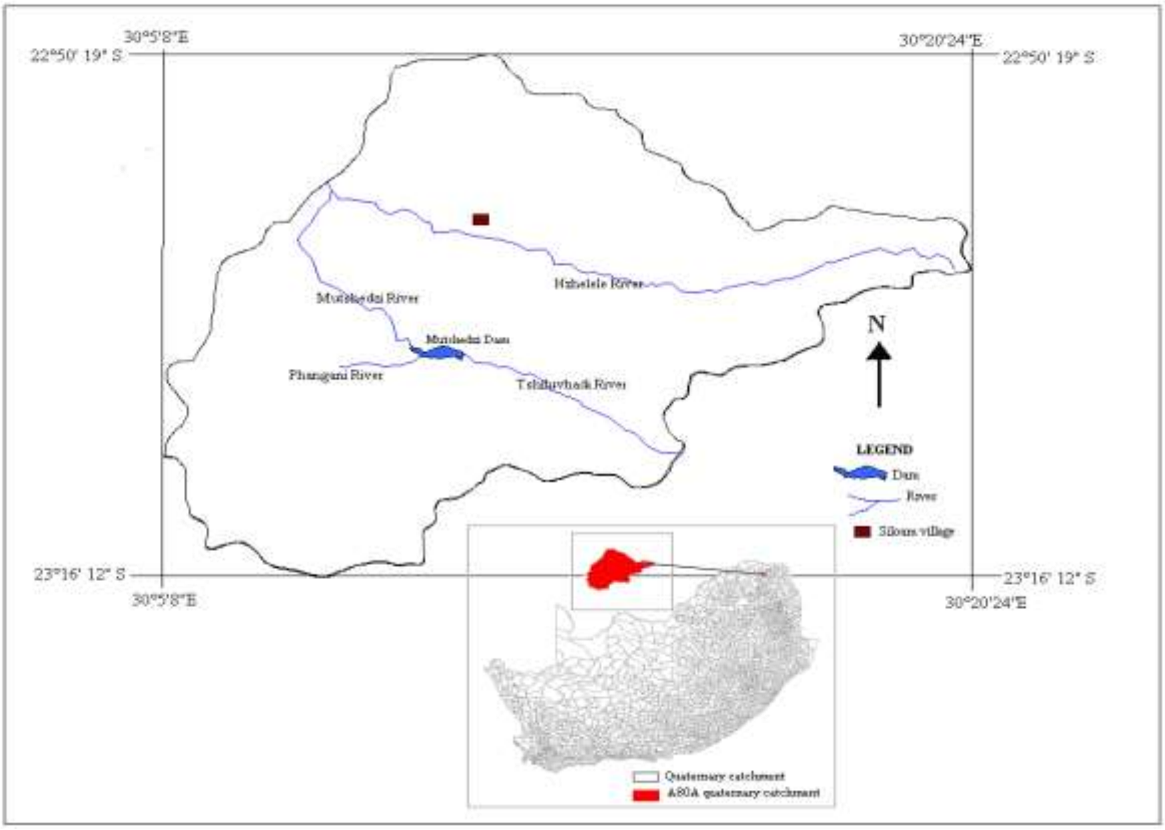


Figure 3.1: Location of Siloam Village in A80A quaternary catchment

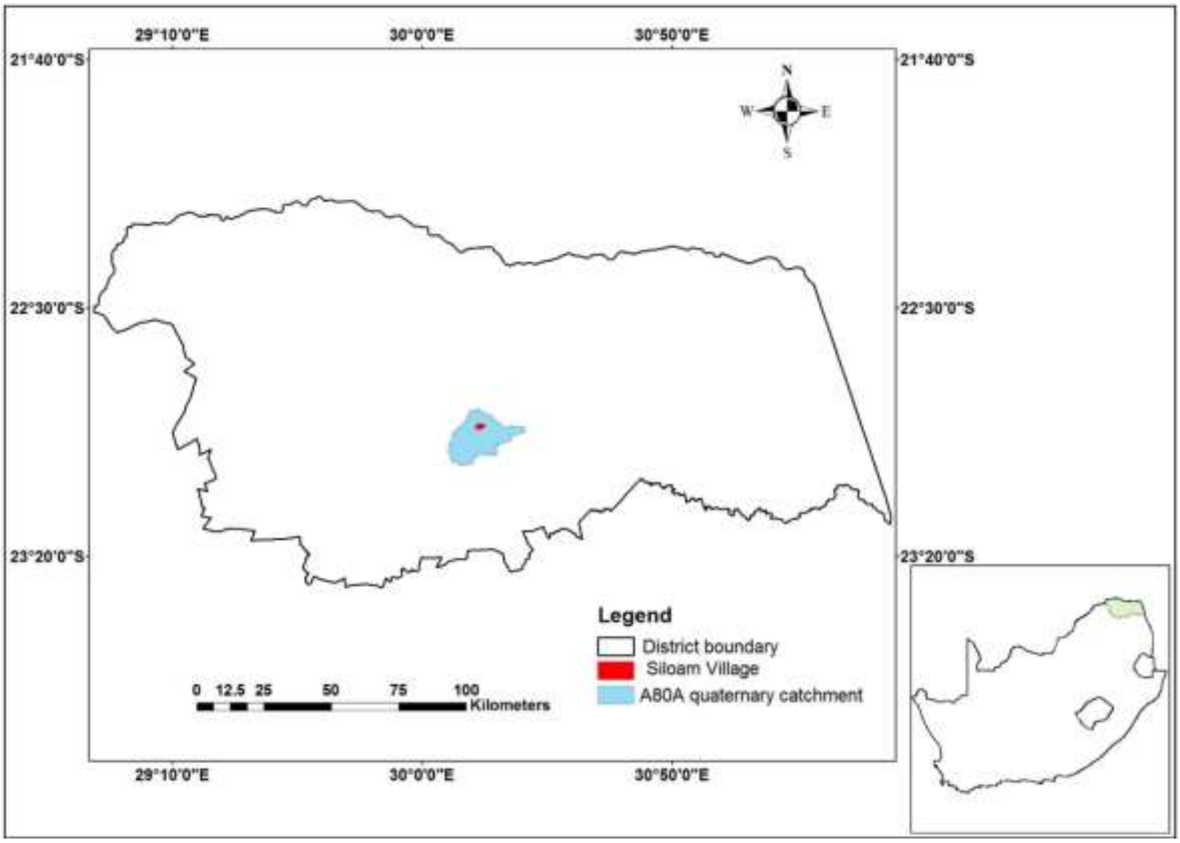


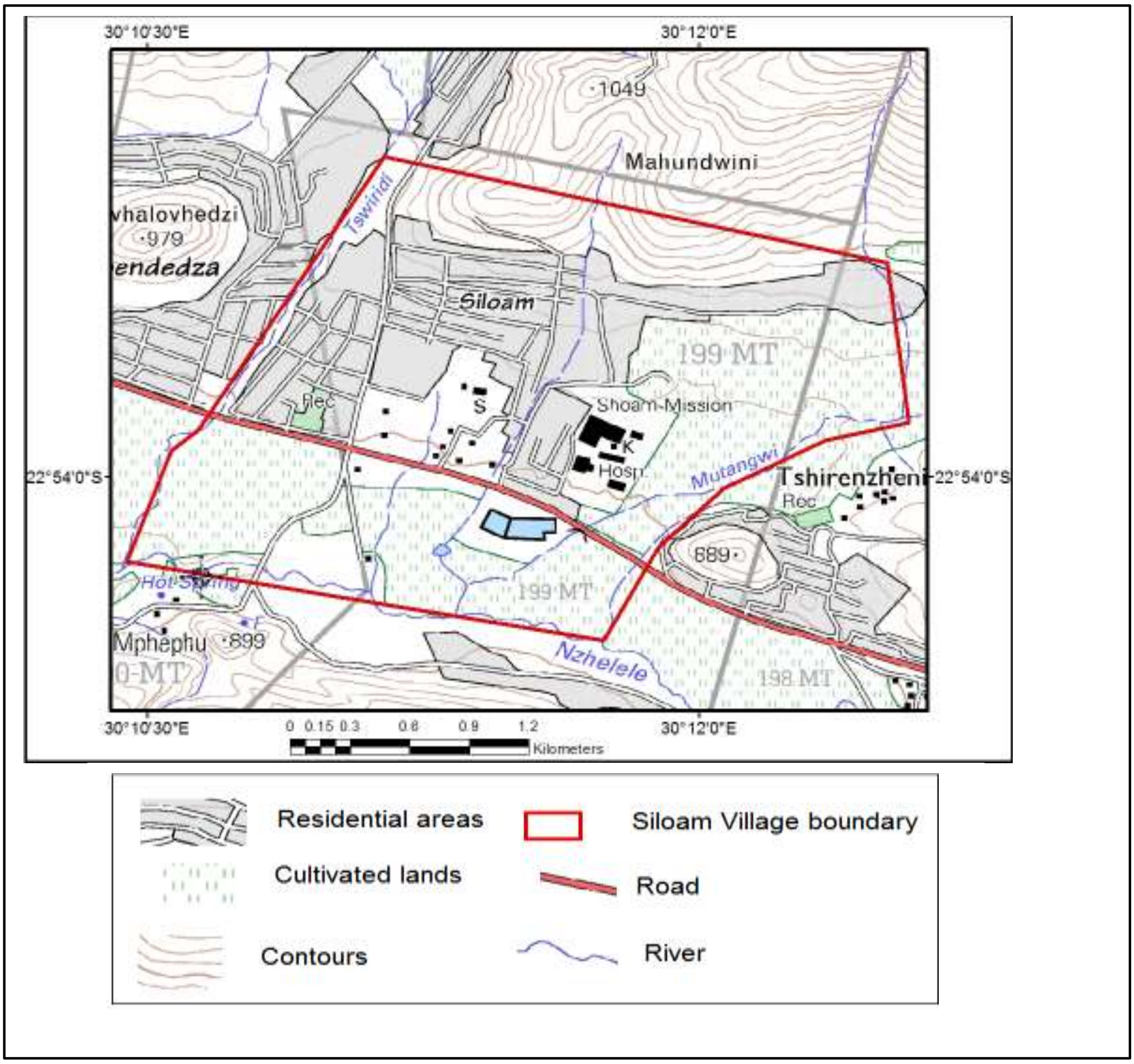
Figure 3.2: Location of Siloam Village within Vhembe District Municipality

3.2.2 Hydrology

The main river in case study area is Nzhelele River (Figure 3.1) which originate from the Soutpansberg Mountains and flows north to Limpopo River. The flow pattern of Nzhelele River is highly variable because of low and unpredictable rainfalls (average 350-400 mm/annum) in the catchment and the river is not perennial (Ashton *et al*., 2001), with no water for several months. The rainfall is seasonal and occurs during summer months from October–March. During dry years the river can remain without water for several months. Summer rainfalls cause dramatic increase in flows, though the tributary streams are mostly episodic and only contain water after rainfalls (Ashton *et al.*, 2001). The mean annual evaporation varies from 1300-1400 mm (Makungo *et al.*, 2010). Limited rainfall and high evaporation indicates the semi-arid nature of the study area which result in scarcity of surface water. Nzhelele area is characterized by high temperature variations with summer temperatures range from 22°C and 40° C while winter temperatures range from 16°C to 22°C.

3.2.3 Topography and soils

Siloam Village is found within the Nzhelele River valley with topography ranging from 800 to1100 m around its neighbourhood (Figure 3.3). The study area consists of medium loamy or coarse sand to sandy clay loam with clay content ranging from 4-40% (Institute of Soil, Water and Climate, 1994). The soil is highly susceptible to cracking. The weathering of igneous and sedimentary is the origin of the soil in the study area (Kabanda, 2004). The soil within Siloam Village have porosity and bulk density ranging from 57 to 58% and 0.8to 1.1 g/cm^3, respectively (Ndwambi, 2015). Reddish soil which covers the area is attributed to the results of weathering of iron-bearing basalt.



3.2.4 Land use and water supply

The study area is dominated by human settlements and subsistence agriculture (Figure 3.3). Mutshedzi Dam is the only dam in the quaternary catchment and it located on Mutshedzi River which is a tributary of Nzhelele River (Figure 3.1). Siloam is one of the villages supplied with water from Nzhelele River weir within the Nzhelele Regional Water Supply Scheme. Mutshedzi Dam also supply the village when there is no water supplied from the Nzhelele weir (Makungo, 2008). Water from Nzhelele weir and Mutshedzi Dam is pumped to the main line passing through other villages and stored in reservoirs. Due to increase in water demand and frequent drought conditions surface water from Nzhelele Regional Water Supply Scheme is inadequate to meet the demand. Due to this a number of residents have drilled boreholes within their homesteads to augment the surface water supply.

3.2.5 Geology

At a regional scale, Siloam Village is located within the younger cover of the Limpopo Mobile Belt (LMB). The LMB (Figure 3.4) of southern Africa is an extensive high-grade terrain that can be subdivided into three lithologically and structurally distinct zones, which are the northern, central and southern marginal zones. The LMB was formed as a result of a collision between the Kaapvaal craton (KC) and the Zimbabwe craton (ZC). The 250 km, ENE-WNW trending LMB is thought to represent a Himalayan-style collision event between the KC and ZC in the north (Bejaichund *et al.*, 2009). The oblique nature of this collision is believed to have initiated or re-activated major transcurrent fault systems, resulting in important structures such as the Thabazimbi-Murchison lineament, which prepared the craton for the development (2600-2100 million years ago) of the Transvaal and Griqualand West basins (Singh *et al.*, 2009).

Soutpansberg depositional basin was formed between two major crustal blocks, (e.g. the Kaapvaal craton in the south and the Limpopo Belt in the north) as an east-west trending asymmetrical rift or half-graben along the Palala Shear Belt (Brandl, 2003). Its rocks rest unconformably on gneisses of the Limpopo Belt and Bandelierkop Complex. Bumby *et al.* (2002) suggested that the Soutpansberg Group may have been related to a half-graben bound to the south by a northwards-dipping normal fault, perhaps associated with orogenic collapse of the Limpopo Belt. The major faults which trend East North East through the Soutpansberg region almost certainly represent reactivated basement fractures (Mason, 1973).

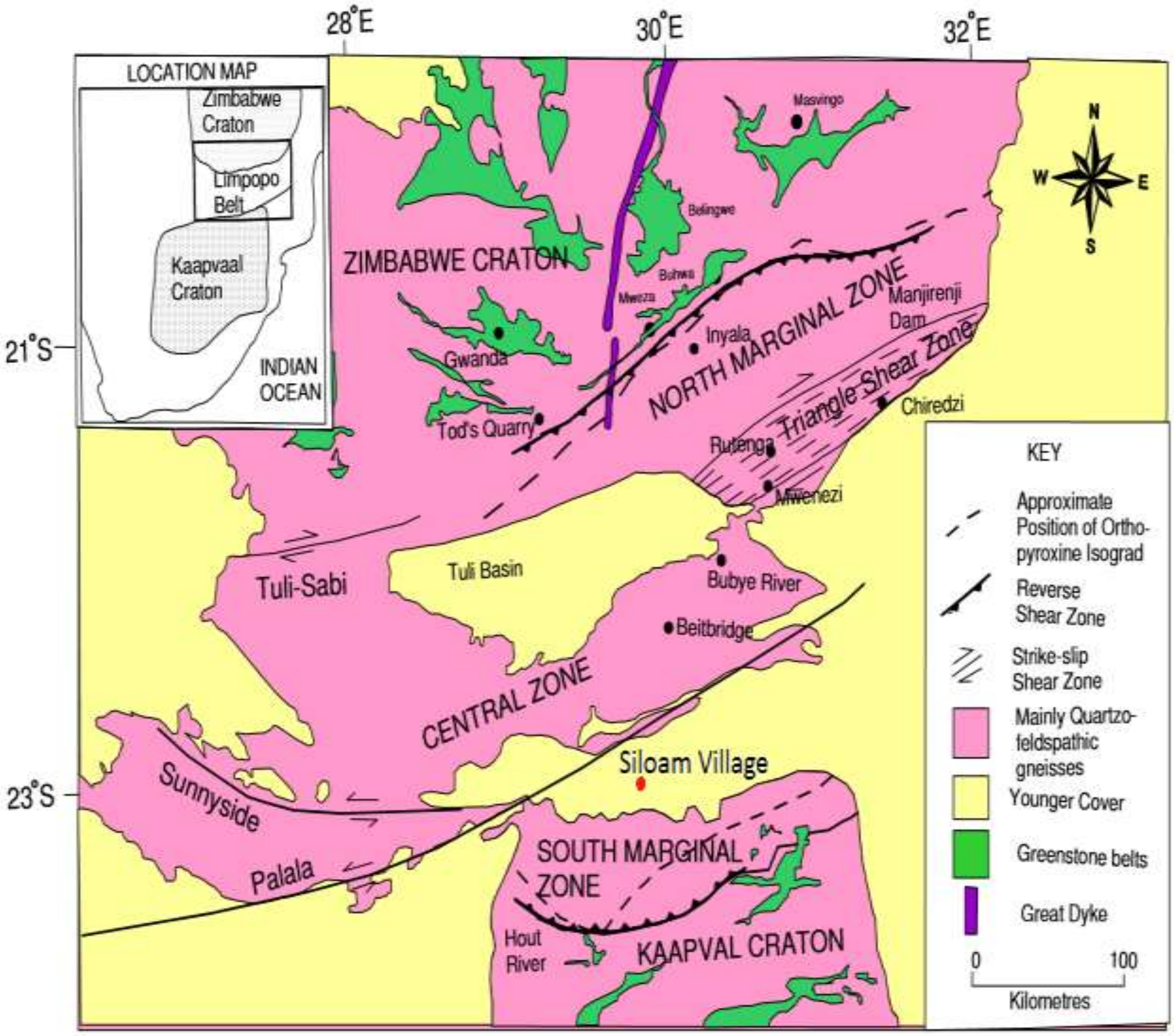


Figure 3.4: Simplified geological map of Limpopo Mobile Belt (Chinoda *et al.*, 2009)

At a local scale, the case study area is situated within the severely faulted Soutpansberg Group of the Mokolian age (Figure 3.5). The Soutpansberg Group forms part of crystalline basement aquifers of Limpopo Province in South Africa. Crystalline basement rocks are usually semi-confined (fractured bedrock) with water-table aquifers (the matrix-regolith) situated on top of them (Holland, 2011). Soutpansberg Group emerges as a large east-west trending mountain range (escarpment) from the Kruger National Park in the east to Vivo in the west. Dykes and sills of diabase are plentiful in the Soutpansberg Group (Brandl, 2003). It has 7 formations which are Tshifhefhe, Sibasa Basalt, Fundudzi, Nzhelele, Wylliespoort, Stayt and Mabiligwe (Figure 3.5). Sibasa Basalt, Fundudzi and Nzhelele formations are the ones that are present in the study area.

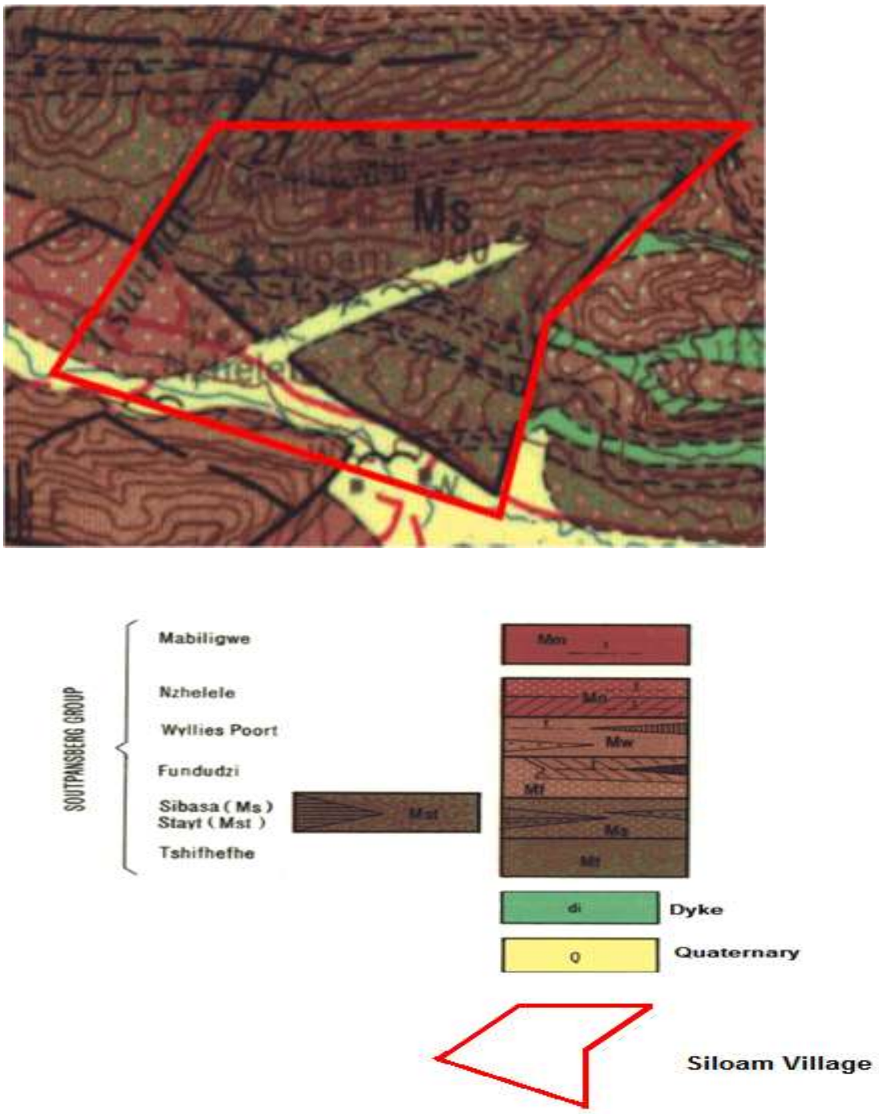


Figure 3.5: Local geology

Sibasa formation consists predominantly of lava (Figure 3.6) with minor intercalations of sedimentary and tuffaceous rocks (Brandl, 1981). The volcanic rocks comprise of repetitive sequence of erupted basalt (Barker *et al.*, 2006). Sedimentary rocks, which include shale, quartzite and conglomerate, generally tend to be more persistent along strike in the upper part of the succession (Brandl, 1981). Argilaceous rocks, interbedded with sandstone, represented by brownish or purple micaceous sandy shale, grey or dark-red shale and thinly laminated dark grey siltstones dominate the Fundudzi Formation (Brandl, 1981). The Nzhelele formation, which is the uppermost unit of the Soutpansberg Group (Figure 3.5), consists of a volcanic assemblage at the base followed by red argillaceous and arenaceous sediments together with several thin, though fairly consistent layers of pyroclasic rocks (Brandl, 1981; Barker *et al.*, 2006). Siloam fault trends from west-north-west to north-west and is estimated to have a vertical displacement of 1500 m. Large, fairly thick alluvial deposits are found along Nzhelele River (Brandl, 1981).

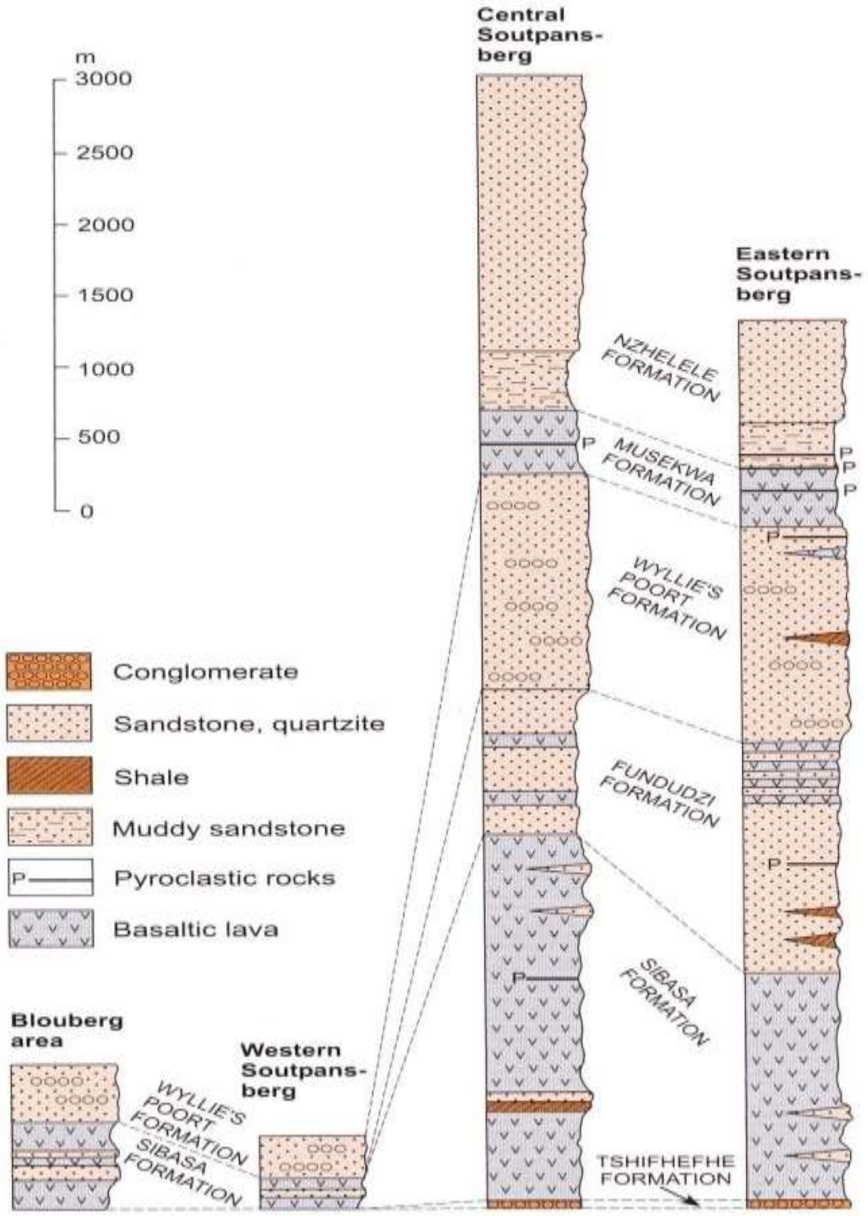


Figure 3.6: Stratigraphy of the Soutpansberg Group in the western, central and eastern Soutpansberg areas, and Blouberg area (Barker *et al.*, 2006)

3.3 Data used

Data used in this study included digital elevation map (DEM), pumping test, rainfall, evaporation and groundwater levels. Advanced Spaceborne Thermal Emission and Reflection

Radiometer Global Digital Elevation Map (ASTER GDEM) (Figure 3.7) covering the area of the study was obtained from USGS Land Processes Distributed Active Archive Center. ASTER GDEM is an easy-to-use, highly accurate DEM covering all the land on earth, and available to all users regardless of size or location of their target areas (Konecny, 2012). This was used in the delineation of the groundwater resource unit for the study area. The groundwater resource unit provided the basis for groundwater levels modelling based on the GW-PITMAN model.

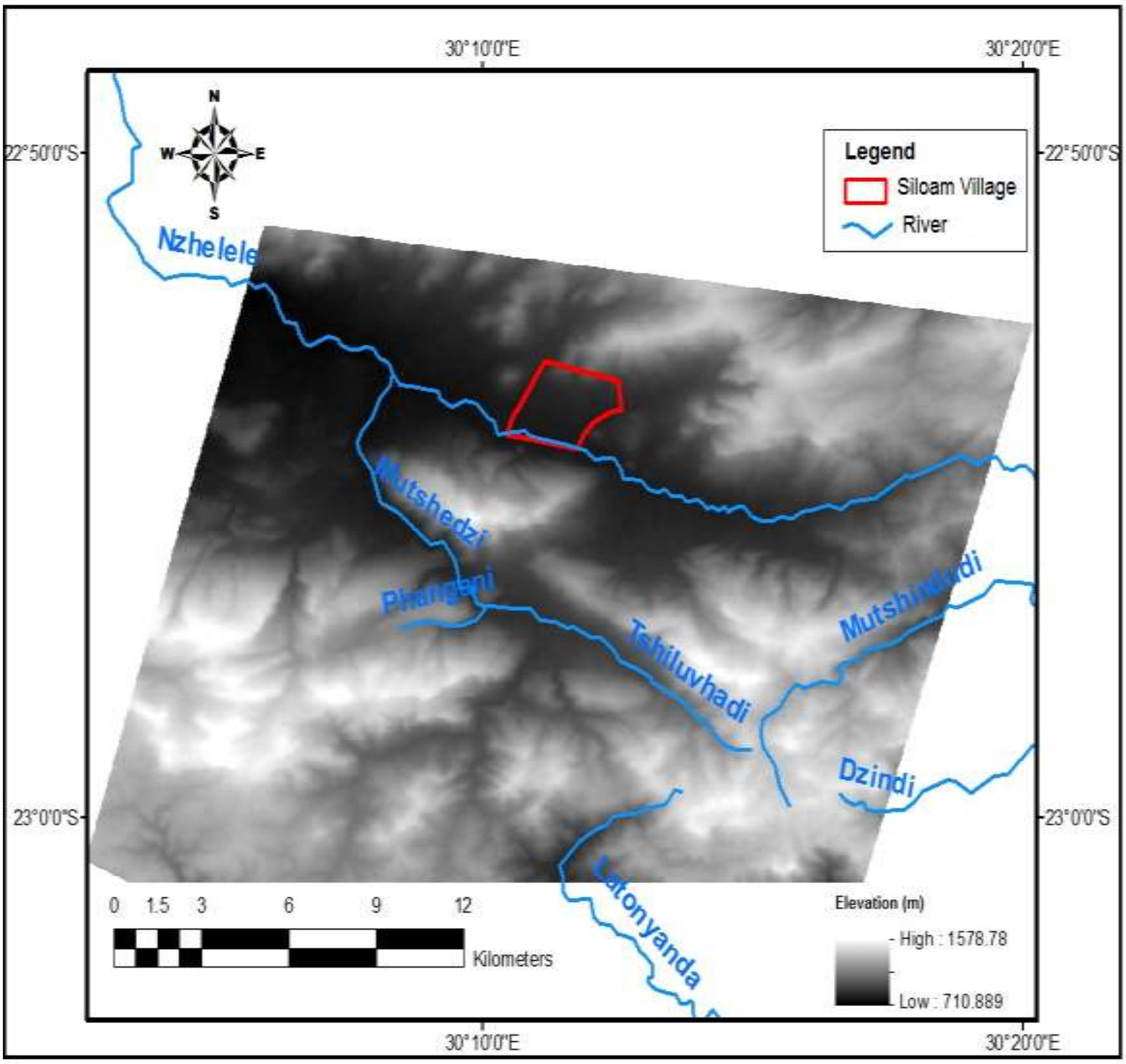


Figure 3.7: DEM covering the study area

Pumping test data for boreholes located within the study area (Figure 3.8) were obtained from VSA Leboa Consulting Pty Ltd. The data are for single-well tests that were conducted as part of the Limpopo Groundwater Resources Information Project (GRIP) aimed at determining sustainable abstraction rates for rural groundwater supply schemes. Test dates, constant pumping rates, test durations and groundwater levels in the boreholes before pumping for the pumping tests are provided in Table 3.1. Pumping test data were used to estimate aquifer storativity, transmissivity and hydraulic conductivity (K_h) based on the aquifer test solution. Storativity and transmissivity constituted inputs into GW-PITMAN model. Analysing pumping test data also aided in identifying the type of aquifer dominating the groundwater resource unit. This was useful in understanding the complex nature of the geologic environment where groundwater is stored as well as the limitations of the groundwater storage modelling approach used in the study area.

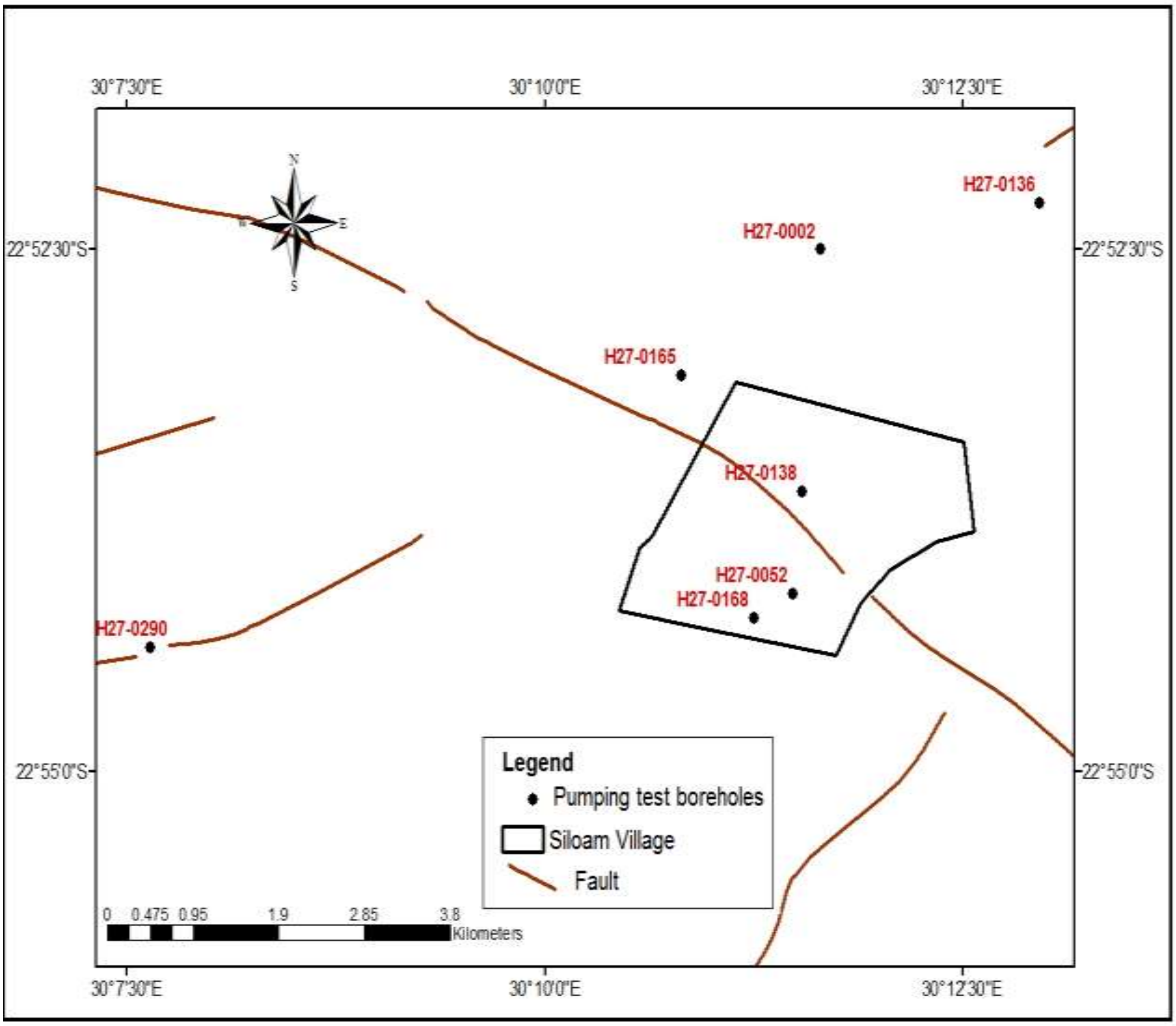


Figure 3.8: Pumping test boreholes locations

Table 3.1: Field measured characteristics of pumping test boreholes

Borehole	Test date	Constant pumping rate (l/s)	Test duration (min)	Groundwater level before pumping (m)
H27-0002	1998/02/01	2.26	1440	3.13
H27-0052	2003/02/15	1.02	630	6.37
H27-0136	2005/07/08	0.31	478	5.26
H27-0138	1998/02/12	2.01	1350	0.00
H27-0165	1998/02/04	3.04	1500	2.05
H27-0168	2005/05/06	2.52	1200	6.97
H27-0290	2005/05/10	5.06	2902	15.70

Locations available rainfall, evaporation and weather stations in A80A quaternary catchment are in Figure 3.9. Rainfall data for station 0766324 for the period 1903/10/01-2000/07/31 (Figure 3.10) were obtained from Lynch (2003). These were the only data available since the station was closed in the year 2000. Rainfall for the period of January 2012-December 2013 (Figure 3.11) was obtained from the University of Venda weather station installed at Siloam police station in Siloam Village. Thus, there was a gap in the data from August 2000 to December 2011, creating the need to infill it. Mutshedzi rainfall and evaporation station (A8E004), located in the same quaternary catchment (A80A) as the University of Venda weather station at Siloam Village (Figure 3.9), had rainfall data from 1991/07/01 to 2012/01/12 (Figure 3.12). Rainfall data from station A8E004 was used to infill data for station 0766324.

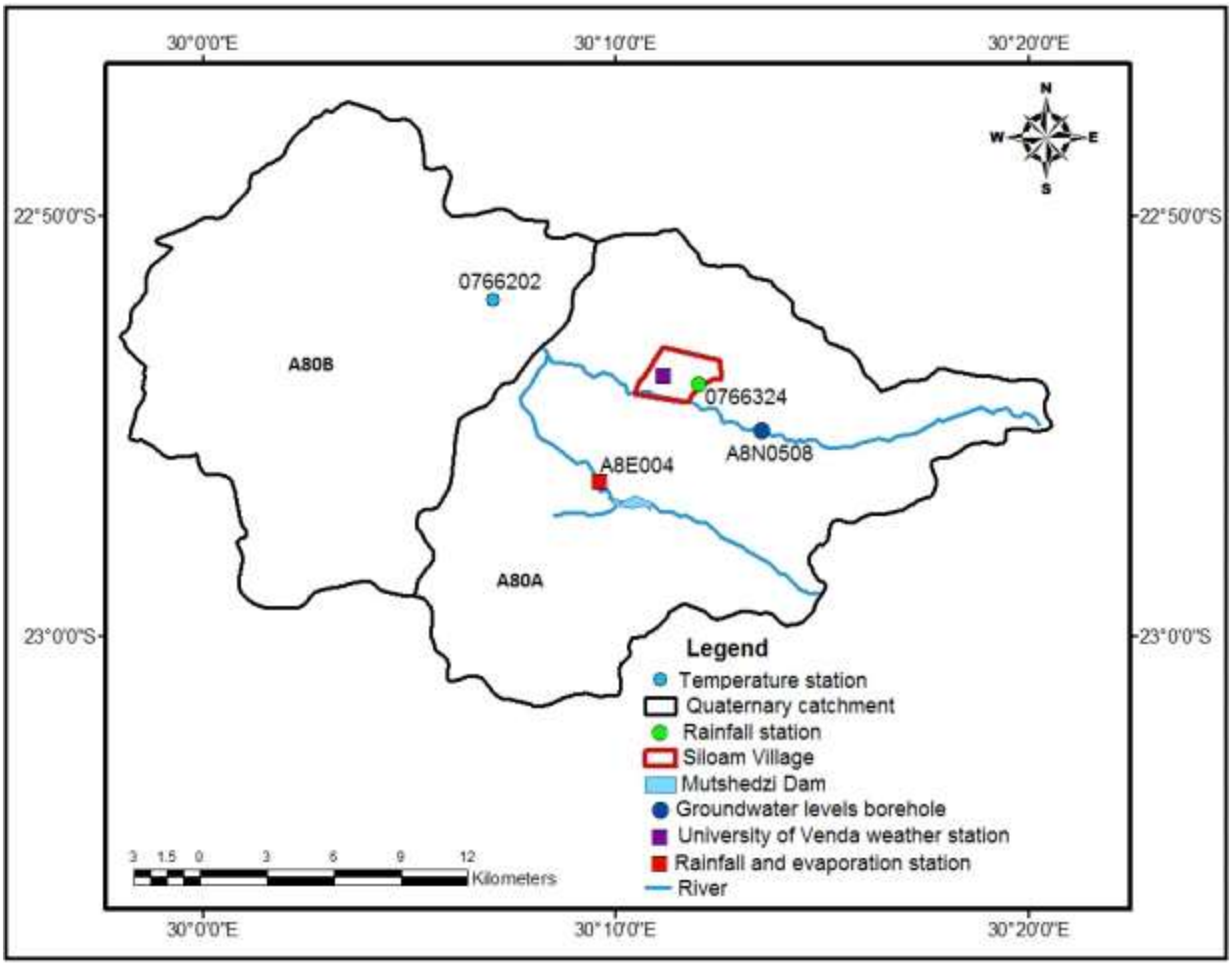


Figure 3.9: Location of rainfall, evaporation, temperature and weather stations

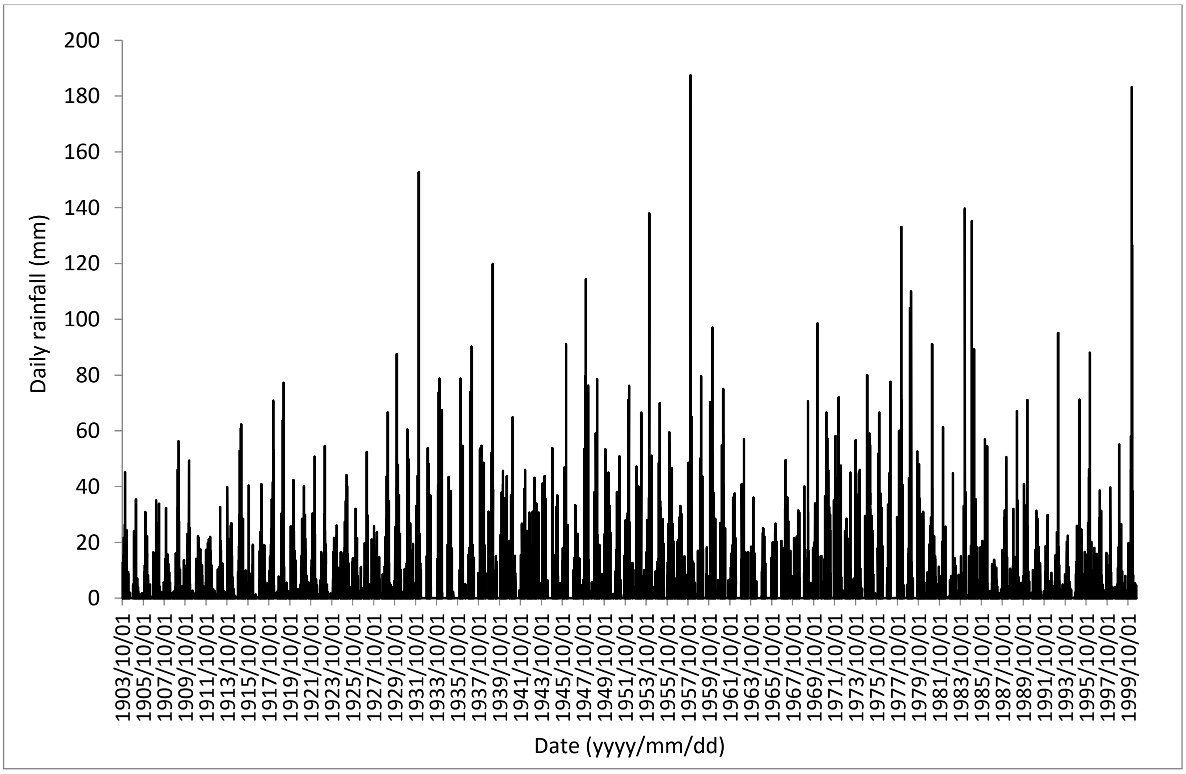


Figure 3.10: Rainfall data from 1903/11/01-2000/07/31 for station 0766324

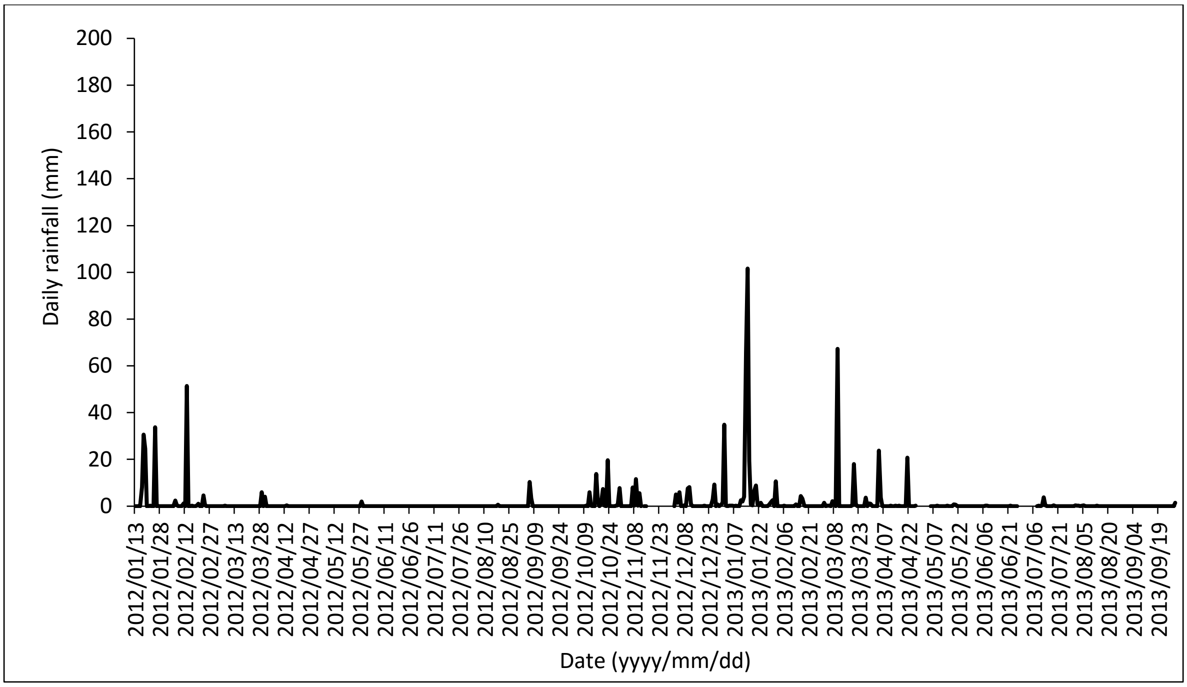


Figure 3.11: Rainfall data from 2012/01/13-2013/12/31 for University of Venda weather station

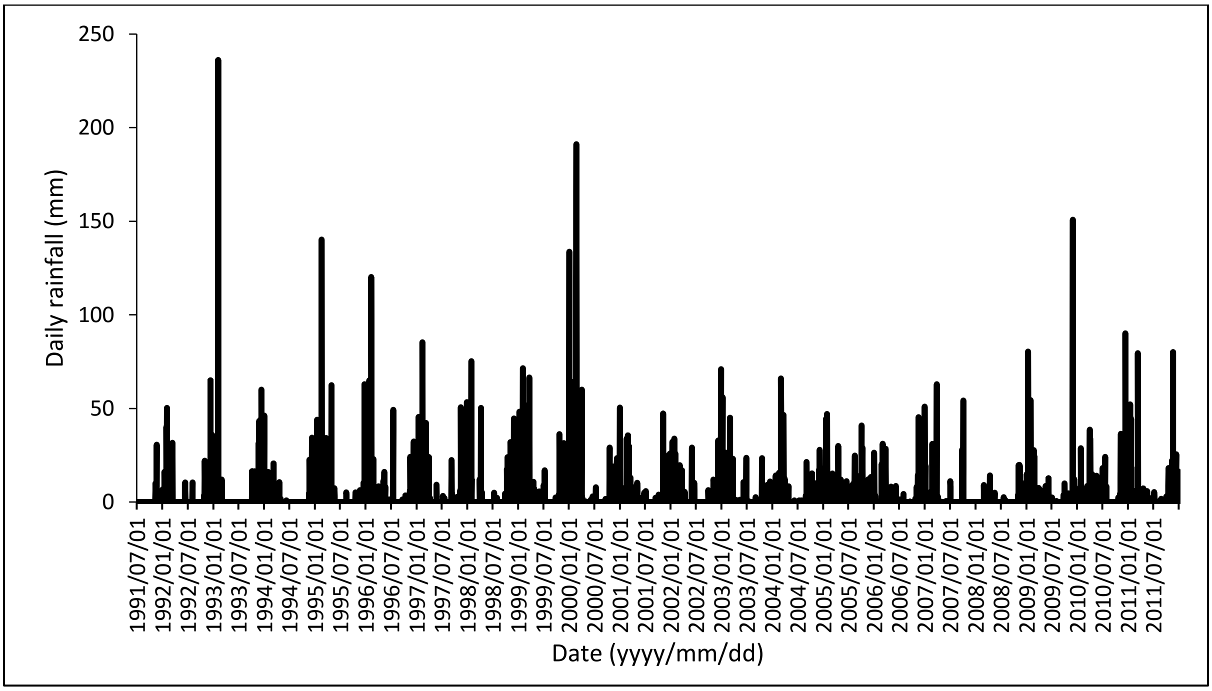


Figure 3.12: Rainfall data for station A8E004

Extended evapotranspiration data computed using the Hargreaves-Samani method for the period 1980-2000 (Figure 3.13) was obtained from Makungo (2009). The extension could only be done for this period since it was the only period when temperature data was available. Makungo (2009) obtained temperature data for Rabali station (0766202) located in the neighbouring quaternary catchment A80B (Figure 3.9) from the WRC temperature database developed by Schulze and Maharaj (2003). Temperature data were used to extend the evaporation data for station A8E004. Evaporation data for the period 2000/08/01 to 2012/01/12 were adopted from station A8E004 which had patched data from 1991/07/01 to 2012/01/12 (Figure 3.14). Evaporation data from station A8E004 were used to compute evapotranspiration for the period 2000/08/01 to 2012/01/12 (Figure 3.15) based on the pan evaporation method (Equation 3.1 described in Allen *et al.* (1998)).

$$ET_0 = E_P \times k_c \tag{3.1}$$

ET_0 is evapotranspiration, E_p is evaporation and k_c is the pan coefficient. The standard pan coefficient of 0.7 was used. Evapotranspiration data from 2012-2013 (Figure 3.15) were available from University of Venda weather station at Siloam Village.

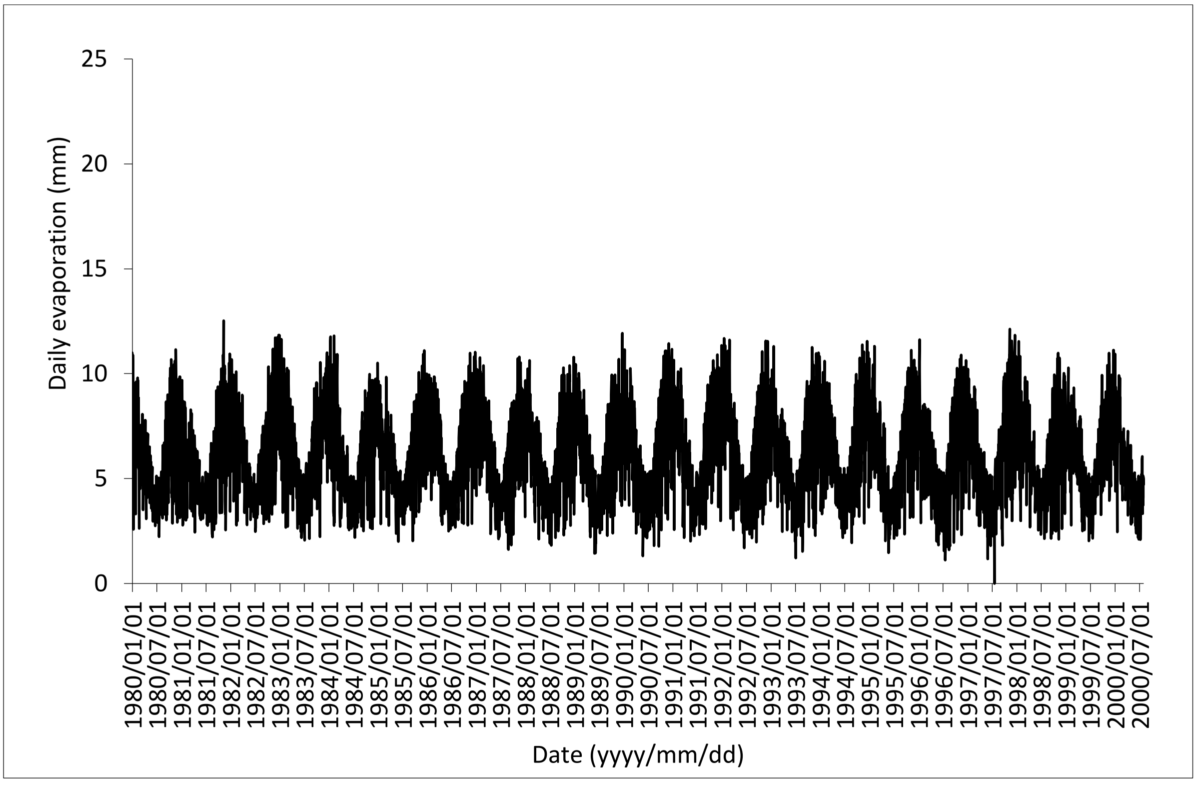


Figure 3.13: Evapotranspiration data from 1980-2000 for station A8E004

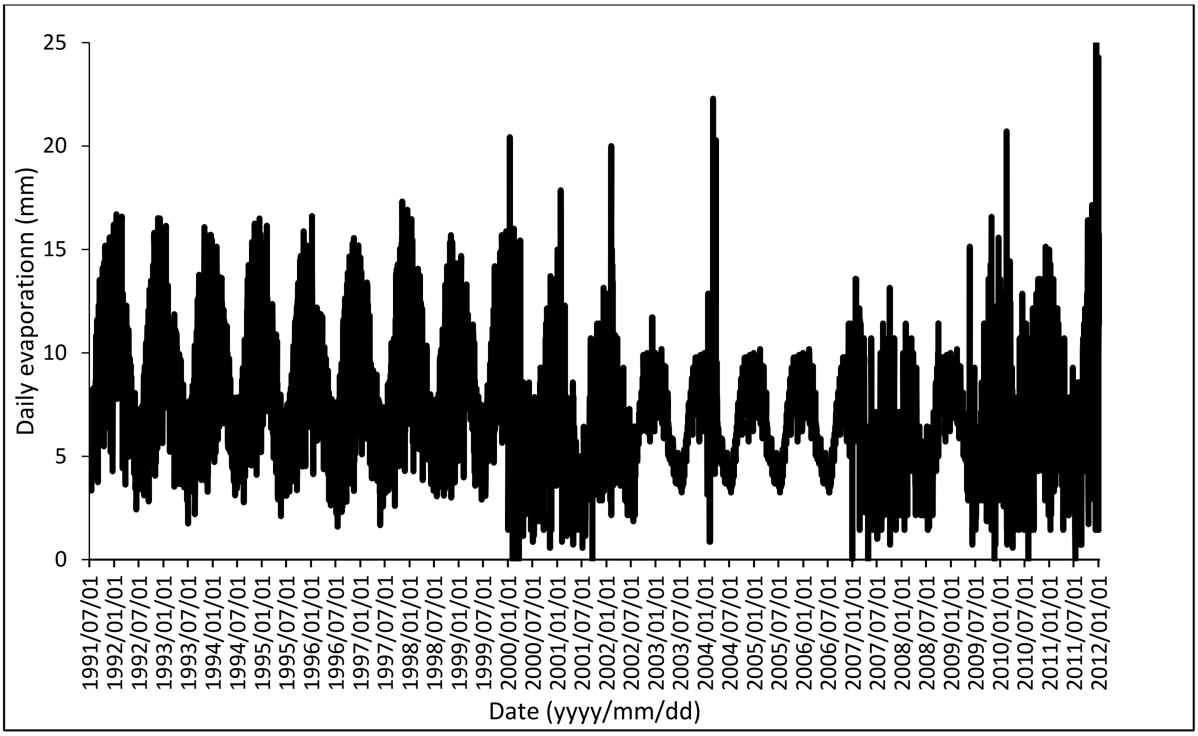


Figure 3.14: Evaporation data for station A8E004 for the period 1991/07/01 to 2012/01/12

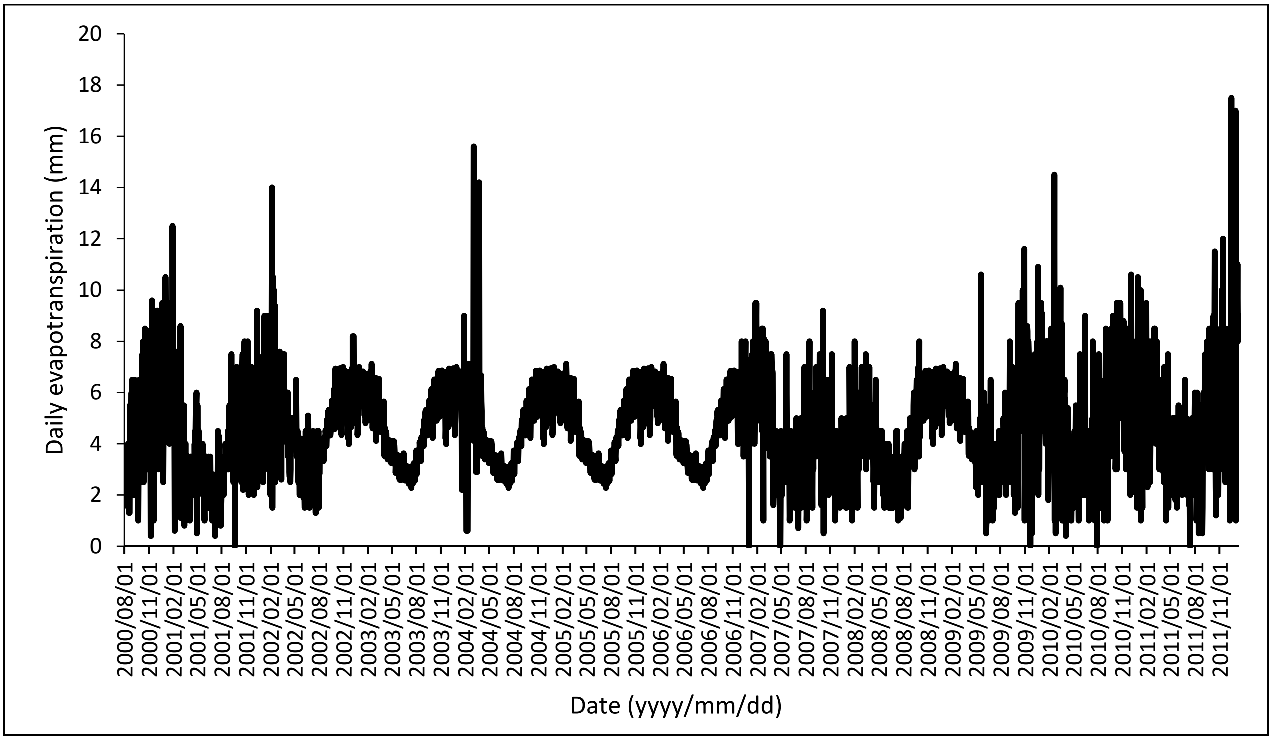


Figure 3.15: Evapotranspiration data for station A8E004 for the period 2000/08/01 to 2012/01/12

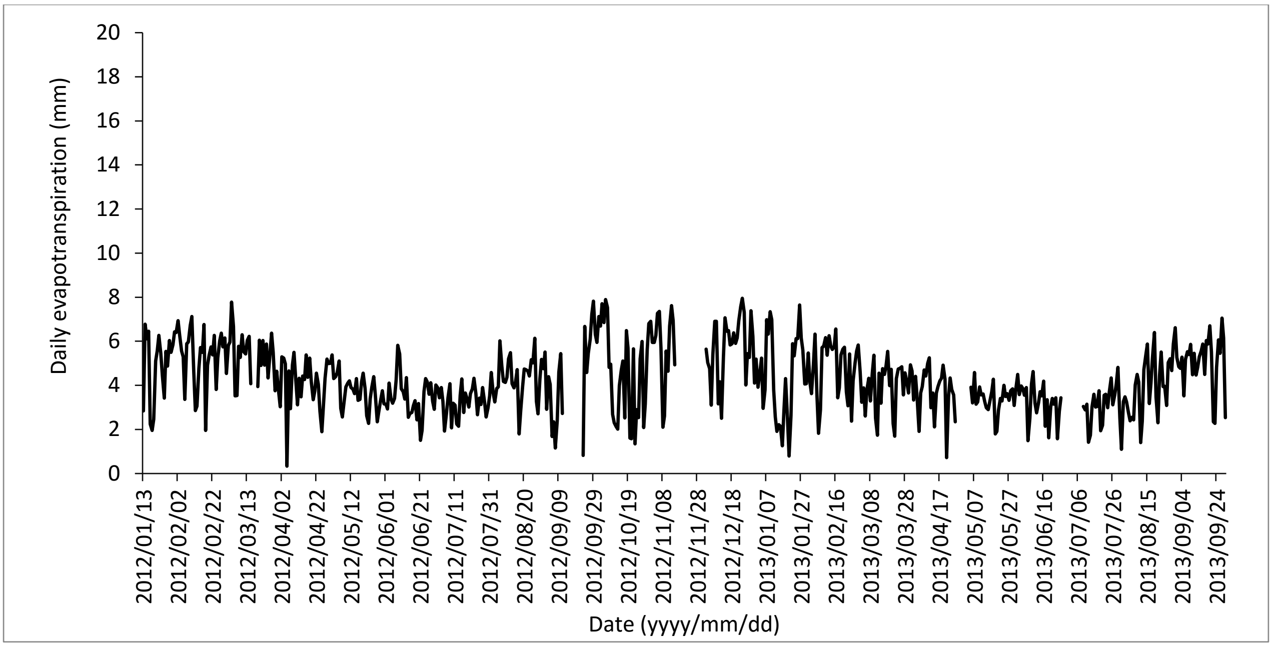


Figure 3.16: Evapotranspiration data from 2012-2013 for University of Venda weather station at Siloam Village

There were no groundwater levels data in the study area. Borehole A8N0508 shown in Figure 3.8 was selected as a representative borehole and its groundwater levels adopted for use in calibrating the GW-PITMAN model. This borehole was selected based on comparable characteristics of its location with that at the river boundary of Siloam Village (Table 3.2). These characteristics include topography, geology, hydrogeology and mean annual precipitation. In addition, the borehole A8N0508 is in the same quaternary catchment as Siloam Village. The observed groundwater levels for this borehole covered the period 2005/07/20 to 2012/11/25 (Figure 3.17).

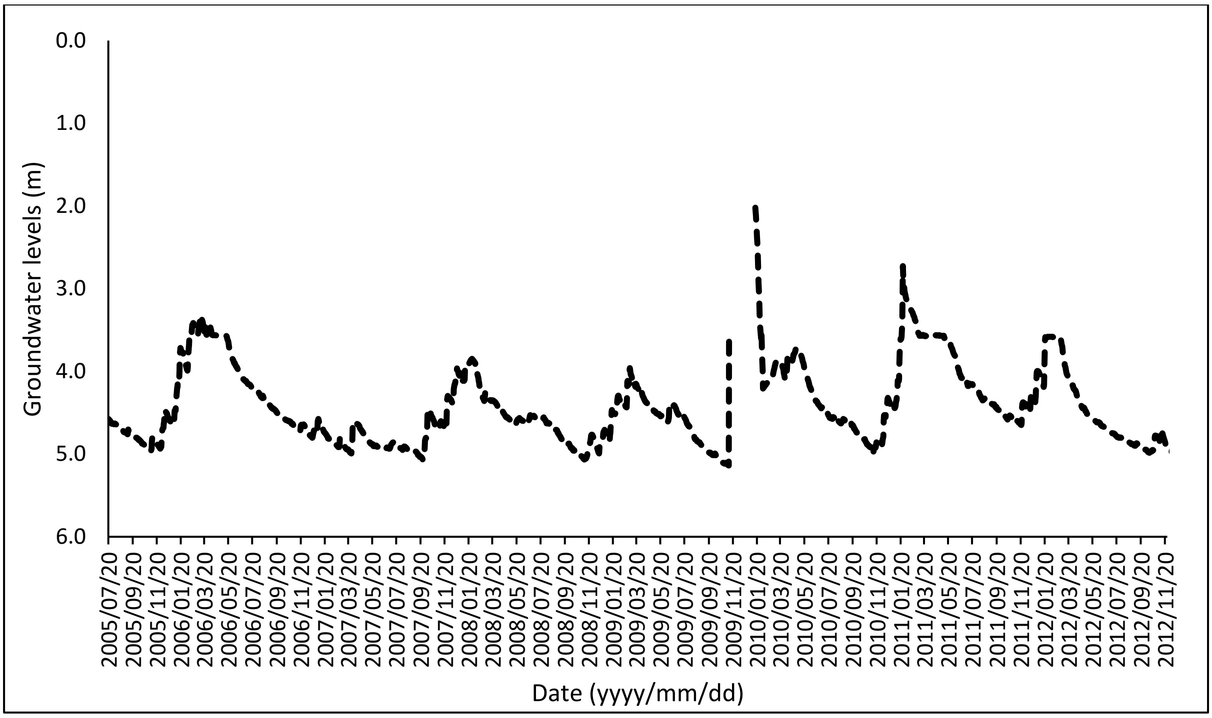


Figure 3.17: Observed groundwater levels for borehole A8N0508

Table 3.2: Borehole and study area characteristics

Feature	Nzhelele River boundary	A8N0508 (Mandala)
Topography	780	810
Geology	Alluvium	Alluvium
Mean Annual Precipitation (mm)	300-400	300-400
Hydrogeology	Class b3 (Fractured aquifers, yield ranging from 0.5-2 l/s)	Class b3 (Fractured aquifers, yield ranging from 0.5-2 l/s)

Time series of rainfall, evaporation and groundwater levels were essential for efficient calibration of GW-PITMAN model and generating the stochastic inputs required in development of the groundwater operating rules. Once-off groundwater level measurements from Limpopo GRIP were obtained to determine the relationship between topography and groundwater levels, and to estimate the initial values of the groundwater gradient. Assessing the relationship between topography and groundwater levels was essential to determine if groundwater in the study area is controlled by topography. This was aimed at showing whether the concept of groundwater divide is applicable in the study area. Initial values of the groundwater gradient were required by the GW-PITMAN model. The location of the GRIP boreholes in the study area is shown in Figure 3.18.

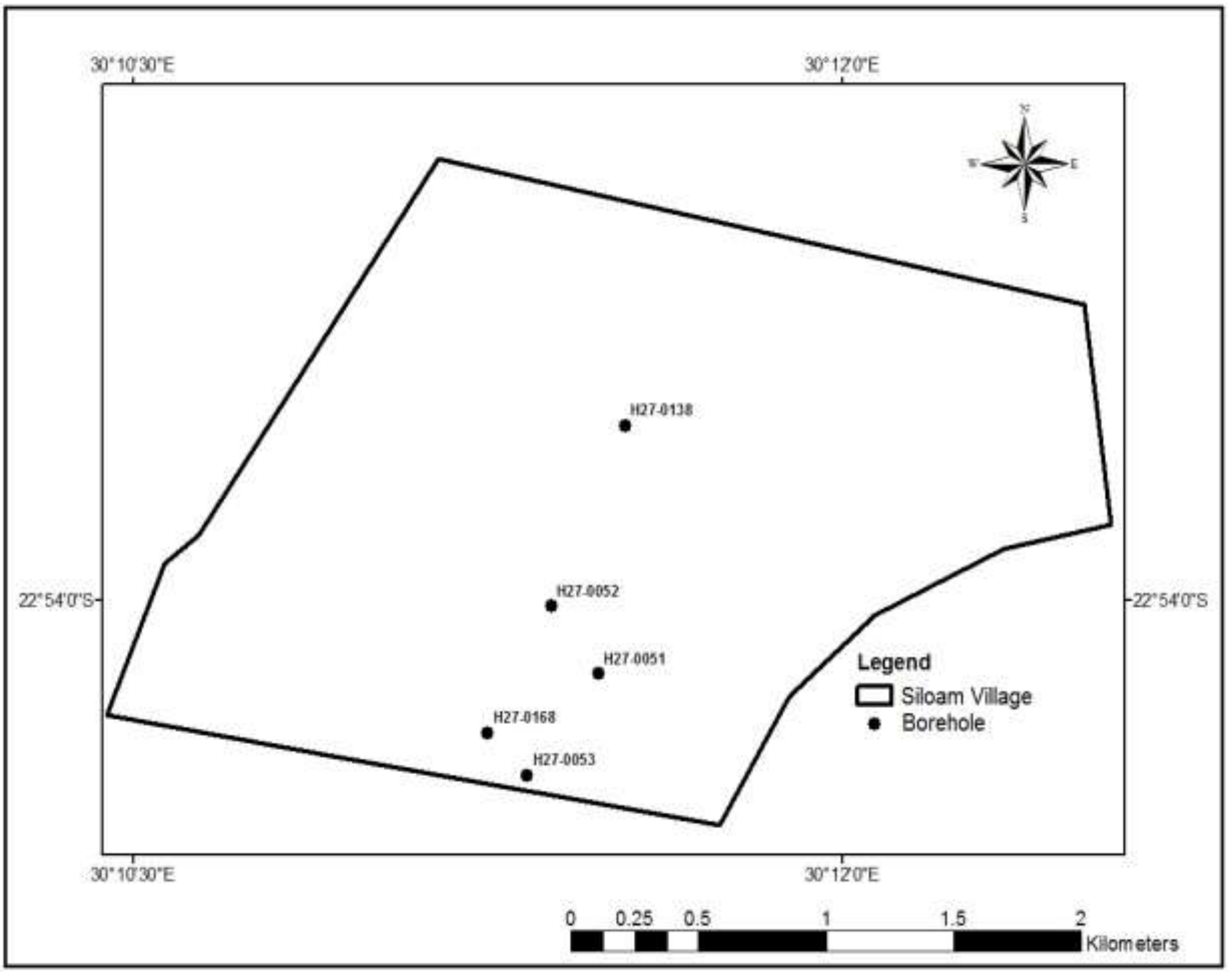


Figure 3.18: Limpopo Grip groundwater levels boreholes

3.4 Summary

This chapter described the characteristics of the case study area, the basis for its selection as well as available data. Describing the characteristics of the study area aids in providing its environmental setting and physical attributes which assists in justifying its selection as a case study. Describing data available in the study area informed the selection of methods used in its analysis (described in Chapter 4) with the aim of achieving the objectives, research questions and hypotheses of the study. This chapter indicated some of the data required for the study (rainfall and groundwater levels) are limited by presence of gaps and short duration. In addition groundwater levels data from a representative neighbouring borehole was adopted for use in calibrating the GW-PITMAN model. This was due to lack of data in the groundwater resource unit.

CHAPTER 4: METHODOLOGY

4.1 Preamble

This chapter presents the methods that were used to analyse the data in order to meet the objectives of the study. A flow chart of the steps followed in the methodology is provided in Figure 4.1. The steps involved delineation of groundwater resource unit (step 1) followed by development of its hydrogeological conceptual model and hydraulic characterisation (step 2). Since the study area is data-scarce, some of the data (rainfall, evapotranspiration and groundwater levels) had considerable gaps. This informed the selection of methods used for infilling and/or extending some of the data sets (step 3 in Figure 4.1). This was followed by calibration and validation of the GW-PITMAN model (step 4) and stochastic generation of rainfall, evaporation and groundwater levels data (step 5). The last step in the methodology involved stochastic groundwater base yield analysis and development of risk-based operating rules.

Methods used for groundwater resource unit delineation and its characterisation, infilling and/or extension of data sets, groundwater levels modelling using GW-PITMAN model, generation of stochastic sequences, groundwater base yield-recurrence interval and development of risk-based groundwater operating rules have been described in this chapter.

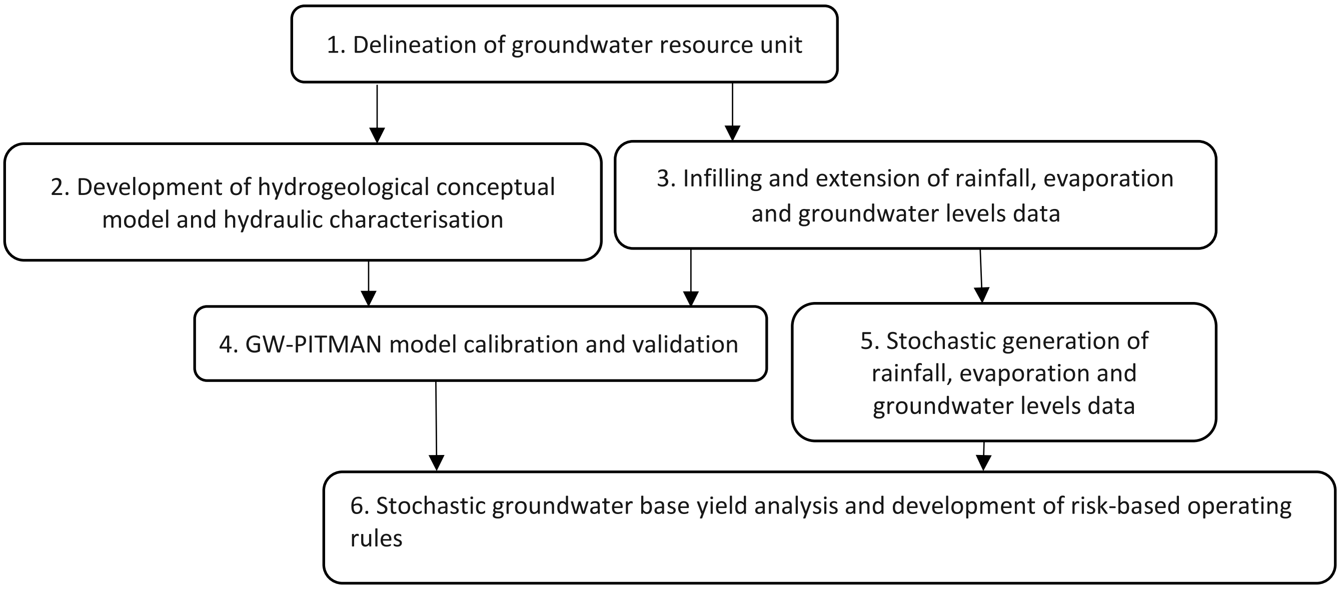


Figure 4.1: Steps followed in the methodology

4.2 Delineation procedure for the groundwater resource unit, development of hydrogeological conceptual model and hydraulic characterisation

The study delineated a groundwater resource unit to define a groundwater reservoir which formed the basis for generating groundwater levels using the GW-PITMAN model, and generation of operating rules. Linear features including faults and lineaments which were not visible on 1: 500 000 hydrogeological and 1: 250 000 geological maps were identified and delineated from a digital elevation model (DEM). The DEM was processed using the procedure described in Abdullah *et al.* (2010). The procedure involved production of eight separate hill-shaded relief images with light sources coming from eight different directions which are 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°. Two final hill-shaded relief images were created by compositing the eight hill-shaded relief images. Each image was composited from the images with four azimuth angles of the light sources that are 0°, 45°, 90° and 135°, and 180°, 225°, 270° and 315°, respectively. The advantage of hill-shaded imagery is the freedom to select illumination from any angle (Henderson *et al.*, 1996).

Lineaments were extracted from hill-shaded relief image using LINE algorithm of PCI Geomatica software. Lineament extraction algorithm of PCI Geomatica software consists of edge detection, thresholding and curve extraction steps (PCI Geomatica, 2001). The delineated linear features were verified from geological and hydrogeological maps of the study area. Faults and lineaments within the vicinity of the study were digitised in ArcMap 10.3. These were used in defining the boundaries of the groundwater resource unit.

The mountain which forms a groundwater divide was used to define the upper boundary of the groundwater resource unit. Groundwater divide separates areas where water flows in one direction from areas where it flows in another. The concept of groundwater divide has been used for delineation of groundwater units in studies such as Sheets and Simonson (2006). The relationship between groundwater levels and topography was used to establish whether topography controls groundwater levels and hence verify if the mountain can be defined as a closed boundary.

A hydrogeological conceptual model was developed from geological cross-sections. Geological cross-sections were constructed following Illinois State University (2013). Three geologic cross-sections which cut across the groundwater resource unit were constructed to capture the main geologic and structural features that control groundwater in the groundwater resource unit. Thus, it aided in providing understanding of the nature of the groundwater environment which is essential when developing a groundwater simulation model.

Aquifer Test Solver (AQTESOLV) Pro version 4.5 was used for automatic curve matching to identify appropriate aquifer models and test solutions for estimating hydraulic characteristics following Duffield (2007). The software automatically selects an aquifer model (i.e. confined, unconfined, leaky and fractured) and aquifer test solution that gives the best fit of observed and estimated drawdowns. This aids in improving the accuracy of estimated hydraulic characteristics. Non-linear least squares fitting was used to obtain the best model parameters. Moustafa (2011) also used non-linear least square fitting method to fit different type curves to the measured drawdown by searching for the best model that matches the observed data.

Residual standard deviation (σ_e), residual variance (σ^2_e) and residual mean ($\bar{e}$) (Equations 4.1-4.3) were used to evaluate the fitted models. The residual (e_i) was calculated using Equation 4.4.

$$\sigma_e = \sqrt{\frac{RSS}{n-p}} \tag{4.1}$$

$$\sigma_e^2 = \frac{RSS}{n-p} \tag{4.2}$$

$$\bar{e} = \frac{\sum_{i=1}^{n} y_i - \hat{y}}{n} \tag{4.3}$$

$$e_i = y_i - \hat{y}_i \tag{4.4}$$

y_i is the i^{th} observation, $\hat{y}_i$ is the estimate of y_i computed by aquifer test solution, p is the number of estimated parameters and n is the total number of observations. Standard deviation and variance are the most common measures of dispersion (variation) for continuous data (Boslaugh and Waters, 2008). Residual mean values for a model with a perfect fit are zero, since there are no estimation errors. Aquifer hydraulic characteristics were computed based on the aquifer test solution model selected by the software for each set of pumping test data. Hydraulic conductivity was computed from the relationship:

$$K_h = \frac{T_r}{b} \tag{4.5}$$

where, K_h is hydraulic conductivity, T_r is transmissivity and b is aquifer thickness. Krasny (1993) classification of T_r transmissivity (Table 4.1) was used to infer groundwater supply potential.

Table 4.1: Classification of the magnitude of transmissivity (Krasny, 1993)

Transmissivity (m^2/day)	**Designation of magnitude**	**Groundwater supply potential**
>1000	Very high	Withdrawal of great regional importance
100-1000	High	Withdrawal of lesser regional importance
10-100	Intermediate	Withdrawal of lesser regional supply (small communities and plants)
1-10	Low	Withdrawal for local water supply (private consumption)
0.1-1	Very low	Withdrawal for local water supply with limited supply
<0.1	Imperceptible	Sources for local water supply are difficult

Derivatives plotted in AQTESOLV were used to identify presence of fracture dewatering from the pumping test data. This was essential in understanding risks such as drying out of boreholes and surface water resources as well as fracture and well clogging which are associated with fracture dewatering.

4.3 Procedures for infilling and extension of data required for groundwater storage computations

4.3.1 Extension of rainfall data

This study applied locally weighted scatter smoother (LOWESS), which is a kernel non-parametric regression (NPR) method, with tricube kernel weighting function and two polynomial degrees (Cleveland, 1979) to estimate extend daily rainfall data. NPR was selected for extending rainfall data in this study because it is a data driven model which does not require knowledge of the physical behaviour of the system and hence requires minimal input data. Thus, it can be applied in data-scarce areas with limited number of rainfall stations, such as the study area. In addition, NPR captures both linear and non-linear relations (Cao *et al.*, 2008), and is capable of solving problems where the structure of the data is complex. Robust LOWESS NPR was also selected because it is the most commonly used method (see Chandler and Scott, 2011). In addition, it is resistant to the "far out" response variables (outliers) at the upper borderline of the plot. Thus, it is not influenced by far out observations (outliers) (Härdle, 1989). Details of robust LOWESS fitting procedure are found in Cleveland (1979), Cleveland and Devlin (1988), Cleveland and Loader (1996) and Cohen (1999).

The tricube kernel weighting function was used in this study since it is the kernel function that is usually specified in the LOWESS procedure (Buskirk *et al,* 2013). The general NPR model is written in a similar manner as non-linear regression (Equation 4.6), but the function, *f(x),* is left unspecified (Fox, 2002; Fox and Weisberg, 2011).

$$y = f(x) + \varepsilon \tag{4.6}$$

where *y* and *x* are dependent and independent variables, respectively, and Ɛ is the random error. The focus of NPR was therefore to determine $\hat{f}(x)$ which is an estimate of *f(x)* (Equation 4.7).

$$y = \hat{f}(x) + \varepsilon \tag{4.7}$$

The procedure followed involved selection of the window width enclosing the nearest neighbours to each data observation. This encompasses data used in local polynomial regression. Within the window width, $\hat{f}(x)$ was approximated by a local polynomial regression with two degrees:

$$\hat{f}(x) \approx \beta_0 + \beta_1(x_i - x_0) + \beta_2(x_i - x_0)^2 + \varepsilon_i \tag{4.8}$$

The local polynomial regression at x_i minimises:

$$\sum_{i=1}^{n}\left(y_i - \sum_{j=0}^{p}\beta_j(x_i - x_0)^j\right)^2 w_i \text{, with respect to } \beta_0, \beta_1 \ldots, \beta_p. \tag{4.9}$$

where β_0, $\beta_1 \ldots \beta_p$ are regression coefficients and w_i is the weighting function. A window width including 50% nearest neighbours was selected in the current study. Data within the window width was weighted based on Equation 4.10. Observations that fell outside of the window width received zero weight.

$$w_i = K(z_i) = K\left(\frac{x_i - x_0}{h_i}\right) \tag{4.10}$$

where K is the tricube kernel weighting function, z_i denotes the scaled distance between the x value for i^{th} observation and the focal (target value), x_0; h_i is half the window width of the local polynomial regression centered at the focal x_0; where x_0 corresponds to the point of interest where local polynomial regressions is being fitted. A window width that is too small will result to insufficient data, and hence a small h_i. This will produce a local polynomial regression curve with a lot of noise (bias). A very large window width results to a large h_i which produces an overfitted and biased local polynomial regression curve. This means selection of the window width is an important factor in NPR which has an influence on local polynomial regression curve. K was computed from:

$$K = \begin{cases} \left(1-|z|^3\right)^3 & for|z|<1 \\ 0 & for|z|\geq 1 \end{cases} \tag{4.11}$$

The local polynomial regression for each x_i value is derived separately. Fitted values from local polynomial regressions were connected to produce a NPR curve. Missing rainfall data was estimated based on the derived NPR curve. The computations were automatically done in XLSTAT, which is an add-in component of Excel Spreadsheet. Data for the period 1991/07/01 to 2000/07/31 was used in model calibration and validation. This was the period in which data was available for both A8E004 (neighbouring) and 0766324 (target) rainfall stations. Sixty and 40% of data points were used for model calibration and validation, respectively. These were the percentages that gave the best model performance after several trial runs. The data points used for calibration and validation were randomly selected within the period 1991/07/01 to 2000/07/31 by the NPR model.

The performance of the model was evaluated based on correlation coefficient (COR), coefficient of determination (R^2), Nash Sutcliffe coefficient of efficiency (NSE), relative error (RE) and root mean square error (RMSE). COR and R^2 describe the degree of co-linearity between simulated and measured data (Moriasi *et al.*, 2007; Obiero *et al.*, 2011). R^2 determines the proportion of *in-situ* variance that can be explained by the model (Rathjens and Oppelt, 2012). RE describes the difference between model simulations and observations in the units of the variable. RMSE indicates errors of the constituent of interest, which aids in analysis of the results (Moriasi *et al.*, 2007). NSE represents the complement to unity of the ratio between the mean square error of observed versus predicted values and the variance of the observations (Ritter and Muñoz-Carpena, 2013).

Scatter plots were also used in identifying overestimation and underestimation of observed rainfall and further validation of the model. Scatter plot is ideal for comparing model performance at low, medium, and high magnitudes (Bennet *et al.*, 2013). Daily rainfall data for Mutshedzi weather station for the period 2000/08/01 to 2012/01/12 was used to estimate missing rainfall data for Siloam weather station for the same period based on the calibrated NPR model.

4.3.2 Infilling and extension of groundwater levels data

A coupled linear polynomial Output-Error (OE) and non-linear Hammerstein-Wiener (NLHW) system identification model (Ljung, 1998; 2014) for estimating groundwater levels was used in this study. This model is referred to as the Output Error-Non-linear Hammerstein Weiner (OE-NLHW). Linear polynomial OE model structure (Figure 4.2) is one of the various linear model structures that provide different ways of parameterising the transfer functions of linear input-output polynomial model within system identification software (Ljung, 2014). The OE model is based on Equation 4.12.

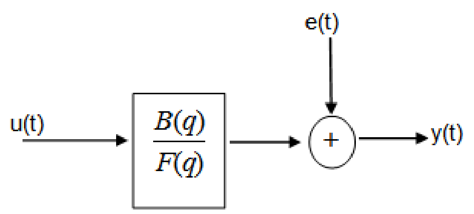


Figure 4.2: Structure of linear polynimial OE model

$$y(t) = \frac{B(q)}{F(q)} u(t - n_k) + e(t) \tag{4.12}$$

$$B(q): nb = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$
$$F(q): nf = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}$$

nb and *nf* are orders of the polynomials *B* and *F*, respectively, and n_k is the delay from input to output in terms of number of samples, q^{-1} is time-shift operator, *u* is the input, *y(t)* is the output at time (*t)* and *e(t)* is model error. NLHW model (Figure 4.3) represents dynamics of a system by a linear transfer function and captures the nonlinearities using nonlinear functions of inputs and outputs. Detailed description of OE and NLHW models is provided in Ljung (1998, 2014).

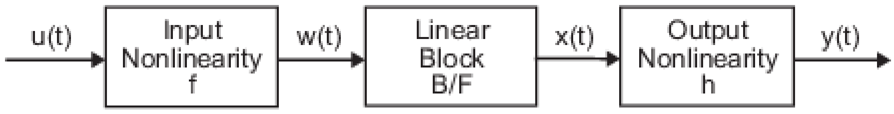


Figure 4.3: Structure of NLHW model

w(t) = f(u(t)) is a nonlinear function transforming input data, *u(t)* at time, *t*. *x(t) = (B/F)w(t)* is a linear transfer function. B and F are similar to polynomials in the linear polynomials OE model (Equation 4.12), and *f* and *h* are scalar functions for input and output channels, respectively. The output, *y(t)*, of NLHW model was computed by:

$$y(t) = h(x(t)) \tag{4.13}$$

Wavelet network (Equation 4.14) was selected as nonlinearity estimator.

$$g(x) = \sum_{k=1}^{n} \alpha_k k(\beta_k (x - \gamma_k)) \tag{4.14}$$

g(x) is wavelet network, β_k is a row vector such that $\beta_k\,(x - \gamma_k)$ is a scalar. The linear polynomial OE model was used to initialise the NLHW model. The initialisation configures the NLHW model to use orders and delays of the linear model, and polynomials as the transfer functions (Ljung, 2014). This initialisation aids in improving the fit of the model.

Daily rainfall from station 0766324, and evaporation data from Mutshedzi station constituted input into the OE-NLHW model. These were the stations within the vicinity of borehole A8N0508 (Figure 3.8). Using climatic data within the vicinity of each borehole overcomes the effects of spatial climate variability (Knotters and Van Walsum, 1997). Rainfall and evaporation were the explanatory variables of the model. Manzione *et al.* (2009) also incorporated precipitation and evapotranspiration as exogenous variables into a model when mapping water table depths since they are the most important driving forces of water table fluctuations.

Data used for model calibration and validation covered the period 2005/07/20-2012/11/25, which corresponded to the period in which observed groundwater levels data for borehole A8N508 was available. Knotters and Van Walsum (1997) used data with lengths of 4-10 years to develop time series models that enabled simulation of water table depths of extensive length (for example, 30 years). Seventy and thirty percent of the data were used to calibrate and validate the model, respectively. Model performance was evaluated using graphical fits, COR, R^2, RMSE, NSE and RE. A combination of graphical results, error statistics (RMSE and RE), and goodness-of-fit statistics (COR, R^2, and NSE) is essential to ensure accurate verification of the model (see Ritter and Muñoz-Carpena, 2013).

The calibrated OE-NLHW model was used to extend groundwater levels based on rainfall and evapotranspiration data covering the period 1980/01/01 to 2005/07/20. Groundwater levels for the periods 2009/11/11 to 2010/01/15 and 2012/11/26 to 2013/12/31 were infilled since the observed data (for the period 2005/07/20 to 2012/11/25) had gaps in these periods. The extended groundwater levels for the period 1980/01/01 to 2005/07/20 were combined with the observed and infilled data to constitute a data set for the period 1980/01/01 to 2013/12/31. This was aimed at generating groundwater levels time series to cover the period of at least 30 years for efficient calibration of GW-PITMAN model and subsequent groundwater base yield-recurrence interval analysis.

4.4 Procedure for generating groundwater levels, model sensitivity analysis, calibration and validation

4.4.1 Procedure for generating groundwater levels for the groundwater resource unit

A program for monthly generation of groundwater levels was coded in FORTRAN based on the GW-PITMAN model developed by Hughes (2004). GW-PITMAN captures the hydrological processes and it incorporates surface-groundwater interactions which improves estimation of average groundwater levels. The GW-PITMAN was selected for use in this study because it is applicable to ungauged areas and has been widely applied in southern Africa. The configuration of the geometry of the groundwater module of GW-PITMAN allows computation of storages. Figure 4.4 shows main components of GW-PITMAN model and relevant parameters. Figure 4.5 shows the configuration of the groundwater resource unit for GW-PITMAN modelling in this study. For typical applications, the input data required for the model includes monthly rainfall, evaporation, and streamflow data for model calibration and verification. For the application in this study, there were no streamflow records and model calibration and verification used groundwater levels data. These data sets have been described in section 3.3. As a result of data limitations including presence of gaps and short duration of record, the data sets were infilled and/or extended as described in section 4.3. The infilled and/or extended data used in model calibration and validation covered a period of 34 years (1980-2013).

The main components of the GW-PITMAN include runoff, reservoir, irrigation and groundwater modules. The runoff and groundwater modules were the only ones relevant in this study since there are no major agricultural activities and no reservoirs in the study area. A brief description of the equations used to compute the components of the groundwater balance is given to enable explanations of the similarities and deviations of the approach used in this study to that of the original PITMAN model (Pitman, 1973).

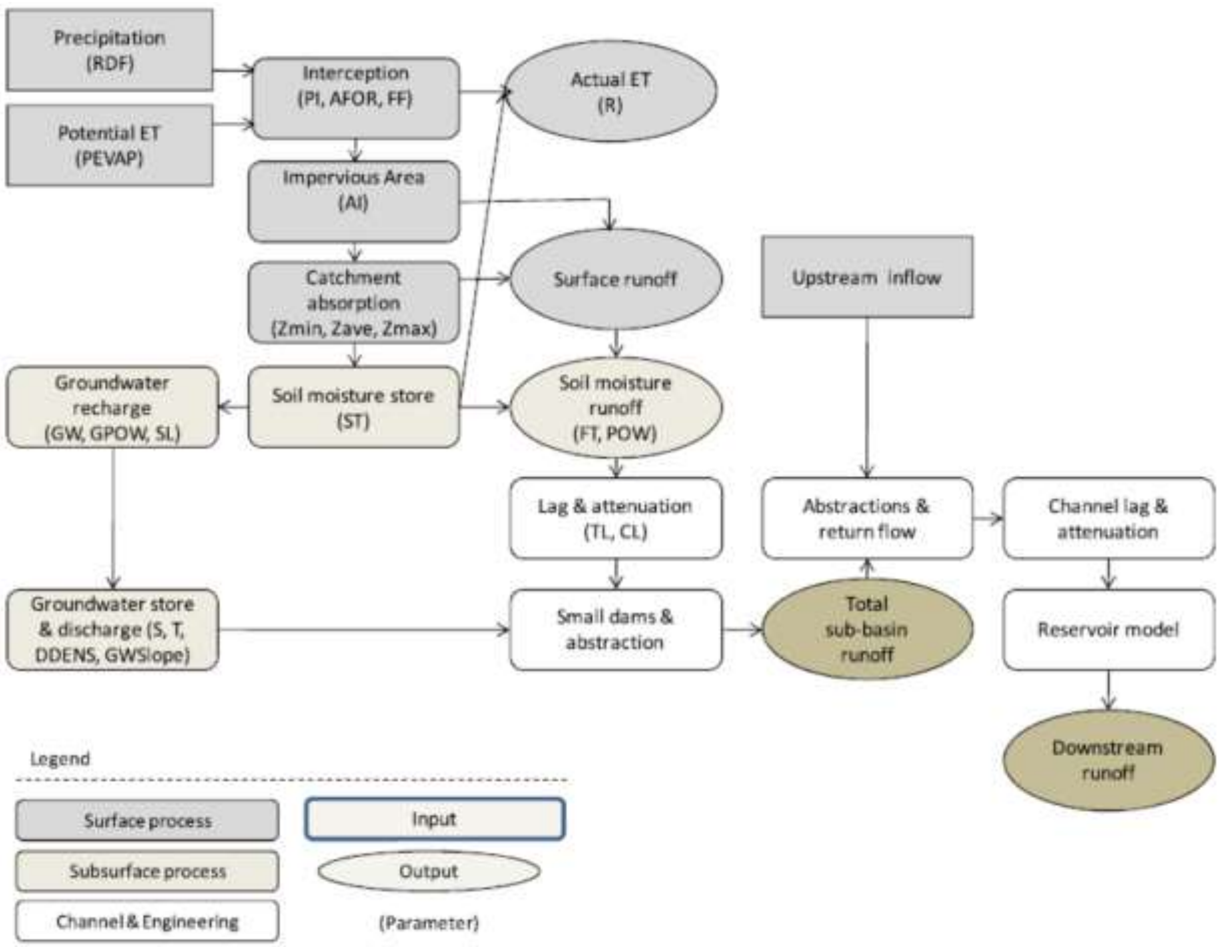


Figure 4.4: Flow diagram of the GW-PITMAN model showing the main model components and their relevant parameters in brackets (Tshimanga and Hughes, 2014)

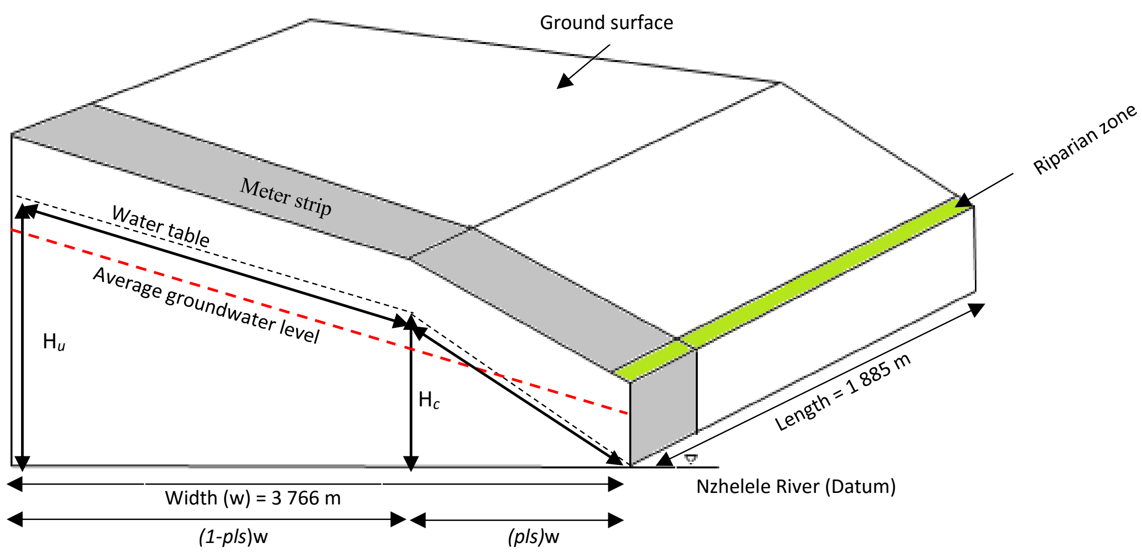


Figure 4.5: Configuration of groundwater resource unit for GW-PITMAN modelling

- **Runoff module**

Rainfall

The GW-PITMAN handles data in monthly time steps, but the water balance is solved at weekly interval. Equation 4.15 was used to account for monthly distribution in rainfall.

$$W = -2 + 1.3732(P + 1.6)^{0.8} \tag{4.15}$$

where *W* is maximum deviation of cumulative rainfall above and below the line representing the average rate, *P* is total precipitation for a month. Cumulative/total rainfall curve was calculated as:

$$y = x^n / \left(x^n + (1 - x)^n\right) \tag{4.16}$$

x is the cumulative/total time, *n* is an exponent related to *W* which was calculated as:

$$n = 1.28 / \left(1.02 - W / P\right)^{1.49} \tag{4.17}$$

Weighted rainfall (WP) was computed as:

$$WP = y \times P \tag{4.18}$$

Interception

It was assumed that monthly interception is allocated within the months in proportion to the weekly rainfall. Total interception (I) was computed from:

$$I = a\left(1 - e^{bp}\right) \tag{4.19}$$

P is total precipitation for a month (mm), *a* and *b* are constants computed as follows:

$$a = 13.08 PI^{1.14}$$

$$b = 0.00099 PI^{0.75} - 0.011$$

Weighted interception (*WI*) was computed using:

$$WI = \frac{I \times WP}{P} \tag{4.20}$$

Surface runoff

Surface runoff was assumed to be derived from runoff from impervious areas and runoff resulting from rainfall not absorbed by the soil (surface runoff from pervious areas (SR_{per})). Direct surface runoff from impervious areas (SR_{imp}) was computed as:

$$SR_{imp} = A_{imp} \times AWP \tag{4.21}$$

Where A_{imp} is the proportion of the groundwater resource unit area which is impervious and *AWP* is the available weighted precipitation. *AWP* is the difference between *WP* and *WI*. For SR_{per}, the following were considered:

$$If \begin{cases} AWP < Z_1 \; then \; SR_{per} = 0 \\ Z_1 < AWP < Z_2 \; then \; SR_{per} = (1 - A_{imp}) \times \left(\frac{2}{3} \frac{(AWP - Z_1)^3}{(Z_3 - Z_1)^2} \right) \\ Z_2 < AWP < Z_3 \; then \; SR_{per} = (1 - A_{imp}) \times \left(AWP - Z_2 + \frac{2}{3} \frac{(AWP - Z_1)^3}{(Z_3 - Z_1)^2} \right) \\ Z_3 < AWP \; then \; SR_{per} = (1 - A_{imp}) \times (AWP - Z_2) \end{cases} \tag{4.22}$$

$$Z_2 = 0.5 \times (Z_1 + Z_3) \tag{4.23}$$

Z_1, Z_2 and Z_3 are the minimum, mean and maximum catchment absorption rates, respectively.

Sub-surface runoff and evaporation from soil

The equation for sub-surface runoff (SR_{sub}) is based on the direct relationship between SR_{sub} and soil moisture:

$$SR_{sub} = FT \left(\frac{(S - SL_1)}{(ST - SL_1)} \right)^{POW} \tag{4.24}$$

FT is the runoff at soil moisture equal to ST, *S* is the actual soil moisture, *ST* is total soil moisture capacity, SL_1 is the minimum moisture storage below which no groundwater recharge occurs in the recharge equation and *POW* is the power of runoff soil moisture storage curve. Initial value of the S was assumed to be $0.5 \times ST$. The equation for evaporation from the soil (*EvapS*) is:

$$EvapS = beS + ce \tag{4.25}$$

$$bes = \left(\frac{0.25S}{ST}\right) \times \frac{PE}{\left(1 - R \times \left(\frac{1 - PE}{PE_{\max}}\right)\right)}$$

$$ce = 0.25 \times PE \left(1 - \frac{1}{\left(1 - R \times \left(\frac{1 - PE}{PE_{\max}}\right)\right)}\right)$$

S is actual soil moisture, *PE* is potential evapotranspiration, R is the evaporation-moisture storage relationship parameter and PE_{max} is maximum potential evapotranspiration.

- **Groundwater module**

The groundwater module estimates the groundwater storage within the groundwater resource unit based on the water balance equation. The spatial geometry of the groundwater resource unit was represented by a rectangle to reduce complexity of the model following Bailey and Pitman (2016). The length and width of the groundwater resource unit are 3766 and 1885 m, respectively (Figure 4.5). In solving the groundwater balance, the slope of the groundwater table was partitioned into upper and lower slopes representing the portions of the groundwater resource unit that are far and closer to the river, respectively. Bailey and Pitman (2016) assumed the upper and lower slopes of the GW-PITMAN model to constitute 60% and 40% of the total slope, respectively. In this study, the proportions of the lower and upper slopes were defined as *pls* and *1-pls*, respectively. *pls* was considered to be a model parameter which was calibrated to aid in determination of its realistic value. *pls* is the proportion of the groundwater resource unit with lower side groundwater table (closer to the river) (Figure 4.5). The groundwater balance for the upper and lower slopes were therefore estimated separately.

The steps followed in computing groundwater storage based on the groundwater balance within each model iteration as summarised from Bailey and Pitman (2016) included:

- Calculation of recharge and volume of water added to the upper and lower slopes.
- Estimating the groundwater flow from the lower and upper slopes based on hydraulic gradients from the previous time step.

- Calculation of riparian evapotranspiration abstraction losses from groundwater.
- Calculation of new volumes of water in the upper and lower slopes. These were used to estimate the hydraulic gradients for the next time step.

The components of the groundwater balance considered in this study included volumes of recharge, abstractions, groundwater flow and riparian losses. Equations from GW-PITMAN model for estimating these components were therefore altered considering the definitions of the lower (*pls*) and upper slopes (*1-pls*) and the fact that the groundwater balance was solved in terms of 1 m strip of the groundwater resource unit. The general formula for computing groundwater recharge using GW-PITMAN model is:

$$RE = GW\left(\frac{(S - SL_2)}{(ST - SL_2)}\right)^{GPOW} \tag{4.26}$$

where *RE* is the groundwater recharge, *GW* is the maximum recharge (at a soil moisture equal to *ST*), *S* is the actual soil moisture in storage. SL_2 is the minimum moisture storage below which no groundwater recharge occurs in the recharge equation, *ST* is the maximum moisture storage capacity and *GPOW* is the power of the moisture storage-recharge equation. Recharge from lower (*pRechl*) and upper (*pRechu*) slopes (Equations 4.27 and 4.28) were obtained by incorporating *pls* and *1-pls*, respectively, into Equation 4.26.

$$p\mathrm{Re}ch_l = pls \times RE \tag{4.27}$$

$$p\mathrm{Re}ch_u = (1 - pls) \times RE \tag{4.28}$$

The volumes of recharge per 1 m strip of the groundwater resource unit on lower (RE_l) and upper slopes (RE_u) were computed based on Equations 4.29 and 4.30, respectively, while the total groundwater recharge *(TRE)* per 1 m strip of the groundwater resource unit was computed using Equation 4.31. A factor of 0.001 is included in Equations 4.29 and 4.30 since the estimations were done per 1 m strip of the groundwater resource unit. The equations also include the groundwater slopes to account for the variations of hydraulic gradients within the upper and lower slopes as these affect the recharge within each of the slopes.

$$If \begin{cases} GWS_l < IlsMax \;\; and \;\; GwS_u < IhsMax \;\; then \;\; RE_l = 0.001 \times TRE \times pls \times W_{GRU} \\ GWS_u > IhsMax \;\; then \;\; RE_l = 0.001 \times TRE \times W_{GRU} \end{cases} \tag{4.29}$$

$$If\begin{cases} GWS_l < IlsMax \ \ and \ \ GwS_u < IhsMax \ \ then \ \ RE_u = 0.001 \times TRE \times (1 - pls) \times W_{GRU} \\ GWS_u > IhsMax \ \ then \ \ RE_u = 0.001 \times TRE \times W_{GRU} \end{cases} \quad (4.30)$$

$$TRE = p\mathrm{Re}chl + p\mathrm{Re}chu \quad (4.31)$$

GWS_l and GWS_u are the slopes of the groundwater table from lower and upper slopes, respectively, W_{GRU} is the width of groundwater resource unit, $IlsMax$ and $IhsMax$ are maximum lower and upper hydraulic gradients, respectively. Initial values of *IlsMax* and *IhsMax* were based on initial hydraulic gradients based on once-off groundwater levels from boreholes in Figure 3.17. The volume of abstractions on lower ($Abst_{actl}$) and upper ($Abst_{actu}$) slopes per week per 1 m strip of the groundwater resource unit were computed based on Equations 4.32 and 4.33, respectively.

$$Abst_{actl} = \left(\frac{pls \times Abst_i \times 7.609}{L_{GRU}} \right) \quad (4.32)$$

$$Abst_{actu} = \left(\frac{(1 - pls) \times Abst_i \times 7.609}{L_{GRU}} \right) \quad (4.33)$$

L_{GRU} is the length of the groundwater resource unit and $Abst_i$ is the initial value of 1 m^3/day used in searching for maximum abstraction rate during model calibration. The volumes of flow from lower (GWF_l) and upper (GWF_u) slopes to the river were computed as:

$$GWF_l = T_r \times GWS_l \times 7.609 \quad (4.34)$$

$$GWF_u = T_r \times GWS_u \times 7.609 \quad (4.35)$$

T_r is the transmissivity and GWS_l and GWS_u are groundwater table slopes from lower and upper slopes, respectively. The value of 7.609 in Equations 4.32 to 4.35 is a factor for converting daily fluxes to weekly considering 4 periods within a month for a 1 m strip of the groundwater resource unit. The loss from riparian strip (*LRS*) was computed using Equation 4.36.

$$If\begin{cases} H_c > 0 \, then \ LRS = 0.00025 \times PE \times RSW \\ H_c > RWL \ then \ LRS = \dfrac{0.00025 \times PE \times RSW \times (1 + H_c)}{RWL} \end{cases} \quad (4.36)$$

$$H_c = pls \times W_{GRU} \times GWS_l \quad (4.37)$$

H_c is the height of water table at the connection between lower and upper slope, *RSW* is the width of the riparian strip and *RWL* is the rest water level. Net increase in groundwater volumes in lower (*dVL*) and upper slopes (*dVU*) were calculated as:

$$dVL = \mathrm{Re}\,ch_l + GWF_u - Abst_{act} - GWF_l - LRS \tag{4.38}$$

$$dVU = \mathrm{Re}\,ch_u - Abst_{actu} - GWF_u - LRS \tag{4.39}$$

New height of the groundwater table at connection of lower and upper slopes (H_{c+1}), and end of upper slope (H_{u+1}) was computed as:

$$H_{c+1} = \frac{H_c + 2dVL}{STOR \times pls \times W_{GRU}} \tag{4.40}$$

$$H_{u+1} = H_u - \frac{H_c + 2dVU}{STOR \times (1 - pls) \times W_{GRU}} \tag{4.41}$$

$$H_u = H_c + (1 - pls) \times W_{GRU} \times GWS_u \tag{4.42}$$

In addition to incorporating *pls* and *1-pls,* Equations 4.40 and 4.42 include storativity to account for the fact that it is a parameter that controls groundwater storage within the groundwater resource unit. H_u is the height of the water table at end of the upper slope. New groundwater gradients of lower (GWF_{lnew}) and upper (GWF_{unew}) slopes were computed as:

$$GWF_{lnew} = \frac{H_{c+1}}{pls \times W_{GRU}} \tag{4.43}$$

$$GWF_{unew} = H_{u+1} - \frac{H_{c+1}}{(1 - pls) \times W_{GRU}} \tag{4.44}$$

GWF_{lnew} and GWF_{unew} were used in the groundwater balance for the next (new) time step. The study required groundwater levels to derive simple and implementable groundwater operating rules (which would indicate the volume of water available for allocation for a given groundwater level). It was therefore essential to incorporate estimation of groundwater levels in the GW-PITMAN model. Equation 4.45 was therefore introduced to allow estimation of groundwater levels.

$$GWL_{est} = H_c \times \left(\frac{dBH}{pls \times W_{GRU}} \right) \tag{4.45}$$

GWL_{est} is the estimated average groundwater level, H_c is the average monthly height of water table at the connection between lower and upper slope and *dBH* is the distance from the borehole to the river. *dBH* was included in Equation 4.45 to account for that the observed groundwater levels are from a neighbouring borehole. At each time step, the estimated average groundwater level is an average value for the entire GRU.

Comparison of equations in GW-PITMAN model and those of the current study

Equations 4.15 to 4.26 and 4.34 to 4.39 are similar to those in the GW-PITMAN model as described in Hughes (2004). Equations 4.27 to 4.31 provide detailed explanation of groundwater recharge estimations for both upper and lower slopes which are not provided in Bailey and Pitman (2016). In the GW-PITMAN model groundwater abstractions are represented by additional water use parameters while groundwater abstractions for upper and lower slopes, in this study, were estimated from Equations 4.32 and 4.33, respectively. Equations 4.40 to 4.44 provide details for estimation of new H_c values, and groundwater gradients of lower and upper slopes which are also not provided in the GW-PITMAN model.

4.4.2 Sensitivity analysis, model calibration and validation

A hybrid manual-automatic approach was used for calibration of the GW-PITMAN model in the study. In this approach, realistic model parameter ranges were estimated based on available hydrogeological data in the study area and SCE-UA algorithm (Duan *et al.*, 1992) was used to optimise model parameters. Setting of model parameters based on available physical and hydrogeological data, and their optimisation using SCE-UA constituted the manual and automatic components of the hybrid approach, respectively. Ten preliminary calibration runs were done, modifying one parameter at a time, to test the response of the model to a range of different parameter values and identify parameters that can be altered during final model calibration. Each model run was based on independent random sampling of the parameters within specified ranges of values. FT, SL, GPOW, ST, R and POW parameters were selected for sensitivity analysis. This was aimed at identifying a set of model parameters to be used in 1000 model calibration runs. The use of multiple calibration runs was aimed at incorporating uncertainty associated with both model inputs and model parameters.

Model calibration was done using the extended and infilled groundwater levels based on observed data from borehole A8N0508 (Mandala) which is located close to Nzhelele River upstream of the groundwater resource unit. The reasons for its selection are provided in section 3.3. The objective function (*OBF*) used in automatic model calibration was to minimise the sum of squared difference between observed and estimated groundwater levels.

$$OBF = \min \sum \left(GWL_{obs} - GWL_{est} \right)^2 \qquad (4.46)$$

Where GWL_{obs} is the observed groundwater levels and GWL_{est} is the estimated groundwater levels. A physical based approach was also used to estimate initial model parameters for calibration. This was done to test if this would result in an improved model fit. In this approach, the expected parameter values were estimated from physical characteristics of the groundwater resource unit using procedures described in Kapangaziwiri (2007) and Kapangaziwiri and Hughes (2008). The approach uses physical basin properties directly in the quantification of the soil moisture accounting, runoff, and recharge and infiltration parameters and it was developed to aid in improving application of the model in both gauged and ungauged basins.

Model validation was limited since time series of groundwater levels was only available from one neighbouring borehole. The model computes average groundwater levels for the groundwater resource unit and thus calibrating using data from one borehole would only give an indication of the expected behaviour of the system instead of its average behaviour. It was therefore decided that model validation can only be achieved by establishing the realistic nature of the model outputs. Hydrographs of simulated runoff, groundwater recharge and groundwater levels were compared with rainfall to determine if they have similar behaviour. Since rainfall has an influence on these variables it is expected to show whether they have similar behaviour and would therefore indicate the realistic nature of the model outputs. This would give an indication of whether the model is accurately simulating the hydrological processes within the groundwater resource unit.

4.5 Procedure for stochastic generation of rainfall, evaporation and groundwater levels

The variable length block (VLB) stochastic generator (Ndiritu and Nyaga, 2014) was used for simultaneous generation of stochastic time series of rainfall, evaporation and groundwater levels. The VLB model was selected for use in this study because it is a multisite and multivariate generator that adequately replicates historical statistics and reproduces annual serial and cross-correlations. Ndiritu (2011) applied VLB for streamflow generation in South Africa and found that it performed better than STOMSA which has been widely used for streamflow generation. It is also able to obtain stochastic values that are significantly beyond the bounds of the historical ones and thus overcome the main limitation of other non-parametric methods (Ndiritu, 2011). Ndiritu and Nyaga (2014) applied the VLB model for stochastic rainfall generation as follows:

- Variable length blocks of annual time series are generated from historic time series
- Random sampling of the blocks with replacement is done to create an annual stochastic time series of the specified length by random sampling of blocks
- Matching of each of the stochastic time series years with a pair of different years of the historic time series based on the magnitude of the annual flows of the current and the previous year.
- Disaggregate stochastic annual values using the monthly distributions of the pair of matching historic years and incorporation of perturbations.
- Updating of the stochastic annual values after the disaggregation.

Ndiritu (2011) emphasised that the variable length blocks are designed such that they cut across the low flow periods considering that most reservoir systems are mainly designed and/or operated to deal with low flow periods. This is also required in groundwater reservoirs where groundwater is used as a sole source of water supply or as a reserve during low flow periods. The hydrologic year was therefore assumed to start in July to ensure that the year begins and ends in the driest months. Detailed procedure for generating variable-length blocks is fully described in Ndiritu and Nyaga (2014). Table 4.2 shows the inputs into VLB generator used in this study.

Table 4.2: Inputs into VLB generator

Input type	Value
Number of stations	3
Length of historic variables in years	34
Length of the sequences to be generated in years	34
Length of sequences to be used in simulations for operating rules	5
Number of stochastic sequences to be generated	100
Minimum block length (years)	3
Minimum number of segments a stochastic sequence needs	3
Upper limit of low rainfall threshold	60
Lower limit of low rainfall threshold (as percentage of rank)	90
Number of years of warmup period to avoid bias	20

In the study, 100 stochastic sequences with record length of 34 years, similar to the historic one were generated. Historic data was for the period 1980-2013. Five years sequences are typically used in the generation of operating rules for surface water reservoirs. Performance of VLB in generation of stochastic sequences was assessed by comparing single statistics of historic time series located within box plots of the 100 annual and monthly stochastically generated time series. Box plots are ideal for presenting and summarising very large data sets, and they enable comparison of two or more data sets. The box plots also help to assess the variability and to identify the range of the generated statistics (Nyaga, 2014). The statistics used include mean, median, the 25th and 75th percentiles, lowest and highest rainfall, standard deviation, skewness, and serial and cross correlation coefficients following Ndiritu and Nyaga (2014). Replication of performance of a given statistic is judged as good when the historical value falls within the interquartile range of the box plots (Apipattanavis *et al*., 2007; Prairie *et al.*, 2007).

4.6 Procedure for groundwater stochastic base yield analysis and development of risk-based groundwater operating rules

Figure 4.6 shows the configuration of the groundwater resource unit for the purpose of developing groundwater base yield analysis and operating rules. Configuration of the groundwater resource unit was essential because unlike water level measurements in a surface water reservoir that start from the bottom of the reservoir to its surface, measurements in groundwater reservoir start from the ground surface to the groundwater table. In Figure 4.6, **A**, is the head of groundwater from the minimum allowable groundwater head (**C**) and **B** indicates the groundwater table depth/groundwater level. The continuous horizontal line is a datum corresponding to the water level of the stream. The minimum allowable groundwater head was considered to be the height up to which groundwater abstractions are constrained from the datum (corresponding to the water level of the river). It is equivalent to the minimum operating level for surface reservoirs. The minimum operating level is the water level below which water cannot be drawn from a reservoir to meet a target draft (Basson *et al.*, 1994). For groundwater aquifers, minimum allowable groundwater head is used to constrain the abstractions from the aquifer to avoid its depletion particularly during dry periods. An ideal datum to allow yield analysis for the entire groundwater resource unit would be the bedrock. However, information on depth to bedrock is not available in most parts of South Africa particularly in rural areas. In the absence of this information, Nzhelele River was set as the datum to enable yield analysis in this study.

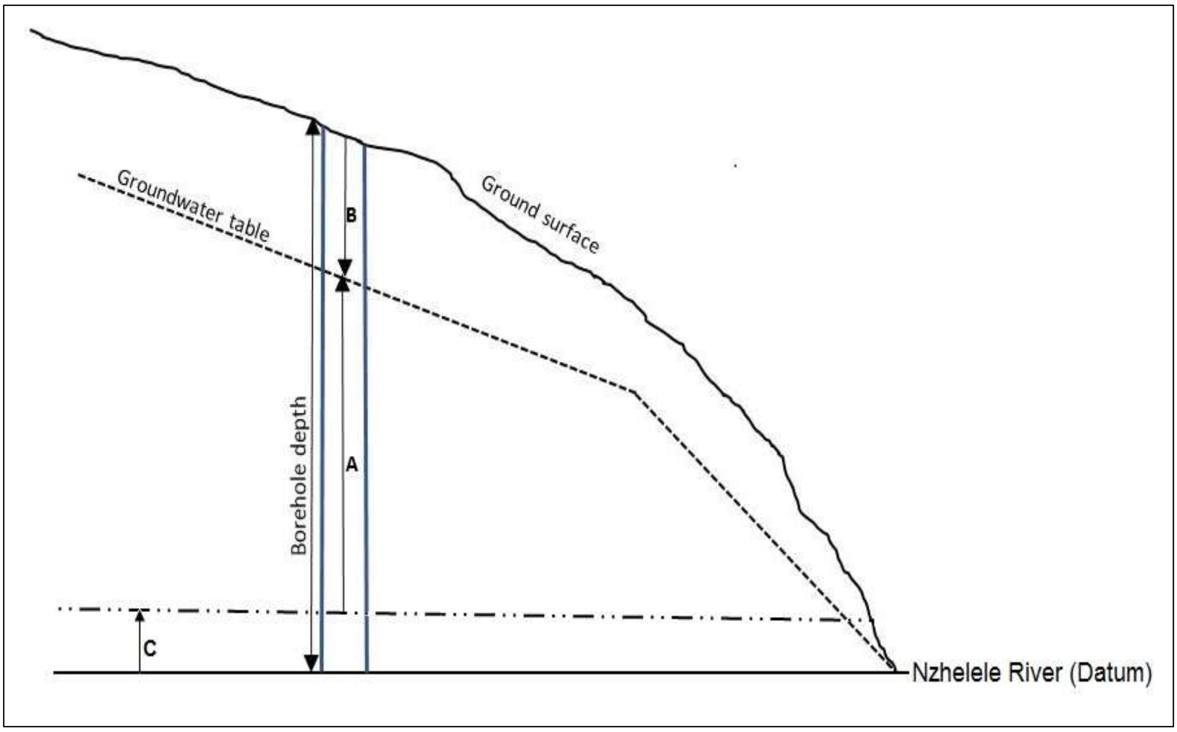


Figure 4.6: Configuration of the groundwater resource unit for groundwater base yield analysis and development of operating rules

The development of risk-based groundwater operating rules followed procedures used to derive probabilistic operating rules for surface water reservoirs. The method followed is described in Ndiritu *et al.* (2017) and is widely used in South Africa (Basson *et al.*, 1994). This approach was found to be suitable for this study because it is a risk-based and provides a detailed evaluation of supply reliabilities to multiple users. The approach also uses SCE-UA optimiser that allows for optimisation of non-linear functions and hence assisted in optimising operating rules for groundwater system.

For surface water systems, operating rules are typically derived from simulations considering reservoir storage levels in terms of percentage of full supply capacity. However, groundwater head was used in this study instead of storage level. This enabled application of procedures used in surface water systems in groundwater systems. It also allowed assessment of the results based on analogy with the simulation behaviour of surface water reservoirs. Groundwater head is equivalent to reservoir storage level for a surface water reservoir and was defined as the depth of water from the set minimum allowable groundwater head to the upper surface of groundwater (groundwater table).

Stochastic groundwater yields were determined by monthly water balance simulation of the GRU based on the calibrated GW-PITMAN model using 100 stochastic sequences of rainfall, evaporation obtained from the VLB stochastic generator. For each run, simulations were conducted starting with a low yield and gradually increasing this yield until the minimum allowable groundwater head got violated in a single period during the simulation. The groundwater heads for the upper and lower slopes were computed from Equation 4.47 and 4.48, respectively. The average groundwater head (GWH_{av}) for the groundwater resource unit was calculated based on average groundwater heads for the lower and upper slopes using Equation 4.49.

$$GWH_l = \frac{1}{2} H_c \tag{4.47}$$

$$GWH_u = \frac{H_c + H_u}{2} \tag{4.48}$$

$$\begin{aligned} GWH_{av} &= (pls \times GWH_l) + (1 - pls \times GWH_u) \\ &= \frac{1}{2}\left[(pls \times H_c) + (1 - pls)(H_c + H_u)\right] \end{aligned} \tag{4.49}$$

GWH_l and GWH_u are the groundwater heads for the lower and upper slopes, respectively. H_c is the height of water table at the connection between lower and upper slope, and H_u is the height of the water table at end of the upper slope. The average groundwater head was used to test if the specified minimum allowable groundwater head has been reached. The model was coded to automatically increase or decrease the target draft (abstractions) to ensure that the minimum allowable groundwater head is exceeded only once in the simulation period.

Information obtained from two recently drilled boreholes in Siloam Village was used to decide on the maximum value of initial groundwater head to use in the simulations. These boreholes were ideal as they are approximately located in the center of the study area and are therefore expected to record average groundwater levels of the GRU. Thus, the boreholes were considered to record average groundwater levels that can be representative of the simulations from GW-PITMAN for the groundwater resource unit (see Figure 4.5). Considering that Siloam Borehole 1 and Nzhelele River are located at altitudes of 830 and 780 m, respectively (Figure 4.7), the maximum initial GWL was considered to be 50 m. This corresponds to the maximum altitude of Siloam Borehole 1 from the river altitude (780 m). Table 4.3 indicates the initial groundwater heads selected for the base yield analysis and corresponding groundwater levels.

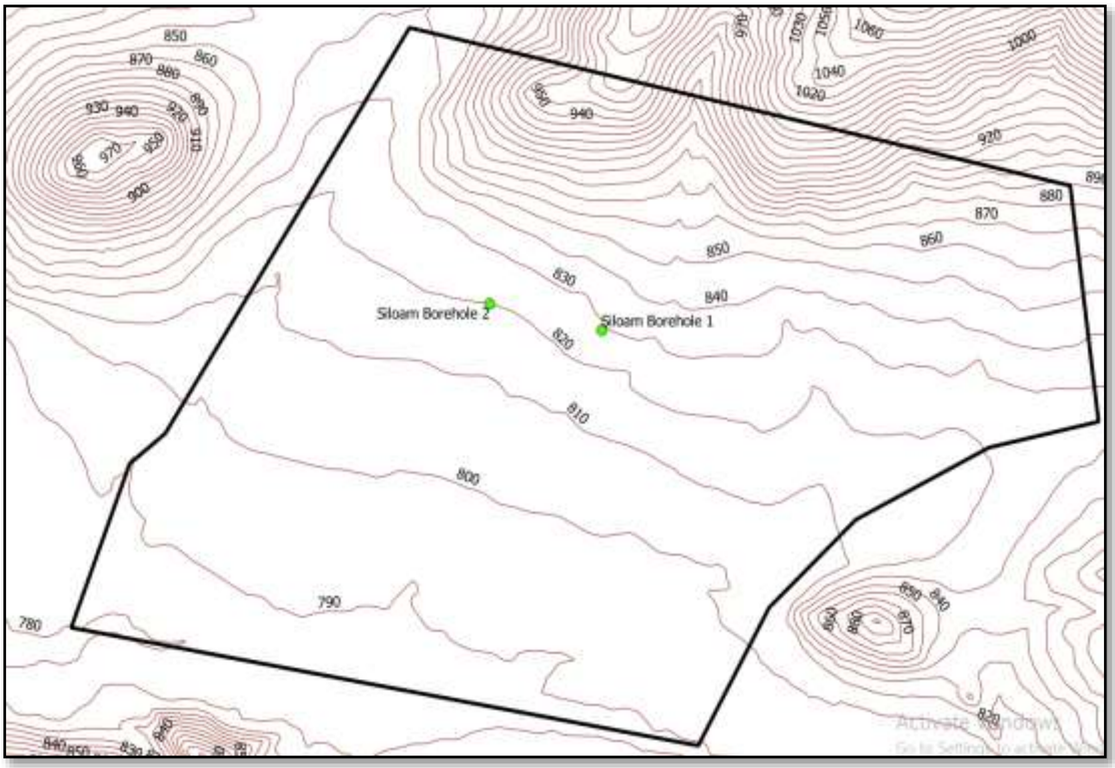


Figure 4.7: Recently drilled boreholes in Siloam Village

Table 4.3: Selected initial groundwater heads and corresponding groundwater levels

Initial groundwater head	Groundwater level
50	0
45	5
40	10
35	15
30	20
25	25
20	30
15	35
10	40
5	45
0	50

The groundwater base yield simulations were run for 5 year stochastic sequences with minimum allowable groundwater head of 5 m. This was selected on the basis that 5 year stochastic sequences are mostly used for short term operational analysis for surface water reservoir systems. Draper (2001) noted that predictions beyond 5 years may have little value as the recurrence of wet years fills reservoirs to capacity, and hence increase reservoir yield within a 5 year period. In addition, FAO (2004) reported that drought events in Limpopo River Basin had an average frequency of occurrence of once every four or five years, and the frequency is expected to increase in the near future due to changes in climatic conditions. Drought periods are mostly followed by floods which recharge groundwater aquifers. For short term operation (with a period of 5 years), it is therefore expected that the aquifer will be recharged to its capacity before the following drought event. This recharged water may last until the end of a dry period if managed sustainably. This further indicates that 5-year long sequences can be regarded as suitable for use in developing the groundwater operating rule curve as used in surface water systems.

For a given initial groundwater head as specified in Table 4.3, 100 simulations were run to generate a time series with 100 base yields. The 100 groundwater base yields obtained for each initial groundwater head were then ranked in ascending order and the probability that any specified base yield is not exceeded was computed using the Weibull plotting position formula (Equation 4.50).

$$p = \frac{m}{n+1} \tag{4.50}$$

p is the probability that the base yield is not exceeded, m is the rank of the base yield and n is the total number of the base yields. The probability of the base yield being exceeded in any year was considered to be *1-p*. The recurrence interval (R_i) (Equation 4.51) was used to quantify the risk of failure to meet the demands in any given year. N represents the planning period (length of sequence) in years (WMO, 2009).

$$R_i = \frac{1}{1-(p)^{1/N}} \tag{4.51}$$

For the purpose of generating practical and implementable risk-based groundwater operating rules, the groundwater level was considered instead of the groundwater head. Groundwater level is the variable that can be measured in practice to determine the status of the groundwater system and facilitate implementation of the developed groundwater operating rules. Developing groundwater operating rules based on groundwater levels aided in simplifying the operating rule curves for ease of implementation by the borehole operators. This means that the task of borehole operators will only involve measuring the groundwater level in the borehole and deciding on the volume of water to be allocated for each user category based on the developed groundwater operating rule curves.

The groundwater base yield analyses were run starting from the decision date of January to obtain the yield that can be available for allocation on this date. The decision date is the date on which the operating rule curve is used to decide on water to be allocated for various uses for a period of one year (starting from a decision date) (Ndiritu *et al.*, 2017). For surface water systems, the decision date is often at the end of the rainy season when most of the rainfall has been received. Considering that groundwater movement, fluctuations and losses due to evaporation are not as much influenced by annual hydrological variations as compared to those of surface water, decision month like January can suffice. The model was coded with January as the decision month.

This study considered that water is typically used for domestic and productive purposes in most rural areas of South Africa as noted by de Mendiguren and Mabelane (2001). Domestic water use includes water used for human consumption (drinking, cooking, personal hygiene, and household cleaning) purposes. Productive water use refer to use of water for economic purposes, in order to ensure food security or to generate income (Ladki *et al.*, 2004). These activities are highly dependent on the availability of secure and reliable water supplies. Vegetable gardens, cattle farming, traditional beer making, hair salons and brick making are examples of some of the productive water uses that serve as sources of income in rural areas (de Mendiguren and Mabelane, 2001). Thus, providing water for these water uses may improve livelihoods of people in rural areas.

The domestic water demand of 189 *l/c/day* for dwellings with house connections in Nzhelele area, where the study area falls, was obtained from DWA (2011). de Mendiguren and Mabelane (2001) estimated productive use to be 40 *l/c/d* for villages with domestic use of 25 *l/c/d*. In absence of data on productive water use in the study area, the water demand for productive use equivalent to domestic water use of 189 *l/c/d* was then calculated by relating it with the estimated value of 40 *l/c/d* for villages with domestic use of 25 *l/c/d* from de Mendiguren and Mabelane (2001) through cross multiplication. This resulted in water demand for productive use of 302.4 *l/c/d*. This enabled determining the percentage of water that can be supplied above the domestic water demand in Siloam Village. This was essential to assess the extent to which groundwater can be used to improve livelihoods in rural areas. The total annual water demands for domestic and productive water uses were calculated based on the population of 2295 for the year 2018 and were converted from *liters per day* to m^3/annum.

The water demands for domestic and productive water uses were divided into specified priority classes (which represent different assurance levels) to ensure that the demands are supplied at required levels of assurance following DWAF (2008b) and DWA 2010). The recommendation was based on the fact that it is not economically feasible to develop and operate water resource systems to meet all the demands at all times in most areas of South Africa (Basson *et al.*, 1994) due to the semi-arid nature of the country which limits water availability. Priority classes therefore determine the portions of water demands that should be supplied at different levels of assurance. Priority classification assisted in achieving the objective of developing operating rules that reduce the risk of failure to supply the demands at required levels of assurance even during dry periods. Three priority classes representing low, medium and high assurance levels, respectively were selected in this study. The low, medium and high levels of assurance corresponded to average recurrence intervals of shortages of 1:10, 1:50 and 1:100 years, respectively, as shown in Table 4.4.

Table 4.4: Priority classification for Siloam demands in percentages

User	**Level of assurance (recurrence interval)**			**Total**
	Low (1:10)	**Medium (1:50)**	**High (1:100)**	
Domestic	10	40	50	100
Productive	30	50	20	100

When deriving operating rules for surface water reservoirs, priority classification is typically adopted after discussions with stakeholders (water users) (DWAF, 2008b; DWA, 2010). However, in this study a theoretical priority classification was developed giving preference to domestic water use, to facilitate development of operating rules. The percentages of domestic, productive, and combined domestic and productive water demands that can be supplied at 1:100 assurance level at varying groundwater levels were computed using Equation 4.52. This was done to determine the ability of the system to meet the demands before curtailments are introduced.

$$\%\ Supply = \frac{AY}{D} \times 100 \qquad (4.52)$$

where % Supply is the percentage of water demand that can be supplied at 1:100 assurance level*, AY* is the available yield at 1:100 assurance level, *D* is the water demand. The demands were cumulatively superimposed on the groundwater base yield curves, starting with the demand portion with the highest assurance (1:100), to determine the ability of the groundwater system to meet the demands at any given groundwater level. For the system to be considered to be able to supply the total demands, the cumulative demands should not exceed the base yield curve. If base yield curve was exceeded the system was considered not to have enough water to supply the total demands and curtailment was introduced, starting with low assurance demands. Superimposing the cumulative demands is aimed at assessing whether the system can meet the cumulative demands without curtailment and the groundwater level and level of assurance of supply at which the curtailment will be required. The curtailed volume of water that can be allocated was calculated as:

$$V_{CUR} = \frac{Y_{LAS}}{D_T} \times D_{LAS} \tag{4.53}$$

V_{CUR} is the curtailed volume of water to be allocated, Y_{LAS}= yield at a given level of assurance, D_T is the total demand for domestic and productive water uses and D_{LAS} is the demand at given level of assurance.

4.7 Summary

This chapter described methods used in the study as well as justifications for selection of each of the methods. The NPR model was selected and used to extend rainfall data while the coupled OE-NLHW model was selected to infill and extend groundwater levels. The GW-PITMAN model was extended to enable its calibration based on groundwater levels using a hybrid manual-automatic approach. This enabled generation of groundwater levels for the groundwater resource unit based on the GW-PITMAN model. Groundwater levels were required in derivation of simple groundwater operating rules. To the knowledge of the author, this is the first attempt to calibrate GW-PITMAN model using groundwater levels and is therefore a significant contribution of this study.

Procedures previously applied for probabilistic surface water reservoir operation were used to obtain stochastic groundwater base yields and to develop risk-based groundwater operating rules. Measurements in a surface water reservoir start from the bottom of the reservoir to its upper surface while in groundwater reservoir they start from the ground surface to the surface of groundwater table. It was therefore essential to reconfigure the interpretation of the groundwater reservoir to allow generation of operating rules based on methods applied in surface water reservoir. Groundwater head which is equivalent to reservoir level for a surface water reservoir was used in the groundwater stochastic base yield- analysis. Groundwater levels were used in the development of risk based groundwater operating rules to enhance their practical applicability. The operating rules were also developed considering the priority classification of the demands to assist in reducing the risk of failure to supply the demands at required levels of assurance.

CHAPTER 5: GROUNDWATER RESOURCE UNIT AND HYDROGEOLOGICAL CONCEPTUAL MODEL FOR THE STUDY AREA

5.1 Preamble

This chapter presents the results of delineation of the groundwater resource unit, interpretation of its hydrogeological conceptual model and implication to groundwater flow and storage. The results on aquifer characterisation based on pumping test data are also presented. The hydrogeological conceptual model and aquifer characterisation were useful when conceptualising the groundwater balance computations while the hydraulic characteristics were essential inputs in the GW-PITMAN model.

5.2 Groundwater resource unit for Siloam Village

Figure 5.1 shows groundwater resource unit delineated from DEM and groundwater divide. The upper boundary of the groundwater resource unit was considered as a no-flow boundary. The mountain forms a groundwater divide at this boundary. Groundwater divides are frequently simulated as no-flow boundaries in groundwater flow models to limit the areal extent of the system being analysed (Reilly, 2001). The relationship between groundwater levels and topography (Figure 5.2) shows that topography controls groundwater levels in the study area and hence groundwater divide can be defined as a closed boundary. This is because in areas where topography controls groundwater levels, groundwater on each side of the divide moves away from the divide and no flow crosses the divide. Figure 5.3 has been plotted using once-off groundwater levels data for boreholes in Table 5.1 obtained from the National Groundwater Archives.

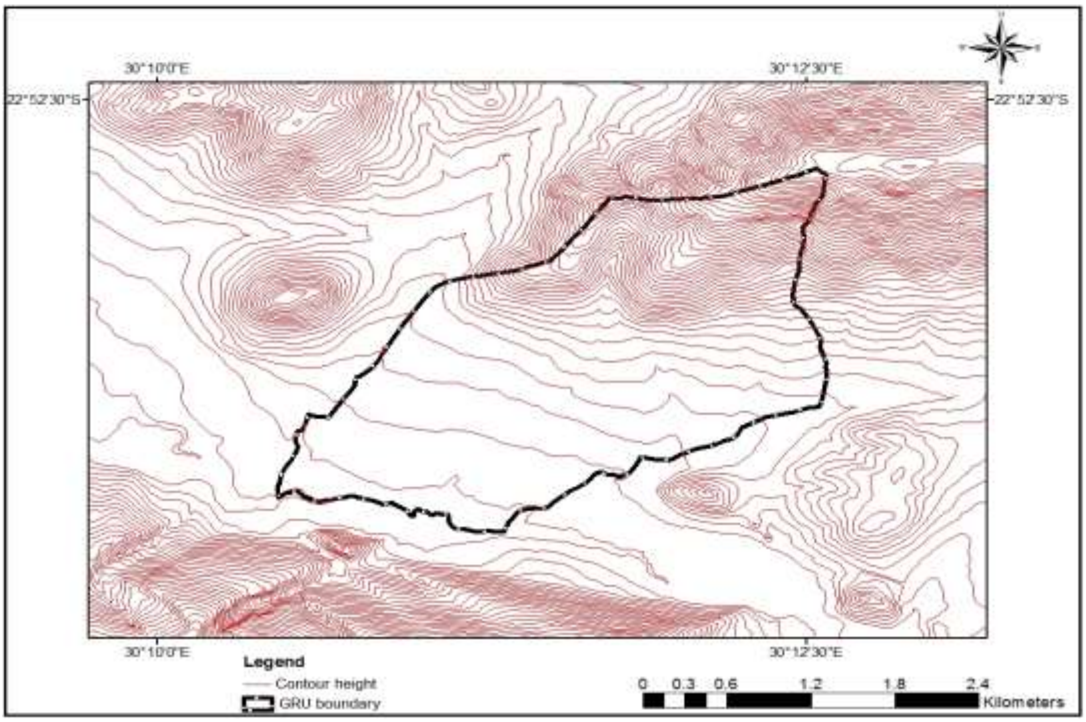


Figure 5.1: Delineated groundwater resource unit for Siloam Village

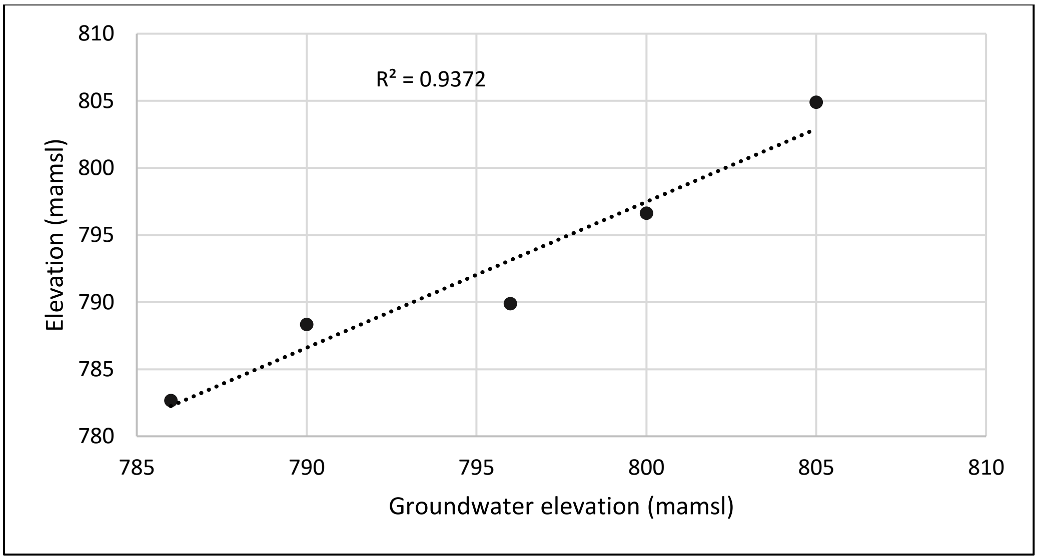


Figure 5.2: Relationship between groundwater levels and topography for 5 boreholes from National Groundwater Archives

Table 5.1: Topography and water level for 5 boreholes from National Groundwater Archives

Borehole	Topography (mamsl)	Groundwater level (m)	Groundwater elevation (mamsl)	Date of measurement
H27-0051	796	6.13	789.87	1998/02/07
H27-0052	800	3.38	796.62	2003/02/14
H27-0053	786	3.35	782.65	1998/02/11
H27-0168	790	1.68	788.32	1998/02/09
H27-0138	805	0.12	804.88	1998/02/03

Interpretation of magnetic data showed low magnetic anomalies/intensities (blue-green) colour (Figure 5.3) around the study area. This indicates that linear structures based on surface magnetic data are not highly pronounced and cannot be defined as no-flow boundaries. Surface magnetic data could therefore not be used in the delineation of the groundwater resource unit. Thus, detailed geophysical investigations are still required to identify and verify the extent and depth of the linear structures. Earlier studies done in Siloam Village (for example Nyabeze *et al.*, 2010; 2011a, b) focused on detailed geophysical investigations only along the Siloam hot spring, thus such data is not available for the rest of the Siloam Village. DEM also showed linear structures on the surface which were used in the delineation of the rest of the groundwater resource unit boundaries. Thus, these boundaries are open flow boundaries.

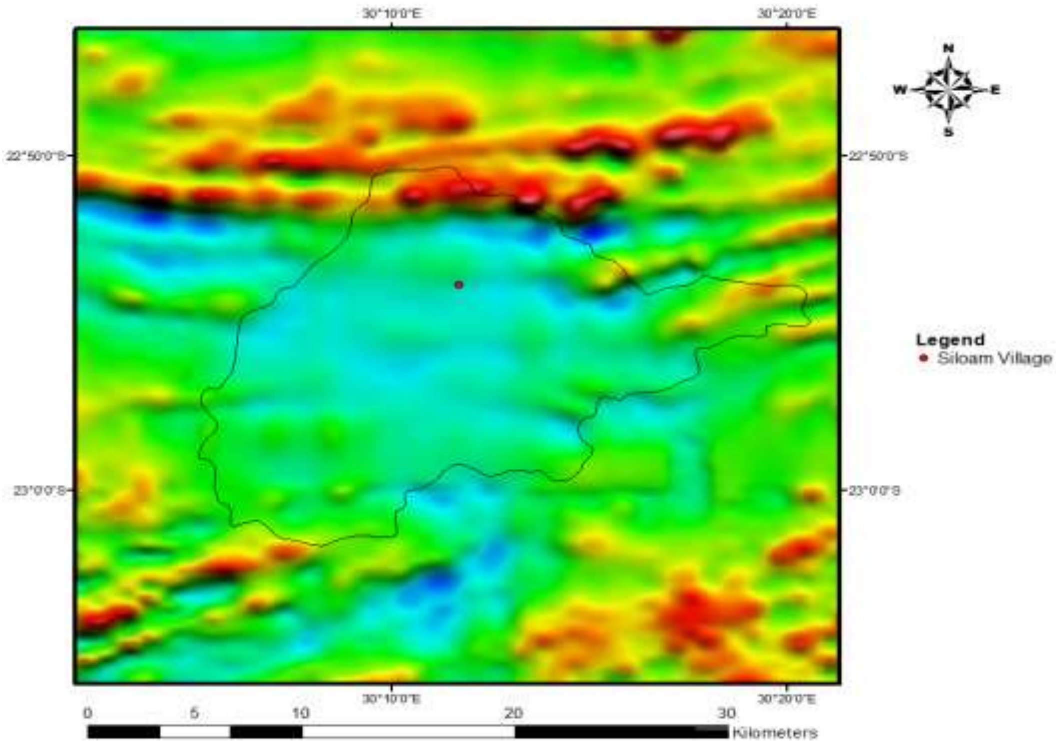


Figure 5.3: Magnetic map for quaternary A80A including Siloam Village

5.3 Geological cross-sections and hydrogeological conceptual model

Figure 5.4 shows cross-section lines A-B, C-D and E-F from which geologic cross-sections were based. Cross-section lines A-B, C-D and E-F were drawn on the map showing geologic formations of the study area. This map has been extracted from the 1:250000 geologic map series 2230 for Messina. The cross-section lines A-B, C-D and E-F were selected because they are perpendicular to major geological features/structures, and they cut across the study area and Nzhelele River. The geologic formations include Sibasa Basalt, Fundudzi and Nzhelele formations and have been described in section 3.2. Figure 5.5 shows the geological cross-sections A-B, C-D and E-F. A closer observation on geological map shows that bedding dip to the north direction with 30, 27 and 20 degrees orientation (Figure 5.4).

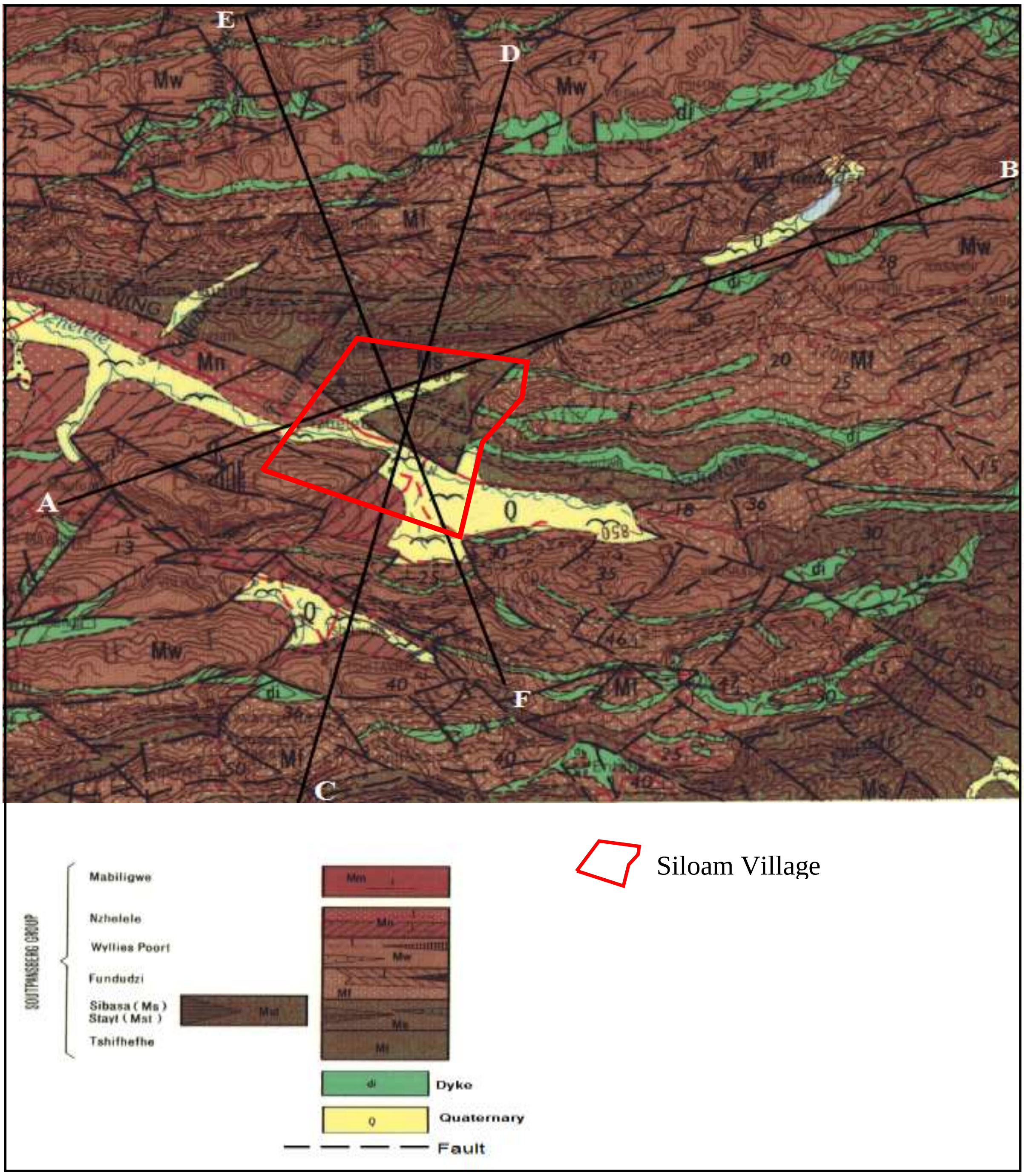


Figure 5.4: Cross-section lines A-B, C-D and E-F in the geological map

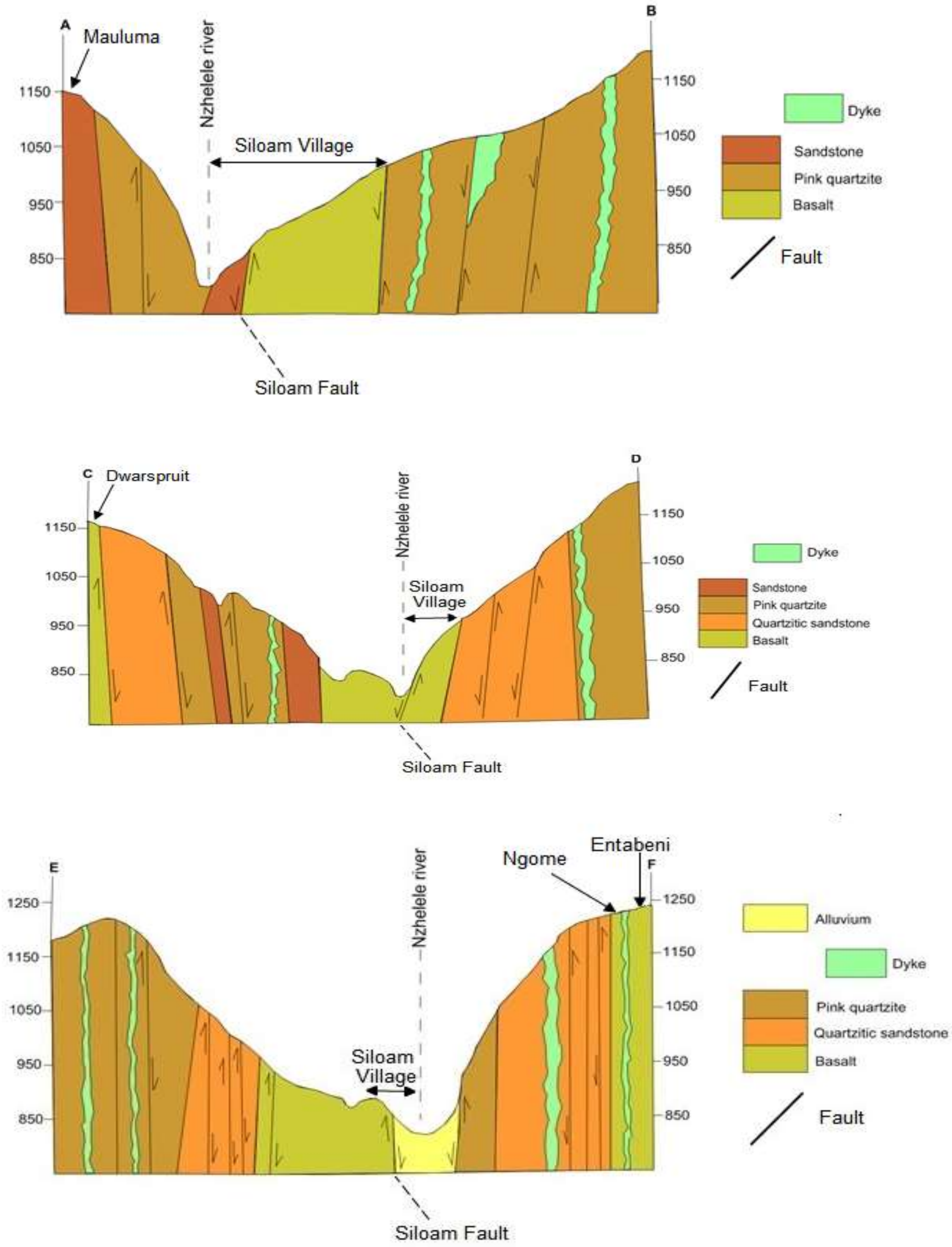


Figure 5.5: Geologic cross-sections A-B, C-D and E-F

Siloam fault (Figure 5.5) which cuts across the study area plays a significant role in controlling groundwater flow and storage. Faults create linear zones of high secondary porosity which may act as preferred channels of groundwater flow (Singhal and Gupta, 2010). Fractures exposed to the surface can also create preferential flow paths, which short circuit the path to the water table (Holland, 2011). The fractures visible on the geological map together with Siloam fault therefore create preferential flow paths of water into the aquifer. It is important to note that the depth and thickness of the fractures is unknown and hence knowledge of the extent to which they influence groundwater flow and storage is limited. Dippenaar *et al.* (2009) reported that in fractured aquifers, flow does not inherently occur in the direction of the fracture alone but it may be restricted to distinct channels within the fracture plane.

Diabase dykes are present within the vicinity of the study area (Figures 5.4 and 5.5). Dykes are prominent landform features that concentrate groundwater flow and storage (Kebede, 2013). They can act either as good conductors of, or as barriers to, groundwater flow depending on the intensity of fracturing associated with the dykes, and their trends in relation to the hydraulic gradient (Babikera and Gudmundsson, 2004). Morel and Wikramaratna (1982) reported that if dykes contain more fractures than the host basement rock, they improve the potential yield of the aquifer. A study by Holland (2012) showed that dykes were important water-bearing features in the Limpopo Plateau. Dykes are also likely to serve as water-bearing features in the study area. Information on thickness and depth of the dykes is not available. Thus, it is not possible to know how deep they are unless exploration drilling is carried out where core samples can be taken for *in situ* testing of hydraulic properties or analyses in the laboratory, which are not within the scope of this study. This also indicates that the extent to which dykes influence groundwater flow and storage is unknown.

Wright (1992) stated that basement aquifers have low permeability and the main groundwater flow systems are relatively localised. In basement aquifers, groundwater occurs in secondary porosity/fractures caused by weathering and fracturing (Adams *et al.*, 2004). The main flow paths in fractured rocks are along joints, fractures, shear zones, faults and other discontinuities (Singhal and Gupta, 2010). Groundwater flow in fractured basement aquifers is only possible along preferred pathways due to heterogeneity in their hydraulic properties (for example, porosity and permeability) (Mohamed *et al.*, 2015). Fractures serve as primary sources that store and allow movement of water in hard rock areas (Sharma and Baranwal, 2005). Since the study area falls within severely fractured Soutpansberg Group, groundwater is likely to be stored in fractures and is expected to flow through preferential pathways. Thus, groundwater flow mostly occurs through interconnected fractures in the study area.

In crystalline aquifers regional flow occurs within the major interconnected fracture systems, while the main groundwater flow systems are relatively localised to the zones between recharge on watersheds to discharge by run-off or evaporation at valley bottoms (Figure 5.6) (Holland, 2011). As explained in section 5.2, groundwater resource unit is bounded by a mountain divide, ephemeral rivers and Nzhelele River. There is also presence of structures that control groundwater flow which are faults and dykes. Based on this, the groundwater resource unit for Siloam Village can therefore be conceptualised as a system whose main groundwater flows are localised (i.e. a localised groundwater system). This is also justified by the relationship between groundwater levels and topography (Figure 5.2) which indicated that topography controls groundwater levels in the study. The groundwater system for Siloam Village can therefore be comparable to the component circled in red in Figure 5.6 within the simplified flow system for crystalline basement terrain.

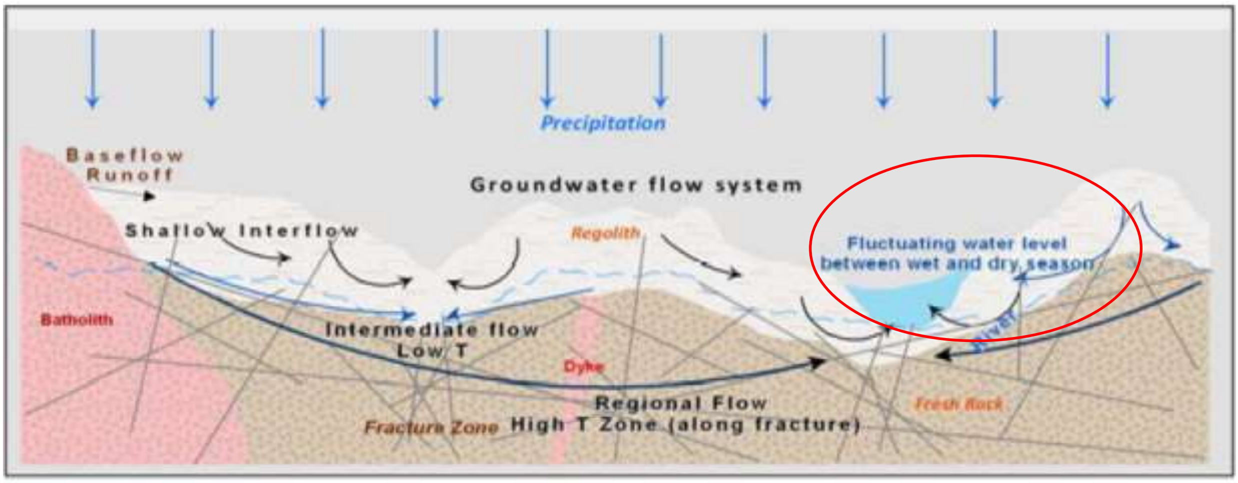


Figure 5.6: Conceptual model for simplified flow system for crystalline basement terrain (Holland, 2011)

Building a conceptual model is an iterative process that can identify gaps in the data, which can be improved with further data gathering. It is expected that there will be continuous updating of the hydrogeological conceptual model of the study area as more data becomes available. The improvement of the conceptual model is important in order to increase understanding of groundwater system and to develop effective planning and control measures. However, it is important to emphasise that fracture network analysis and exploration drilling are required for detailed understanding of the groundwater systems in the study area.

5.4 Groundwater resource unit aquifer characterisation

Non-linear least squares fitted statistics are shown in Table 5.2. Mean residual errors from data for all boreholes ranged from -0.87 to 3.04 m (Table 5.2). These values were all closer to zero, except for borehole H27-0290. This shows that fitted aquifer test solution for the boreholes had least residual errors since mean residual errors closer to zero show less estimation errors. Negative residual means (boreholes H27-0002 and H27-0138) imply that the sum of estimated drawdowns were higher than those of observed values since numerator component of the residual mean equation (Equation 4.3) indicates the sum of the difference between observed and estimated values. H27-0290 had the highest mean residual error, standard deviation and variance. Thus, H27-0290 had residual errors which were spread out over a wide range of data. H27-0052 had the best model fit as shown by low mean, standard deviation and variance. The fitted models for most of the boreholes generally had low statistics indicating least model errors and good model fits. The identified aquifer test solutions can therefore aid in accurate estimation of hydraulic characteristics.

Table 5.2: Non-linear least squares fitted statistics

Borehole	σ_e (m)	σ^2_e (m^2)	$\bar{e}$ (m)
H27-0002	2.09	4.38	-0.10
H27-0052	0.19	0.44	0.04
H27-0136	7.33	2.71	0.71
H27-0138	11.2	125.50	-0.87
H27-0165	5.05	25.53	0.55
H27-0168	1.72	1.31	0.01
H27-0290	62.91	3957.6	3.04

Graphical fits of observed and estimated drawdowns, particularly on log-log scale, were comparable for the majority of the boreholes (Figures 5.7 to 5.10). Good fit of the model to the data suggests a sound conceptual model and increased confidence in the estimated parameters as explained in Holland (2011). Leaky aquifer model was the most commonly identified model indicating that the study area is dominated by leaky aquifer (Figures 5.7 to 5.9 and Table 5.3). Parsons (2004) noted that most aquifers in South Africa are semi-confined. In a leaky aquifer, water comes from storage of the pumped aquifer and the leaky confining unit, during early time of pumping (Kuniansky and Bellino, 2012). The effect of leakage is that the drawdown curves flatten and eventually become constant when steady state is reached (Hemker and Randall, 2013). This has been displayed in log-log graphs of H27-0002, H27-0136 and parts of H27-0165 and H27-0168.

Fractured double porosity behaviour was identified in borehole H27-0052 (Table 5.3). In a double porosity aquifer, matrix blocks have low permeability and high (primary) porosity and storage capacity, only the fractures produce flow directly to the well and matrix blocks act as a source, which feeds water into the fractures (Holland, 2011). Renard *et al.* (2009) explained that early pumping depletes the first reservoir (fractures, for example), which is then partly compensated by a delayed flux provided by a second compartment of the aquifer (second/intermediate stage) and equilibrium is reached at the late time (last stage). A single horizontal fracture model was identified in borehole H27-0138 (Table 5.3). Single fracture model assumes that the pumping well is intersected by a single horizontal fracture. In a single fracture model, the flow towards the well takes place in the fracture only and it is parallel (i.e. linear in the fault) (Holland, 2011). At intermediate times water is supplied by the fracture and matrix.

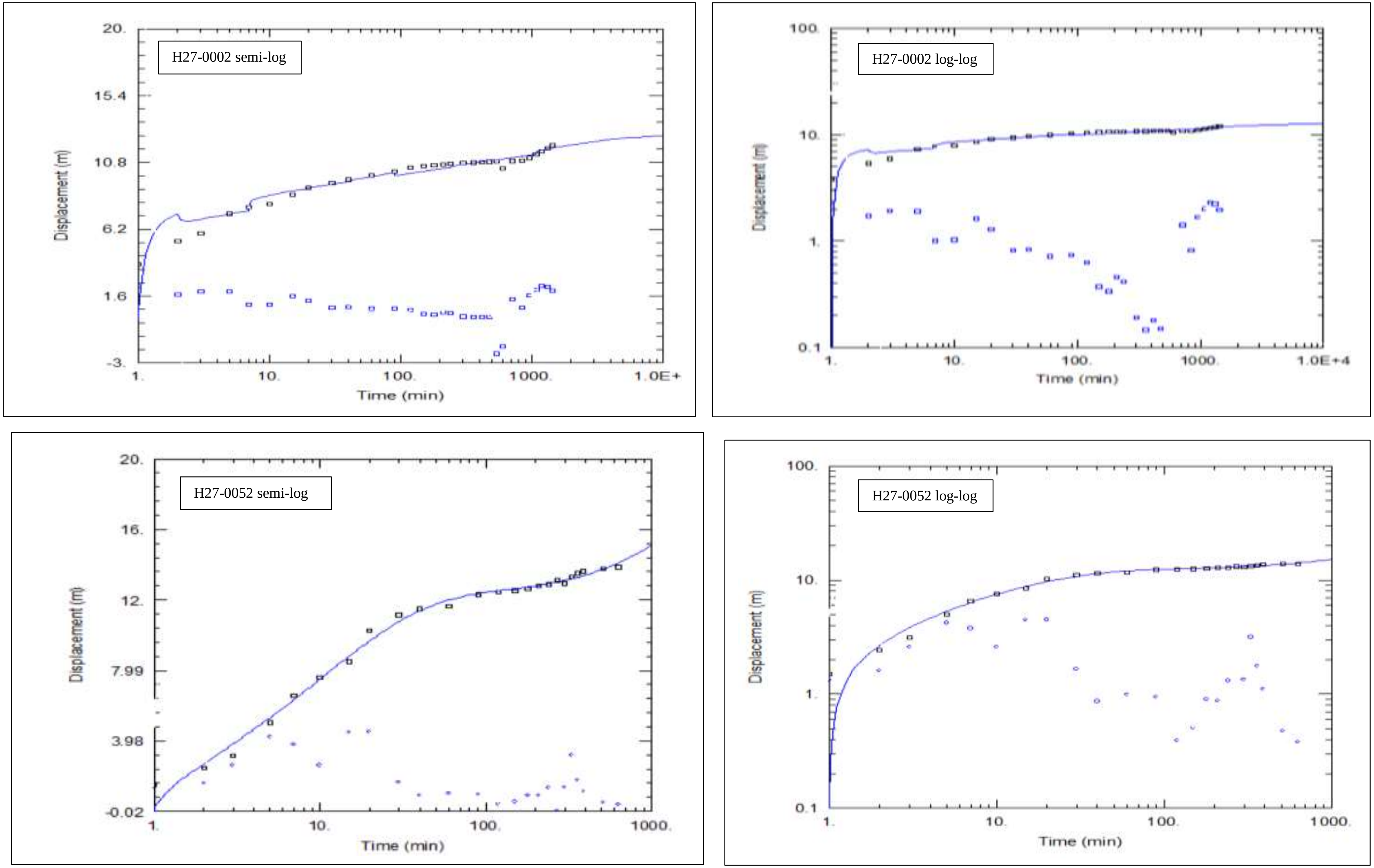


Figure 5.7: Semi-log and log-log diagnostic plots for boreholes H27-0002 and H27-0052

Legend: Derivative Observed drawdown Fitted

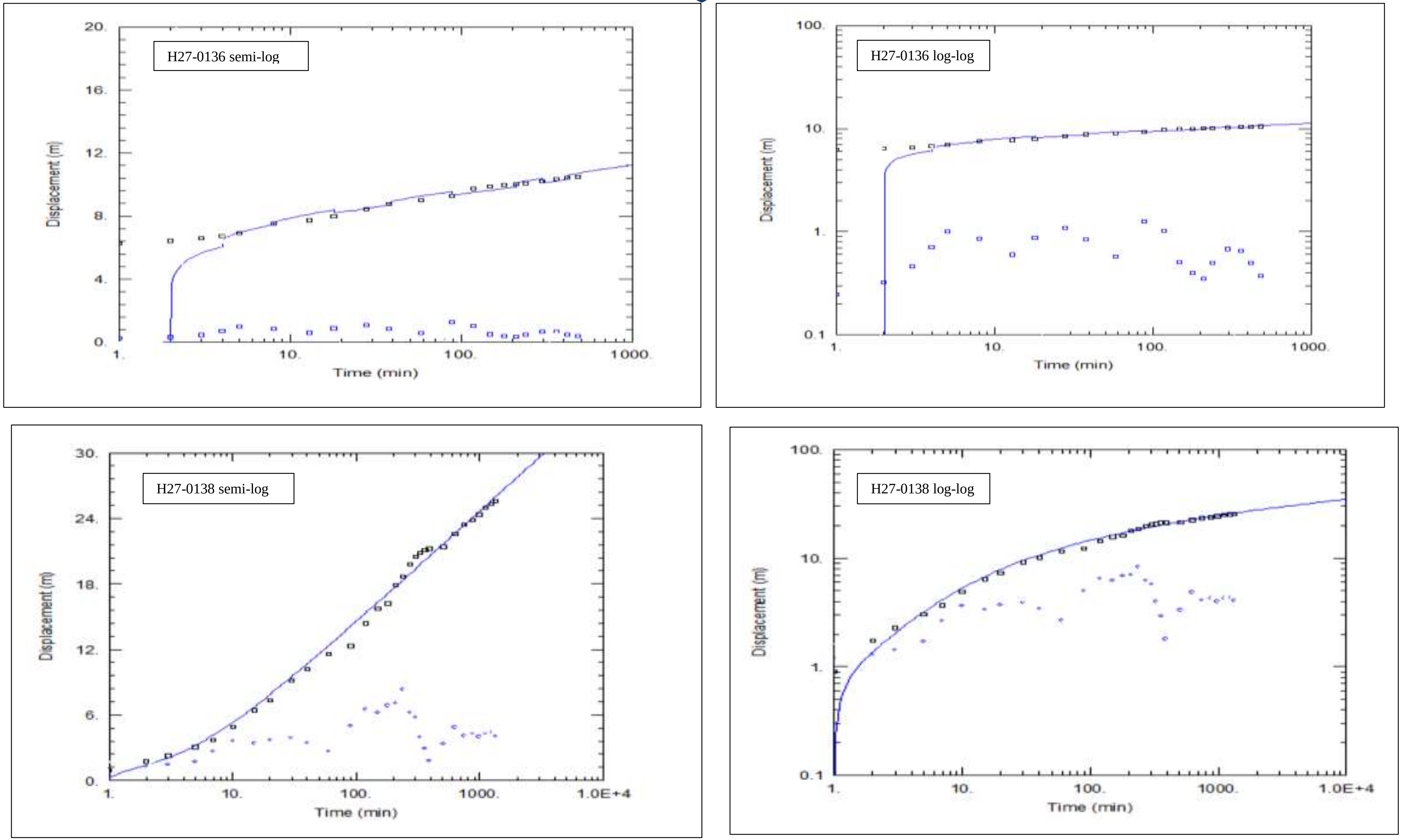


Figure 5.8: Semi-log and log-log diagnostic plots for boreholes H27-0136 and H27-0138

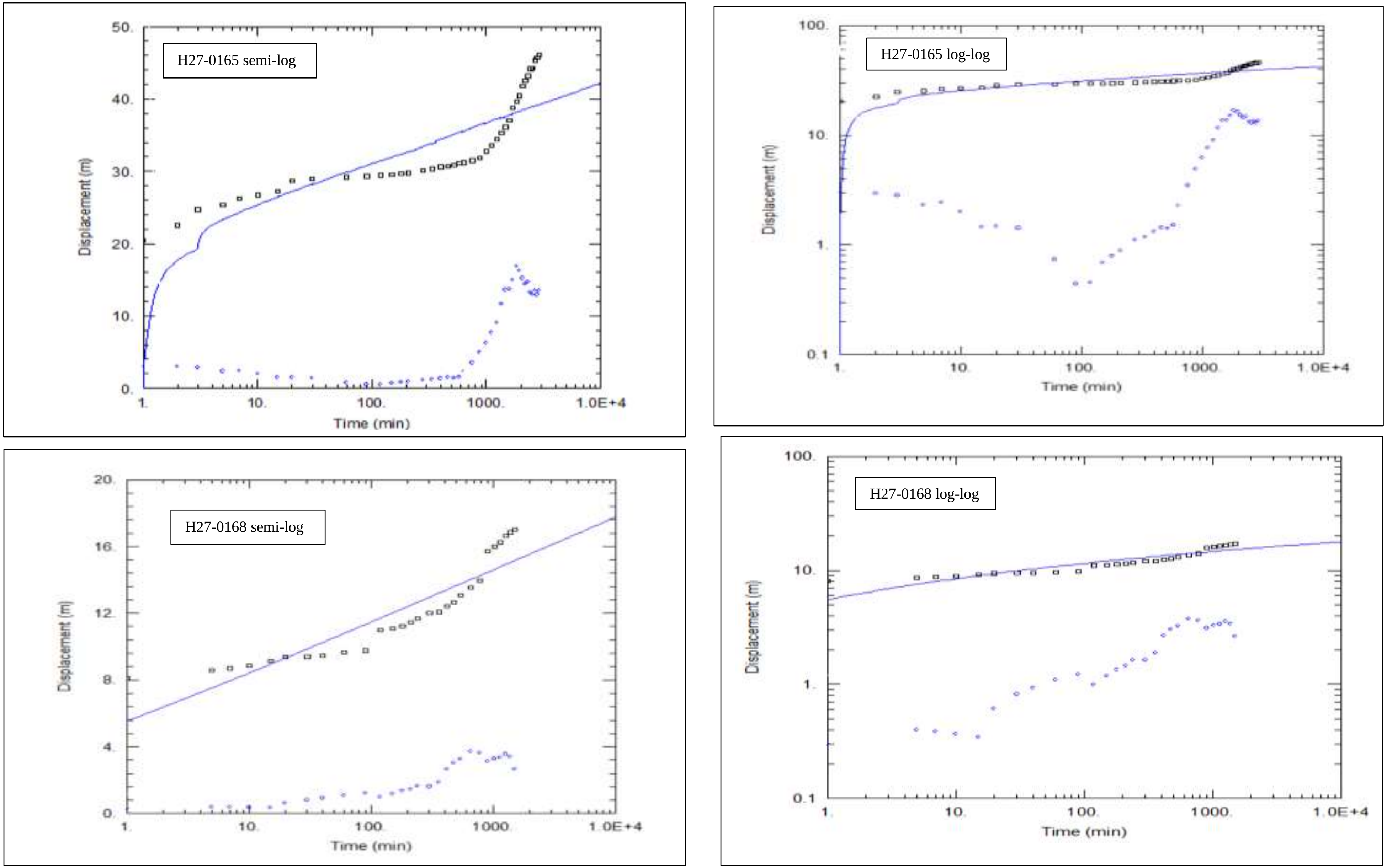


Figure 5.9: Semi-log and log-log diagnostic plots for boreholes H27-0165 and H27-0168

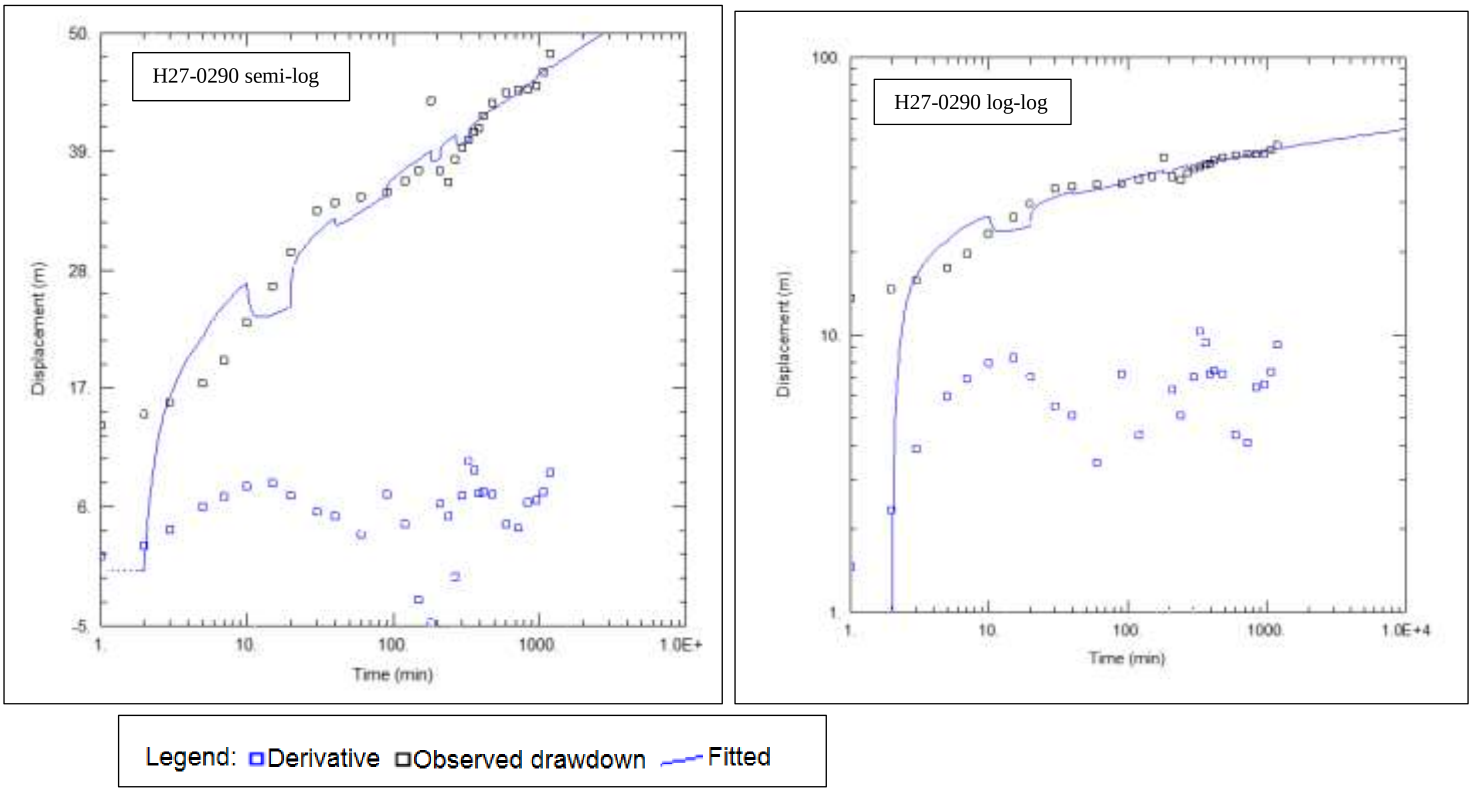


Figure 5.10: Semi-log and log-log diagnostic plots for borehole H20-0290

All boreholes are characterised by fluctuating derivative curves showing high presence of fracture dewatering in the study areas (Figures 5.6 to 5.9). If fractures were dewatered during the test, the derivative curve in the log-log plot drops at the position of the fracture and it rises after that (Van Tonder *et al.*, 2002). According to Lasher (2011), when the pressure in the fracture is released, it causes a change in rate of water level, shown by a drop in the derivative; the fracture contribution then reaches equilibrium with the surrounding matrix, showing unconfined aquifer behaviour causing the derivative to increase again. Hammond and Field (2014) also noted that the derivative formed a sharp peak, probably due to a rapid decline related to dewatering of the first fracture followed by recovery due to leakage in a leaky aquifer in New Hampshire.

The results of this study show that the aquifer in the study area is characterised by fracture dewatering. Fracture dewatering has an effect on groundwater levels depending on the abstraction rate (van Tonder *et al.*, 2002). Jayawardena and Sarathchhandra (1995) also associated drying out of surface water resources to the dewatering cone extending along a linear rock fracture in Sri Lanka. Fracture dewatering should be avoided, whenever possible, because of the danger of mineral precipitation that can cause fracture and well clogging (van

Tonder *et al.*, 2002). The latter study noted that caution should be placed when assigning sustainable yields to boreholes in aquifers characterised by fracture dewatering to avoid well clogging. Thus, operating rules are required in such areas to manage allocation and use of groundwater.

Storativity values for boreholes H27-0002, H27-0136, H27-0165 and H27-0168 (Table 5.3) fall within the range of typical values of 0.00001 to 0.001 for confined and leaky aquifers specified in Hall and Chen (1996). Storage coefficient or storativity indicates the ability of an aquifer to store water (Sen, 2009). Thus, the aquifer within the vicinity of these boreholes has low potential to store water since they have low storativity values. This is typical of fractured basement aquifers since their storage is dependent on the presence of fractures and their connectivity. Boreholes H27-0138, H27-0052 and H27-0290 have relatively high STOR values of 0.006, 0.008 and 0.068, respectively. These boreholes are also on or within the vicinity of faults. Borehole H27-0290 which is located on a fault (Figure 3.7) has the highest value of STOR. Holland (2012) reported that proximity of lineaments increases borehole productivity within Limpopo Province. Subsequently, increased storativity can be linked to the presence of faults. Jeanne (2012) also noted that micro and macro-fractures lead to increase in storativity. High storativity values indicate potential for high water storage by an aquifer (Watson and Burnett, 1995; Dhungel and Fiedler, 2016). The aquifers where boreholes H27-0138, H27-0052 and H27-0290 are located have high potential to store water.

Table 5.3: Hydraulic characteristics obtained in the study area

Borehole	Lithology	Saturated aquifer thickness (m)	Aquifer Model	Solution	Storativity	Transmissivity (m^2/day)	Hydraulic conductivity (m/day)
H27-0002	Tuff, arenite, basalt	11.96	Leaky	Moench (case 1)	0.0003	5.50	0.460
H27-0052	Arenite, basalt, shale	17.23	Fractured (double porosity)	Moench w/slab blocks	0.0080	2.10	0.122
H27-0136	Tuff, arenite, basalt	10.49	Leaky	Hantush	0.0006	0.78	0.074
H27-0138	Arenite, basalt, shale	25.60	Fractured (single fracture)	Gringatern-Ramey w/horizontal fracture	0.0060	4.90	0.191
H27-0165	Tuff, arenite, basalt	46.09	Leaky	Moench (case 1)	0.0007	12.3	0.267
H27-0168	Arenite, conglomerate	19.10	Leaky	Hantush	0.0006	7.40	0.387
H27-0290	Tuff, arenite, basalt	48.00	Leaky	Moench (case 2)	0.0680	6.10	0.127

Transmissivity values ranged from 0.78 to 12.3 m^2/day (Table 5.3). Mean transmissivity value of 15.6 m^2/day from a sample of 339 boreholes from Soutpansberg, which is within the vicinity of the study area, was obtained in a study by Holland (2012). Ishaku *et al.* (2009) reported Tr values ranging from 0.3 to 19.7 m^2/day with an average of 2.90 m^2/day in the basement aquifers in Taraba State, Nigeria. The transmissivity values obtained in the current study are within the range of values obtained by Ishaku *et al.* (2009) though they are lower than the mean value of 15.6 m^2/day obtained by Holland (2012). This may be because Holland (2012) covered a large area (63500 km^2) thereby incorporating a wider range of transmissivity values.

Transmissivity values within the ranges of 0.1-1 and 1-10 m^2/day are classified as very low and low, respectively, according to Kransy (1993). Kransy (1993) also classified these ranges of transmissivity as having potential for very low withdrawals for local water supply with limited consumption and low withdrawals for local water supply (private consumption), respectively. Five of the boreholes (H27-0002, H27-0052, H27-0138, H27-0168 and H27-0290) have low transmissivity values, with potential for local groundwater supply for private consumption. Chilton and Foster (1995) also reported that crystalline basement aquifers are capable of small water supplies which are vital to rural population for domestic use and livestock watering. Borehole H27-0136 has very low transmissivity value with limited water supply. The transmissivity for borehole H27-0165, however, falls in the intermediate classification category by Kransy (1993), and has the potential to contribute to lesser regional supply for small communities and plants.

5.5 Chapter summary and contribution

The results of this chapter have addressed the first specific objective and research question on determining the characteristics of the groundwater resource unit. The groundwater resource unit is bounded by a mountain divide, ephemeral rivers where there are lineaments and Nzhelele River. Plotted groundwater levels and topography showed that groundwater flow is controlled by topography. The main groundwater flow direction is from the upstream of the groundwater resource unit towards Nzhelele River.

Hydrogeological conceptual model indicated presence of faults and diabase dykes in the study area. Faults create preferential flow paths and influence storage of water into the aquifer. Dykes are likely to serve as water-bearing features in the study area as Holland (2012) noted that they are important water-bearing features in the Limpopo Plateau, where the groundwater resource unit is also found. It was, however, noted that knowledge of the extent to which faults and dykes influence groundwater flow and storage is limited since their thicknesses and depths are unknown. The groundwater resource unit for Siloam Village was conceptualised as a system whose main groundwater flows are localised (i.e. a localised groundwater system).

The models fitted from automatic curve matching generally had low statistics for most of the boreholes indicating least model errors and good model fits. This showed that identified aquifer test solutions can aid in accurate estimation of hydraulic characteristics (transmissivity and hydraulic conductivity). The results showed that the study area is dominated by a leaky aquifer. Fractured double porosity and single fracture models were also identified. These findings indicate that the geologic environment where groundwater is stored in the study area is heterogeneous. Leaky aquifer (semi-confined conditions) indicate presence of direct recharge from an unsaturated zone. Double porosity aquifer has two media (matrix blocks and fractures) while water flows along the fracture with higher permeability than that of the rock in a single fractured aquifer. Aquifer hydraulic characteristics estimated in this chapter assisted in defining ranges of input values for the GW-PITMAN model.

Delineation of the groundwater resource unit and development of its hydrogeological conceptual model serves as a motivation to collect additional data and undertake additional studies aimed at updating the hydrogeological conceptual model. Holland (2011) noted that hydrogeology of crystalline aquifers is not yet fully understood. Studies that have been done in Limpopo Province including Dippenaar (2008) and Holland (2011) mostly covered areas within Sand, Luvuvhu and Letaba River Catchments. The hydraulic characteristics of Nzhelele area were still unknown. The heterogeneous nature of fractured crystalline basement aquifers requires that analysis of their characteristics be done at many sites as possible for efficient decision making. The study contributed to knowledge on hydraulic characteristics of basement aquifers.

The groundwater resource unit characteristics including presence of faults and diabase dykes, and heterogenous aquifer types as identified in this chapter make representation of this complex groundwater environment within a groundwater model to be a complicated task. Within the GW-PITMAN model, it is expected that transmissivity and storativity are two of the model parameters which represent the complex nature of the geologic environment where groundwater is stored and its influence on groundwater storage. Transmissivity and storativity describe the ability of an aquifer to transmit and store water, respectively, and these are influenced by presence of faults and diabase dykes as well as type of aquifer. Classification of transmissivity values indicated variable supply potential though potential for local water supply for private consumption was dominant, indicating that the groundwater resource unit may potentially meet the water demands of the study area.

CHAPTER 6: EXTENSION OF RAINFALL AND GROUNDWATER LEVELS DATA

6.1 Preamble

This chapter presents the results and discussions on modelling and extension of rainfall and groundwater levels based on NPR and OE-NLHW system identification models. Long term and continuous data sets generated from the models are required for groundwater modelling using the GW-PITMAN, generation of stochastic sequences, stochastic yield analysis and development of operating rules.

6.2 Modelling and extension of rainfall data

The graphical fits for observed and estimated rainfall for calibration and validation runs are presented in Figures 6.1 and 6.2. Results for the periods 1998/07/04 to 2000/07/04 and 1998/07/01 to 2000/07/01 have been plotted in Figures 6.1 and 6.2, respectively, to improve on their visual inspection. The procedure used for NPR modelling is described in sub-section 4.3.1. As explained in sub-section 4.3.1, the data points used for calibration and validation were randomly selected by the NPR model, though they were within the period 1991/07/01 to 2000/07/31. Random selection avoids biased partition of the calibration and validation data sets which is likely to result in improved and representative the model calibration. The comparisons of observed and estimated rainfall for calibration and validation runs showed a general agreement between observed and simulated rainfall (Figures 6.1 and 6.2). A general visual agreement between observed and simulated constituent data indicates adequate calibration and validation over the range of the constituent data being simulated (Singh *et al.*, 2004). General visual agreement should, however, be supported by statistical techniques for verification of the model performance as recommended by Legates and Mabe (1993). However, the graphs also show underestimation of some of the rainfall events in both calibration and validation runs.

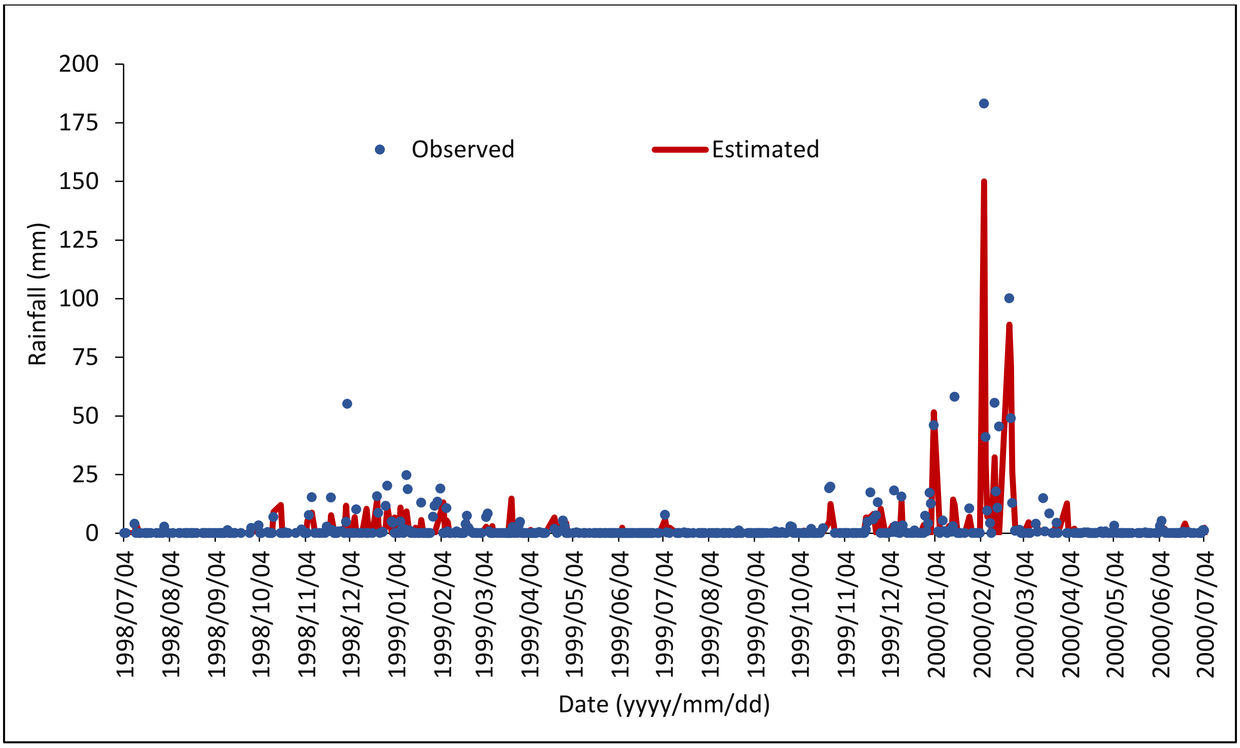


Figure 6.1: Observed and estimated rainfall for calibration run

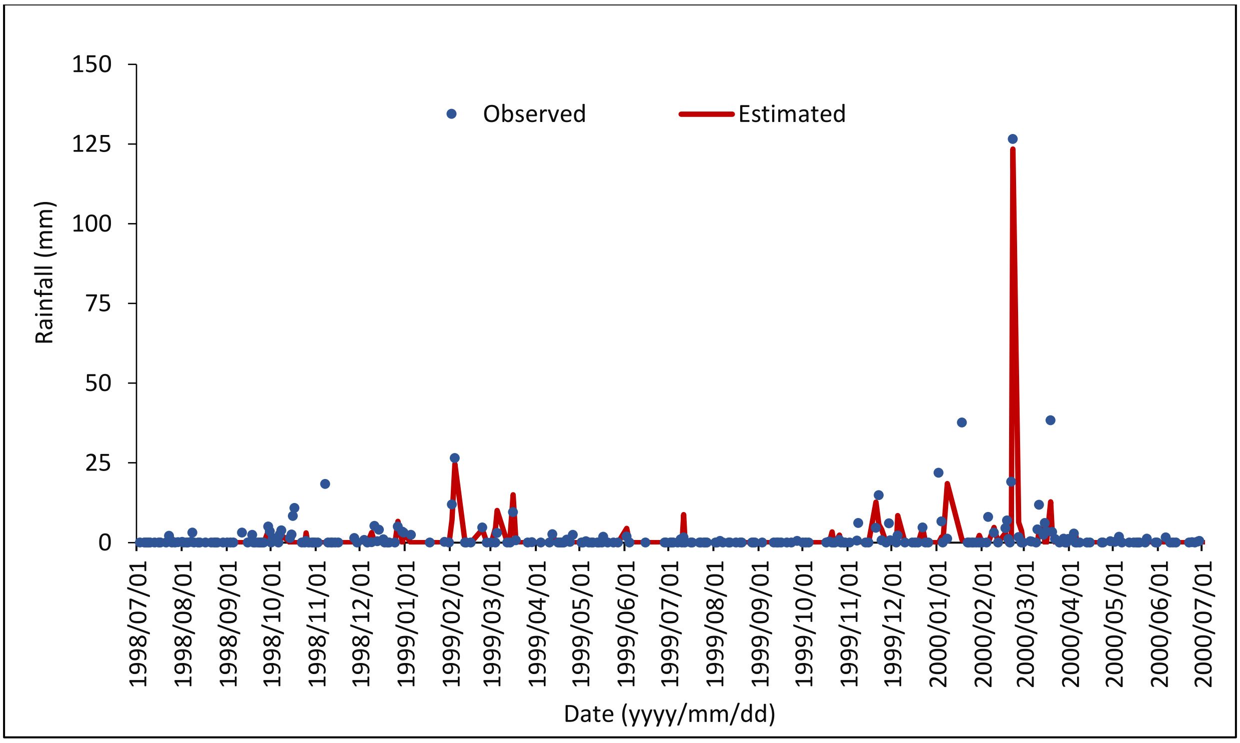


Figure 6.2: Observed and estimated rainfall for validation run

Performance measures for calibration and validation runs together with their acceptable ranges are in Table 6.1. R^2 values for calibration and validation showed very good and good model performance, respectively. R^2 values were within the range of 0.57 to 0.96 obtained by Seo *et al.* (2015) in a study on estimating spatial precipitation using regression kriging and artificial neural network residual kriging. This indicates that the R^2 values obtained from the NPR model are comparable with values obtained from regression kriging and artificial neural network residual kriging. COR values for calibration and validation exceed 0.8 showing satisfactory model performance. RMSE values for both calibration and validation runs are reasonable as they are low and fall within the ranges obtained in other studies. RMSE of 64 mm was obtained in a study by Ali (1998). Kalra and Ahmad (2011) obtained RMSE values ranging from 0.44 to 2.69 inches, which are equivalent to a range of 11.18 to 68.33 mm. RE values for both calibration and validation were satisfactory since they were within the range of ±25 to ±30 (Table 6.1).

Table 6.1: Performance measures for calibration and validation runs

Performance Measure	Calibration	Validation	Criteria
R^2	0.76	0.70	0.65-0.7 Good[a] 0.75-0.85 Very good[a]
COR	0.87	0.84	≥0.80-satisfactory[b]
NSE	0.75	0.68	0.8-0.9 Very Good[c] 0.65-0.75 Good[a]
RMSE (mm)	3.67	3.03	0= Perfect[c]
RE (%)	30	29	±25-±30 Satisfactory[a]

[a]Yan *et al.* (2014); [b]Singh *et al.* (2004); [c]Shamsudin and Hassim (2002)

Makungo and Odiyo (In Press) obtained R^2, COR, RMSE and NSE within the range of 0.52-0.86, 0.78-0.91, 5.96-6.74 and 0.51-0.86, respectively, for stations 0766480, 0723485 and A9E002. Stations 0766480, and 0723485 and A9E002 are located in Nzhelele and Luvuvhu River Catchments in Limpopo Province, South Africa, respectively. The latter study also applied NPR for estimating missing rainfall data. R^2, COR and NSE values obtained for station 0766324 (plotted in Figure 3.9) were within the same ranges though its RMSE values were lower than those of Makungo and Odiyo (In Press). The model performance ranged from good to satisfactory based on the assessed measures of performance which were compared to the criteria in Table 6.1. Thus, the model can effectively estimate missing rainfall for station 0766324.

Scatter plots of observed and estimated rainfall for both calibration and validation runs of the NPR model are in Figures 6.3 and 6.4. Most of the scatter points in Figures 6.1 and 6.2 lie below the best fit lines showing that there was underestimation of rainfall in both calibration and validation runs. The maximum rainfalls for both calibration and validation runs were underestimated. The best fit (1:1) line in a scatter plot indicates where all data points would fall if there was perfect agreement between the models and observations (DeAngelis *et al.*, 2013). However, as a result of inherent model structure and measurement errors in observed, most models are likely not to have perfect fits. This is likely to result in uncertainty in the estimated rainfall values. The stochastic based procedure used for development of operating rules in this study accounts for uncertainty and it is therefore expected to minimize this.

Inherent model structure and measurement errors in observed data are likely to have resulted to underestimation and overestimation of rainfall by NPR model. The shift from the ideal line shows the possibility of systematic errors (Javan *et al.*, 2015). There was more dispersion of scatter points in the calibration run as compared to validation run. This indicated more difference of observed and estimated rainfall in the validation run. Maximum rainfall for the validation run was closer to the best fit line indicating good estimation. Scatter which is closer to the ideal line indicates good simulation (Javan *et al.*, 2015). Rainfall data for the period 2000/08/01 to 2012/01/12 which was estimated using the calibrated and validated NPR model for Siloam Village is shown in Figure 6.5.

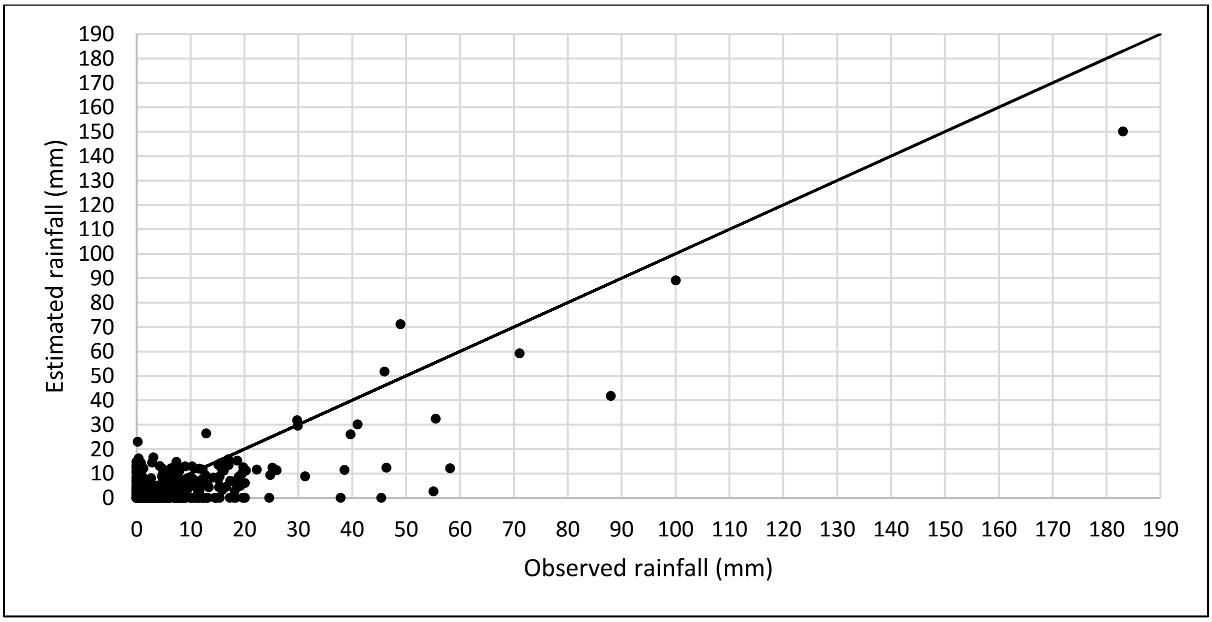


Figure 6.3: Scatter plot for estimated and observed rainfall for calibration run

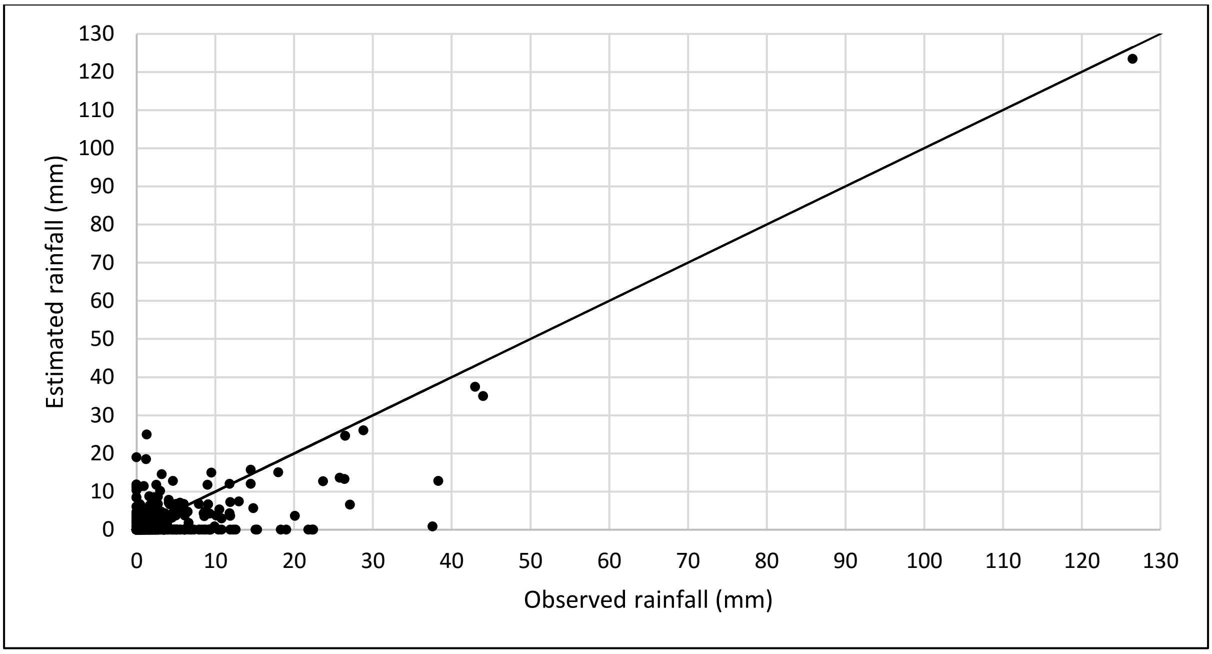


Figure 6.4: Scatter plot for estimated and observed rainfall for validation run

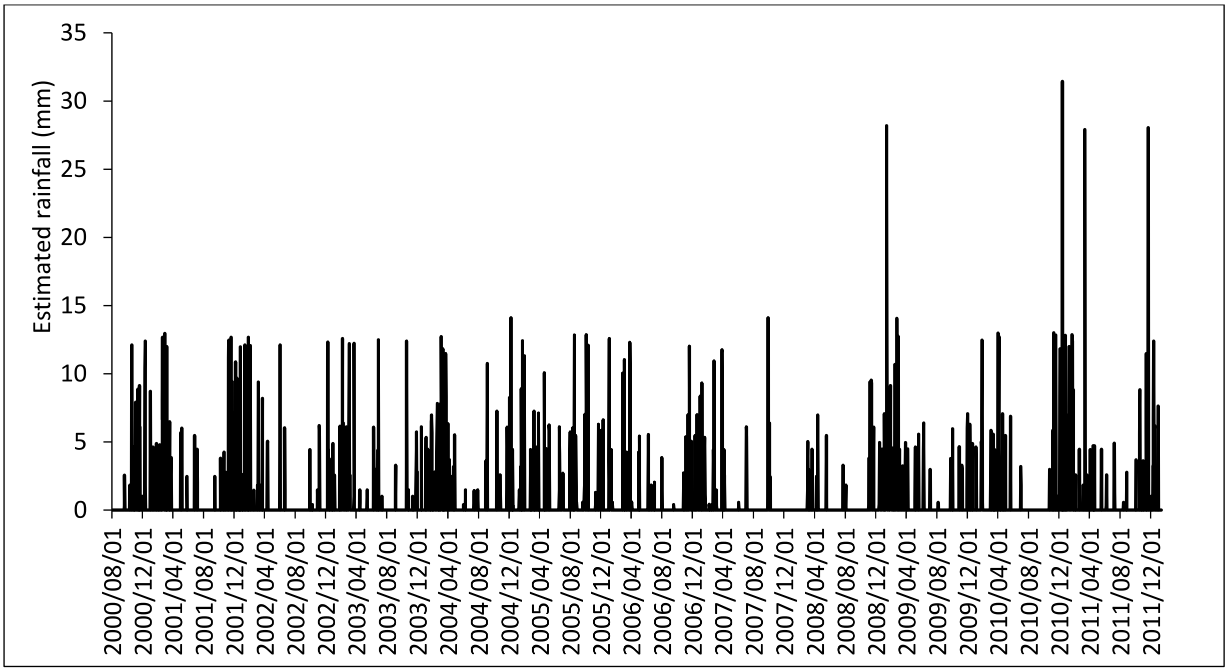


Figure 6.5: Estimated rainfall for Siloam Village

6.3 Groundwater levels modelling and extension

Selected model orders from groundwater levels modelling based on OE-NLHW system identification model described in sub-section 4.3.2 are in Table 6.2. These model orders gave the best results after several trial runs. The model orders are in pairs since two input variables (rainfall and evaporation) were used in the modelling. The model orders define the number of terms in each equation. Model calibration resulted to equations for *B(q)* and *F(q)* polynomials defined in Equation 4.12 and their coefficients. Thus, the model orders and coefficients of the equations are the parameters obtained from the calibration process.

Table 6.2: Model orders used in groundwater modelling

Borehole	Calibrated *nb, nf, nk*	Polynomial B(q)	Polynomial F(q)	Model error (*e(t)*)
A8N0508	4,4; 3,3; 4,4	$B(q_1) = -0.025q^{-4} - 0.029q^{-5} - 0.029q^{-6} + 0.025q^{-7}$ $B(q_2) = -0.0004q^{-4} - 0.006q^{-5} - 0.006q^{-6} + 0.004q^{-7}$	$F(q_1) = 1 - 1.602q^{-1} - 1.06q^{-2} + 0.4568q^{-3}$ $F(q_2) = 1 - 1.444q^{-1} - 0.05654q^{-2} + 0.5003q^{-3}$	0.1359

The shapes of the observed and estimated groundwater levels graphs are mostly similar for both calibration and validation runs (Figures 6.6 and 6.7). General agreement between observed and simulated frequencies for the variable of interest indicates adequate simulation over the range of conditions examined (Singh *et al.*, 2004). In borehole A8N0508, noticeable underestimation of groundwater levels within the period 2011/10/21 to 2011/12/28 and overestimation within the period 2011/12/30 to 2012/02/12 occurred in the validation run. System identification models do not capture the physical processes particularly those that occur during extreme rainfall events and thus, they are likely to underestimate peak groundwater levels (Makungo and Odiyo, 2017). For example, during extreme rainfall events soil saturation occurs quickly thereby accelerating recharge into the aquifer which increases the peak groundwater levels.

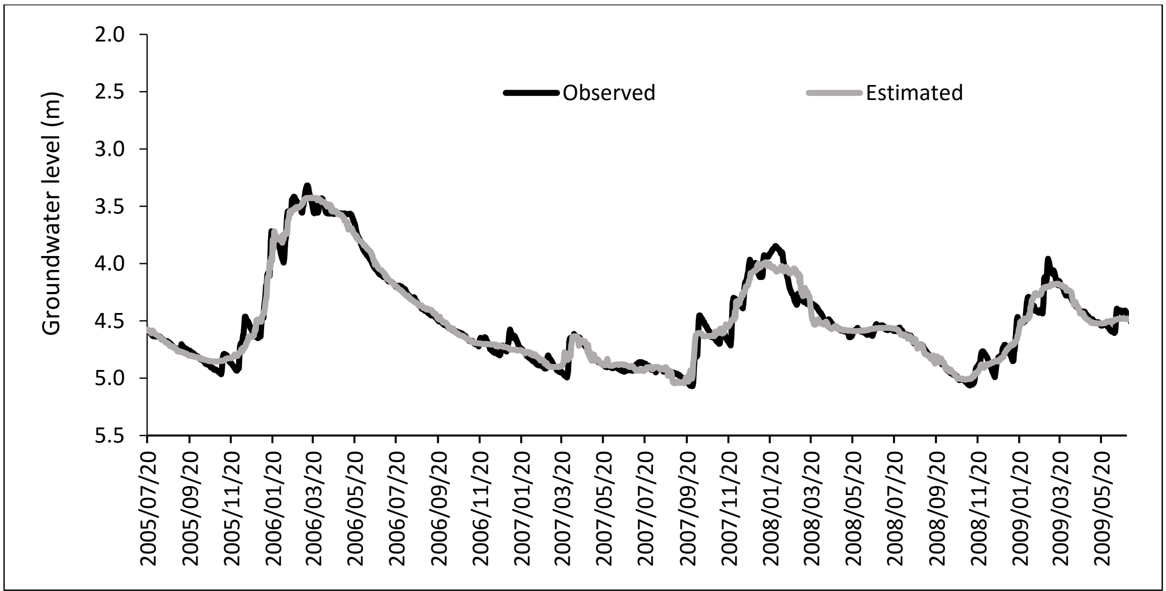


Figure 6.6: Observed and simulated groundwater levels for calibration run

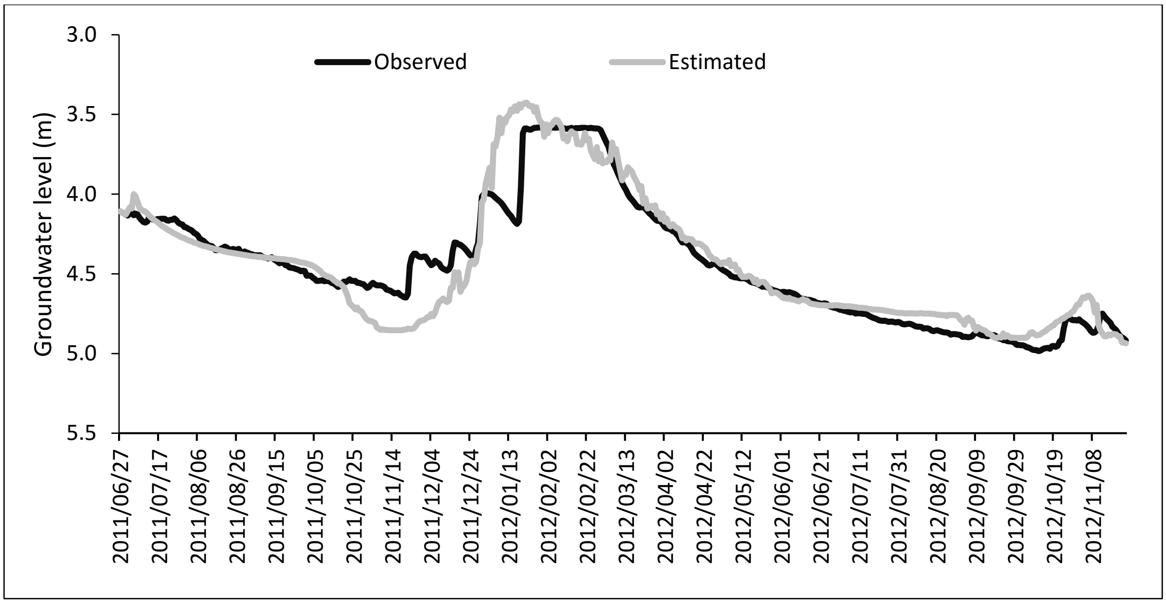


Figure 6.7: Observed and simulated groundwater levels for validation run

Scatter plots of observed and estimated groundwater levels for both calibration and validation runs are presented in Figures 6.8 and 6.9, respectively. Most of the scatter points are close to the best fit line showing agreement of observed and simulated groundwater levels in both calibration and validation runs This has been confirmed by Ritter and Muñoz-Carpena (2013) who indicated that higher the agreement between calculated and observed values, the more the scatters tend to concentrate close to the 1:1 line.

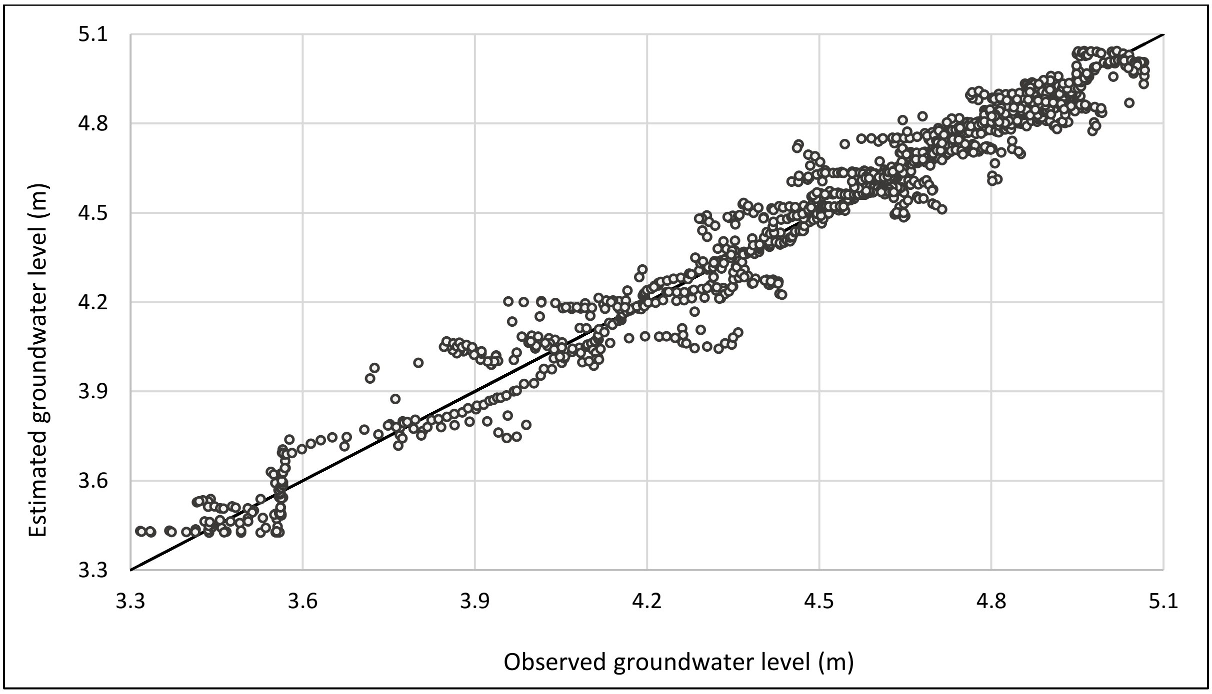


Figure 6.8: Scatter plot of observed and estimated groundwater levels for calibration run

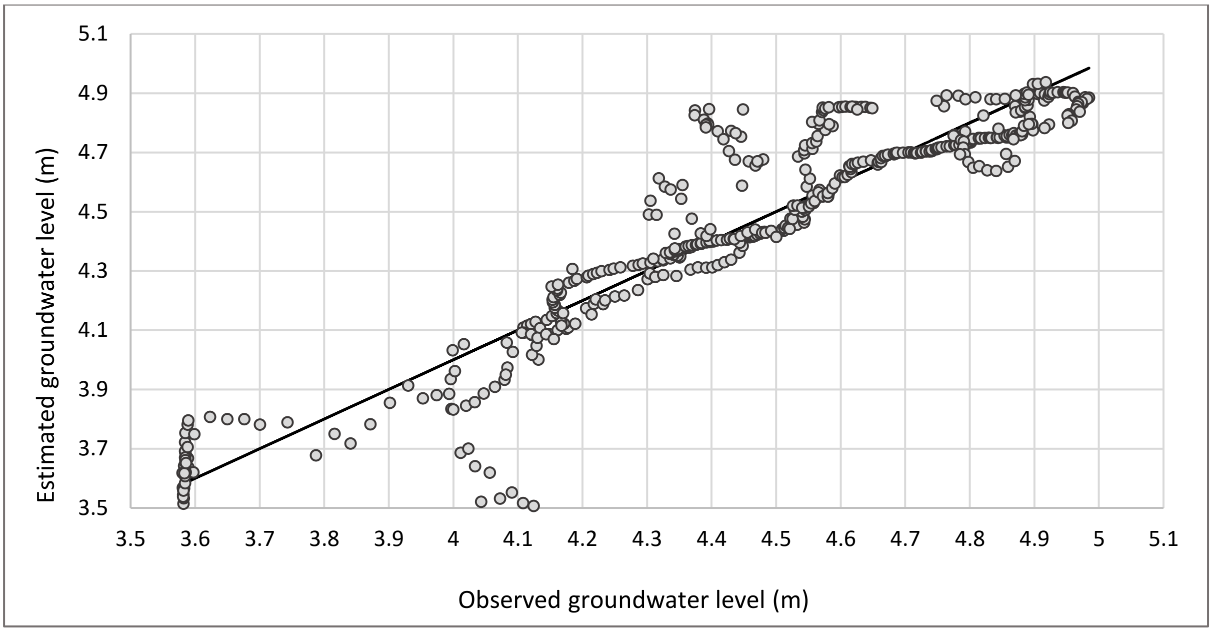


Figure 6.9: Scatter plot of observed and estimated groundwater levels for validation run

R^2 values for calibration and validation runs of borehole A8N0508 were greater than 0.85 showing excellent model performance (Table 6.3). von Asmuth and Knotters (2004) obtained R^2 values of 81.5 and 91.9% on calibrating continuous time TFN system identification model for characterising groundwater dynamics using groundwater levels data. These values are comparable to those obtained in borehole A8N0508 in the current study.

NSE value for calibration run for A8N0508 showed very good performance since it was >0.9. High NSE values indicate less variance error (Van Liew *et al.*, 2003). Validation run for A8N0508, had NSE values within the range of 0.8-0.9, showing good model performance according to Yan *et al.* (2014). NSE values of 0.81 and 0.93 were obtained by Vadillo (2014) when using NLHW to predict spring discharge based on precipitation data. The NSE values are comparable to those obtained in this study. COR values for calibration and validation run of A8N0508 were >0.8, thus showing satisfactory model performance according to Singh *et al.* (2004).

Table 6.3: Computed measures of performance for borehole A8N0508

Performance Measure	Calibration	Validation	Criteria
R^2	0.99	0.86	> 0.85 Excellent[a] 0.75-0.85 Very good[a]
COR	0.97	0.93	≥0.80-satisfactory[b]
NSE	0.99	0.84	0.65-0.70 Good[a] 0.8-0.9 Good[a] ≥ 0.9 Very good[a]
RMSE (m)	0.03	0.01	0 Perfect[c]
RE (%)	0.08	0.11	≤ ±10 Excellent[a]

[a]Yan *et al.* (2014); [b]Singh *et al.* 2004; [c]Shamsudin and Hassim (2002)

RMSE values for A8N0508 calibration and validation runs were closer to zero. RMSE values close to zero indicate perfect fit (Singh *et al.*, 2004). Average RMSE value of 22 cm was obtained in a study by Knotters and Bierkens (2001) on predicting water table depths using regionalised autoregressive exogenous variable model. Knotters and Bierkens (2000) obtained RMSE values ranging from 17.77-23.37 cm on validating an autoregressive exogenous variable model for predicting water table depths. von Asmuth and Knotters (2004) obtained RMSE values of 8.4 and 11.2 cm. RMSE values for A8N0508 of 0.03 and 0.01 m for calibration and validation runs, respectively, are comparable to those of related studies. RE values for both runs were less than ±10% showing excellent model performance (Table 6.3). Values for COR, R^2, RMSE and NSE ranged from 0.8-0.99, 0.63-0.99, 0.08-2.06 m and 0.68-0.99, respectively, for boreholes A8N0515, A9N0018 and A9N0009 in study by Makungo and Odiyo (2017) which also applied OE-NLHW model for estimating groundwater levels. Boreholes A8N0515, and A9N0018 and A9N0009 are located in Nzhelele and Luvuvhu River Catchments, respectively, in Limpopo Province, South Africa. Values obtained for borehole A8N0508 are comparable to those of Makungo and Odiyo (2017), except for the RMSE for the calibration and validation runs which were lower than those of the latter study.

The graphical fits, scatter plots and measures of performance generally show efficient calibration and validation of the model. Thus, rainfall and evapotranspiration can be used to simulate groundwater levels based on the coupled OE-NLHW system identification model. General increases and decreases in groundwater levels which correspond to increases and decreases in rainfall, respectively are indicated in Figure 6.10. In Figure 6.10, both rainfall and groundwater levels were missing during the period 2009/11/10-2010/01/15.

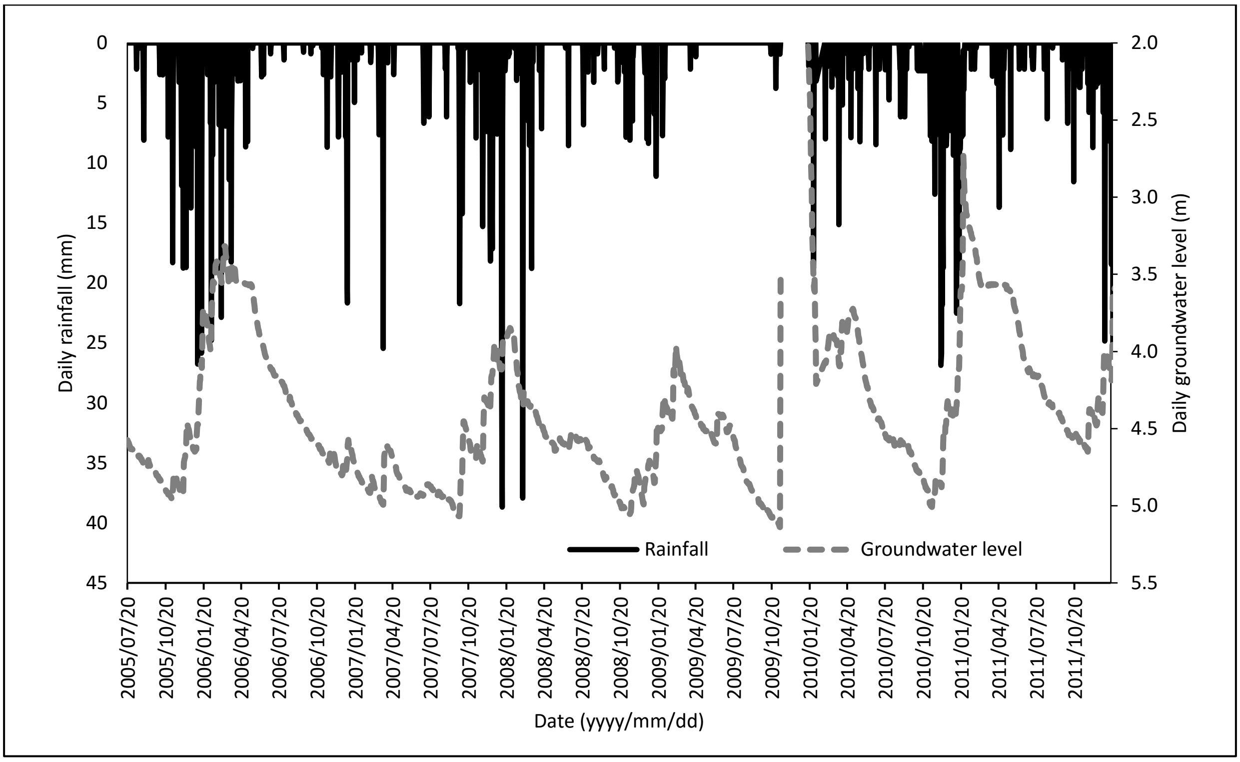


Figure 6.10: Relationship between rainfall for station 0766324 and groundwater levels for A8N0508

Results from multi-site monitoring in a study by van Wyk *et al.* (2012) emphasised that a direct recharge mechanism, which is enhanced by the presence of macro-pore features, exists in fractured hard-rock terrains of South Africa. This was indicated by the short lag-time (1 hour to 5 days) between rainfall events and water table responses (van Wyk *et al.*, 2012). Since, the study area falls within the fractured hard-rock terrains of South Africa, the lag time between rainfall and groundwater levels is also expected to be short. This was accounted for by the *e(t)* component of the model. The *e(t)* in Equation 4.12 accounts for the unknown physical processes that influence groundwater levels. These include the influence of hydrogeologic environment where groundwater recharge and storage takes place, groundwater abstractions and lag time between rainfall and groundwater level rise in the study area. In studies by Knotters and Walsum (1997) and Manzione *et al.* (2009), the noise component of the TFN models were used to describe part of the water table behaviour that could not be explained from the used physical concepts or empirically from the input series. The transfer components of OE-NLHW describe part of the groundwater levels that could be described from an input by linear transformation time series of the input. Transfer component of a TFN model was used to describe the part of the water table depth that could be explained from an input by linear transformation of a time series of that particular input in a study by Manzione *et al.* (2009).

The macro-pores from severely faulted crystalline basement aquifers and their interconnectivity result in good response of groundwater levels to rainfall events in the study area and hence a good relationship between the two. Rainfall is therefore one of the major drivers of groundwater level fluctuations in the study area. Peak groundwater levels mostly corresponded with peak rainfall events and low rainfall events corresponded with drop in water levels (Figure 6.10). This explains good model performance based on performance measures for borehole A8N0508. A study by Ochoa and Reinoso (1997) reported that increments in the water table level could be explained by local precipitation suggesting that the Doñana National Park dune aquifer, in the southwestern coast of Spain is very sensitive to local meteorological conditions. The response of groundwater levels to rainfall particularly during low rainfall periods (Figure 6.10) indicates that groundwater is vulnerable to drought and hence requires operating rules to manage its allocation.

Knotter and van Walsum (1997) noted that use of models to estimate fluctuating quantities is successful only if they adequately describe the relation between input and output series. This implies that the calibration period must be long enough in order to identify appropriate models and to estimate parameters accurately. In their study, analyses for two observation wells indicated that a 4-year calibration period contained all information needed to provide a satisfactory description of the relation between precipitation excess and water-table depth, for semi-monthly time steps and for shallow water-table depths. In the current study, a minimum period of 4 years for daily time step was also used in model calibration. This was assumed to contain all hydrological information needed to provide satisfactory description between input and output variables. Groundwater levels for borehole A8N0508 which were extended based on the calibrated and validated OE-NLHW system identification model are presented in Figure 6.11.

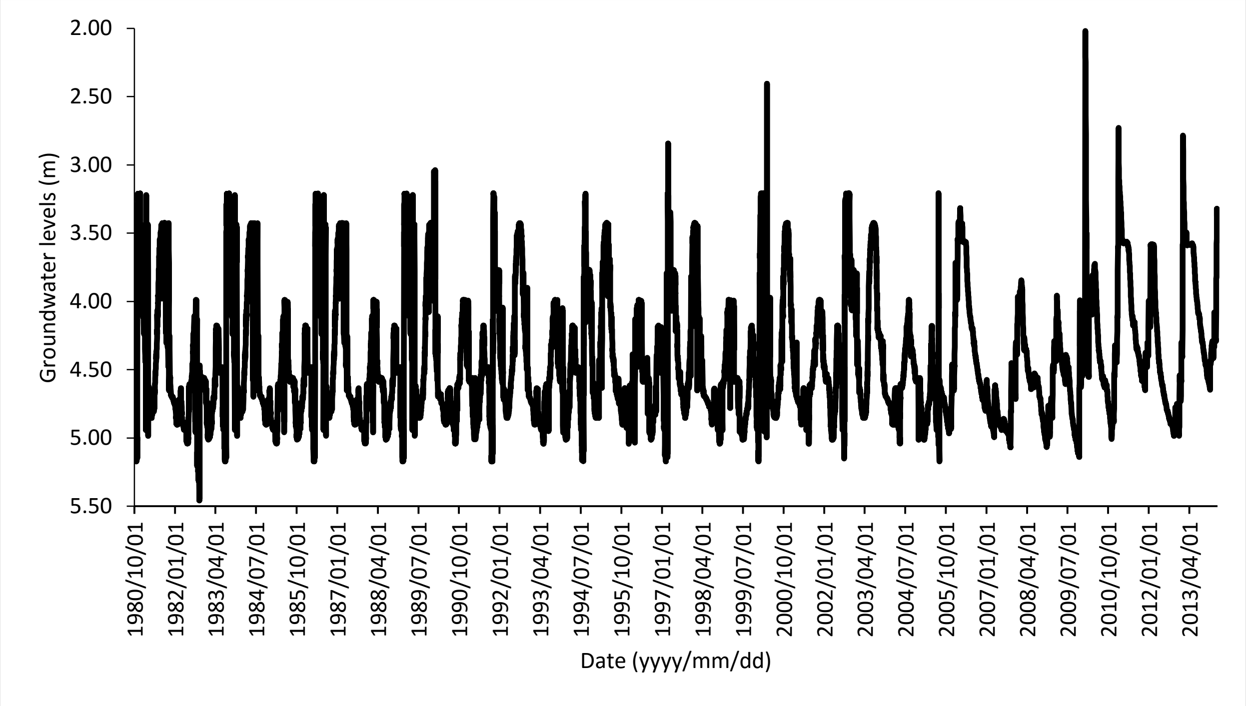


Figure 6.11: Extended groundwater levels

6.4 Chapter summary and contributions

The data infilled and/or extended in this chapter provided some of the inputs required for stochastic groundwater base yield-recurrence interval analysis. They therefore supported achievement of third specific objective and research question. The scatter plots of observed and estimated rainfall indicated underestimation of rainfall in both calibration and validation runs, which was also shown in the graphical fits. However, the graphical fits for calibration and validation runs generally showed agreement of observed and estimated rainfall. The NPR model had acceptable and satisfactory performance based on the assessed measures of performance indicating that the model can be effectively used in extension of rainfall for Siloam weather station.

The tricube kernel weighting function used in the current study has been tested for rainfall estimation by Lee and Kang (2015). The latter study used rainfall data for the same location (station) to estimate missing rainfall data. The current study tested performance of NPR in estimating missing rainfall data at a target station, based on data from a neighbouring station. This aids in estimating missing data for long periods (for example, a month or a year), which cannot be achieved if data for the same location are used. In most developing countries including the study area it is typical to encounter limited rainfall data sets compounded with large data gaps. Thus, testing and validation of NPR in estimating missing rainfall data at a target station, based on data from a neighbouring station with the same characteristics aids in generating long-term rainfall data which can be used for a number of environmental and hydrological applications in developing countries. This approach was tested in estimating missing rainfall data in one and two stations within Nzhelele and Luvuvhu areas, respectively, by Makungo and Odiyo (In Press) and was found to be effective in rainfall estimation. The study by Makungo and Odiyo (In Press) was an expansion of this thesis which was used to validate the approach to include the results for station 0766324.

Except for Makungo and Odiyo (In Press), literature review for this study was not able to identify any application of robust LOWESS NPR for estimating missing rainfall data at a target station, based on data from a neighbouring station, indicating its limited application. This creates the need for more studies to determine its success in rainfall estimation. This study therefore contributed to knowledge on alternative approaches to estimating missing rainfall data. De Silva *et al.* (2007) indicated that the best method for estimating missing rainfall data can vary for different climatic zones depending on their rainfall patterns and spatial distributions. Testing and validation of more approaches for rainfall estimation would provide alternative approaches that can be applied in different climatic zones.

Scatter plots of observed and estimated groundwater levels for both calibration and validation runs of OE-NLHW showed good agreement (Figures 6.8 and 6.9). There was more dispersion of scatter points in the validation run as compared to calibration run indicating more variation of observed and estimated groundwater levels in the former. However, graphical fits, scatter plots and measures of performance generally show efficient calibration and validation of the model indicating that it can be used to simulate groundwater levels. This approach was used in modelling groundwater levels in one and two boreholes within Nzhelele and Luvuvhu areas, respectively, by Makungo and Odiyo (2017) and was found to be effective in groundwater modelling.

As reported in Makungo and Odiyo (2017), the coupled OE-NLHW system identification model had not yet been tested for groundwater levels modelling. The study by Makungo and Odiyo (2017) was an expansion from this thesis which was used to validate the approach and it included the results for borehole A8N0508 which has been covered in this thesis. The latter study noted that the calibrated models can reasonably capture description between input and output variables and can, thus be used to estimate long term groundwater levels.

It is important to note that the approaches used in this study had either limited applications or had not yet been applied for data extension. Assessment of their performances had indicated that they can be successfully applied in extension of rainfall and groundwater levels data. This shows that they have potential for application in other study areas where they have not yet been tested. Thus, this study has contributed to promotion of wide application and testing of these methods and provides alternative methods for data extension in areas where there is limited data.

CHAPTER 7: GW-PITMAN MODELLING AND GENERATION OF GROUNDWATER LEVELS

7.1 Preamble

This chapter focused on results and discussion of the GW-PITMAN modelling that includes sensitivity analysis, model calibration and validation. Results and discussions on model parameters from 1000 calibration runs and their comparisons with the objective function are also included. Comparisons of modelled groundwater levels with runoff and recharge are made to assess the realistic nature of the modelling. Time series of groundwater levels was required for stochastic yield analysis and development of operating rules.

7.2 Results of sensitivity analysis

Ranges of parameter values used in sensitivity analysis are given in Table 7.1. As explained in sub-section 4.4.2, these values were obtained from ten preliminary calibration runs, modifying one parameter at a time, to test the response of the model to a range of different parameter values and identify parameters to be altered during final model calibration. This ensured that only parameters that are influence the hydrological behaviour of the groundwater resources unit and hence have on influence on groundwater levels fluctuation are calibrated. SL, ST, GPOW and R are sensitive parameters since adjusting their values resulted to changes in groundwater levels (Figures 7.1 to 7.6). ST, FT and GPOW were considered as sensitive parameters while calibrating Pitman model for selected case studies in South Africa by Hughes *et al.* (2010), due to the large separations of these parameters' frequency distributions. In Ndiritu (2009), calibration of the Pitman model in the Kafue Basin in Zambia revealed that ST, FT, POW and R were significant model parameters. This was because values for these parameters were reasonably well defined (closer to each other when plotted on a graph), indicating that they are significant model parameters. The results of sensitivity analysis in this study are therefore mostly comparable to those of other studies that applied Pitman model.

FT and POW were the only parameters which resulted in no significant changes in groundwater levels for all ranges of values which were tested (see Figures 7.1 to 7.6). This is because FT and POW do not affect runoff generation in semi-arid areas where sustained baseflow does not exist, as explained in Kapangaziwiri (2007). In Hughes *et al.* (2010), FT was identified as a sensitive parameter in the Breede River Catchment while it was not sensitive in the Sabie, upper Vaal and Gouritz River Catchments which were selected case studies. This was explained to be due to uncertainties in the estimates of farm dam impacts or the extent to which these are reflected in the observed data within the Breede River Catchment (with irrigation supported by a number of farm dams). Irrigation has an effect on sustained baseflows and hence influences runoff generation. Mwelwa (2004) reported that ST and FT were the most sensitive parameters in the Kafue Basin, Zambia. Zambia is located in a sub-humid tropical climate where FT is expected to affect runoff generation making it a sensitive parameter.. Ndiritu (2009) also identified FT as a sensitive parameter during automatic calibration of the Pitman model in the Kafue Basin Basin, Zambia. Ndiritu (2009) noted that the relative importance of the parameters may vary among catchments depending on the dominant hydrologic processes in the catchments. This explains why FT and POW would be considered to be insensitive in semi-arid areas while they are sensitive in sub-humid tropical areas.

Default scenario in Table 7.1 and Figures 7.1 to 7.6 indicate a case with parameters which were defined during the coding of the model in this study. The groundwater levels simulated from the preliminary calibration runs (Figures 7.1 to 7.6) were poorly estimated as expected. This was because preliminary calibration was only aimed at identifying sensitive model parameters and not to optimise the model for improved estimation of the groundwater levels. Improved model calibration was done after sensitivity analysis and is discussed in section 7.6.

Table 7.1: Ranges of GW-PITMAN model parameter values used in sensitivity analysis

Parameter	**Scenario**		**Range of lower limit**	**Range of upper limit**
FT (mm): Runoff from moisture storage at full capacity. Determines the balance between evaporation and runoff in humid areas. Generally zero.	FT0		0.00	0.00
	FT0.5		0.0-0.5	0.5-1.0
	FT10		10-20	20-30
	FT50		50-100	100-150
	Default		25-40	20-80
SL: Lower limit of soil moisture below which no groundwater recharge occurs	S0-0.5	SL1	0-0.5	1-1.5
		SL2	2-5	5-10
	SL1.5	SL1	1.5-3	4-8
		SL2	2-3	1-4
	SL0.5	SL1	0.5-1	8-15
		SL2	8-15	7-20
	Default	SL1	0.5-1	0-5
		SL2	8-15	7-20
GPOW: Power of the moisture storage-recharge equation. Controls rate of recharge from the soil from any given soil moisture state	GPOW1.5		1.5-2.5	3-4
	GPOW3		3-5	4-8
	GPOW0.5		0.5-1	1.5-2
	Original		2-3	1-4
ST(mm): Maximum moisture storage capacity	ST200		200-250	175-450
	ST500		500-650	600-800
	ST50		50-100	150-200
	Default		400-500	350-900
R: Evaporation-moisture storage relationship parameter. Controls the rate at which evaporation reduces soil moisture	R0.1		0.1-0.5	1-2.5
	R0.2		0.2-0.3	0.25-0.35
	R0.3		0.3-0.6	0.8-1
	Default		0.4-0.5	0.1-1
POW: Power of the moisture storage-runoff equation. Controls the rate of runoff from the soil for any moisture state	POW2		2-3	2.5-5
	POW0.1		0.1-0.4	0.45-0.8
	POW0		0-0.05	0.04-1
	Default		1.5-2	1-4

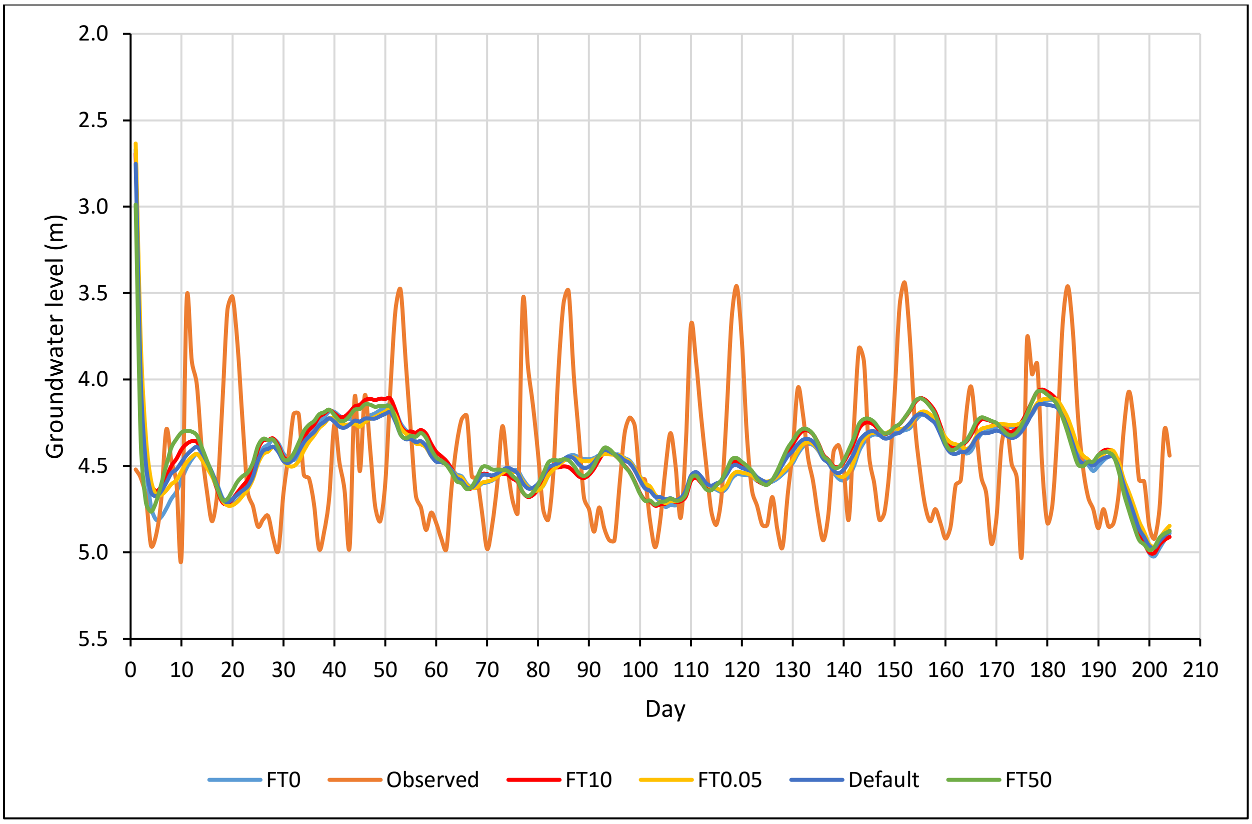


Figure 7.1: Simulated groundwater levels for ranges of FT together with observed values

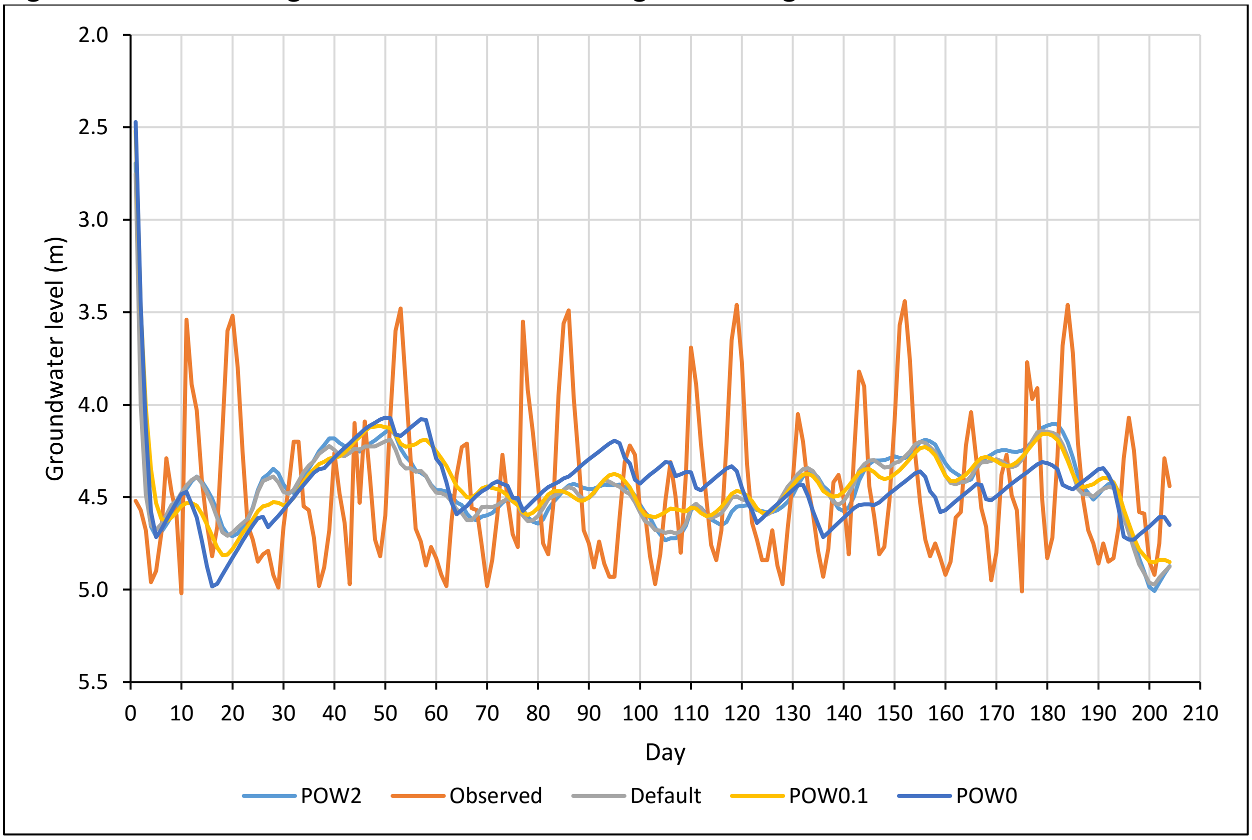


Figure 7.2: Simulated groundwater levels for ranges of POW together with observed values

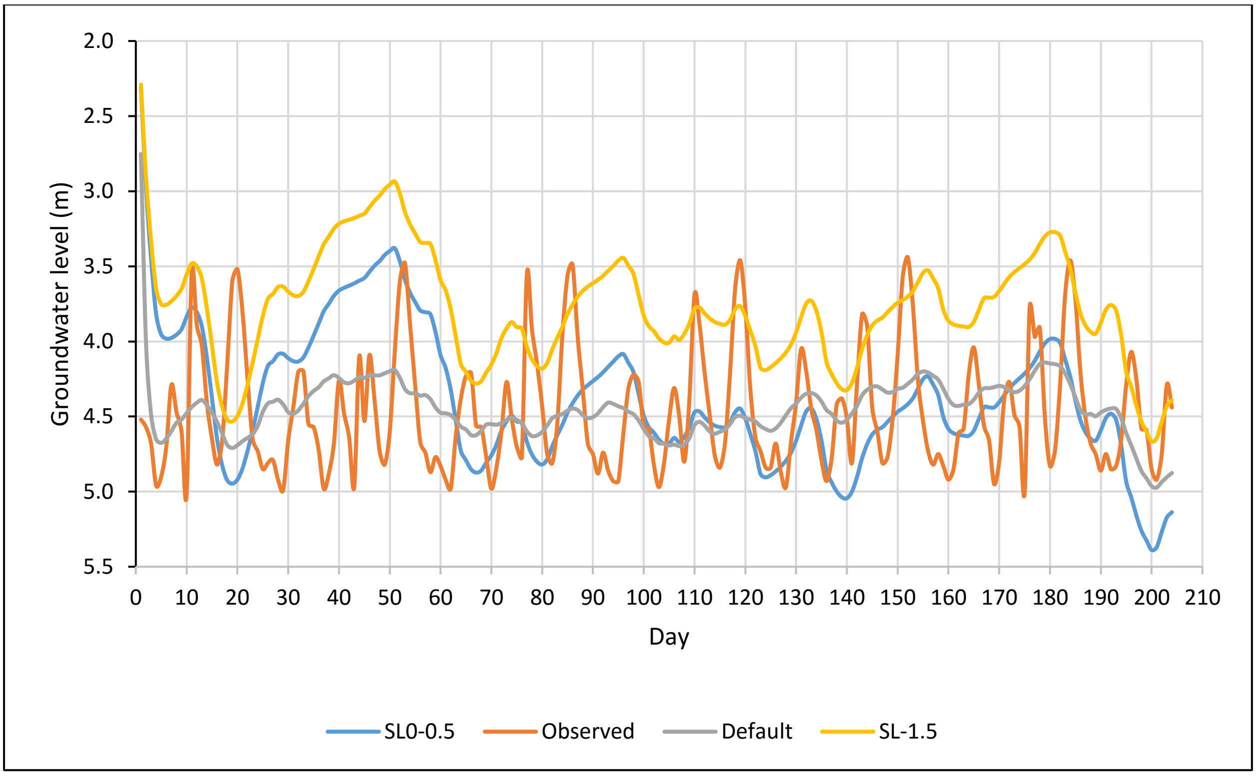


Figure 7.3: Simulated groundwater levels for ranges of SL together with observed values

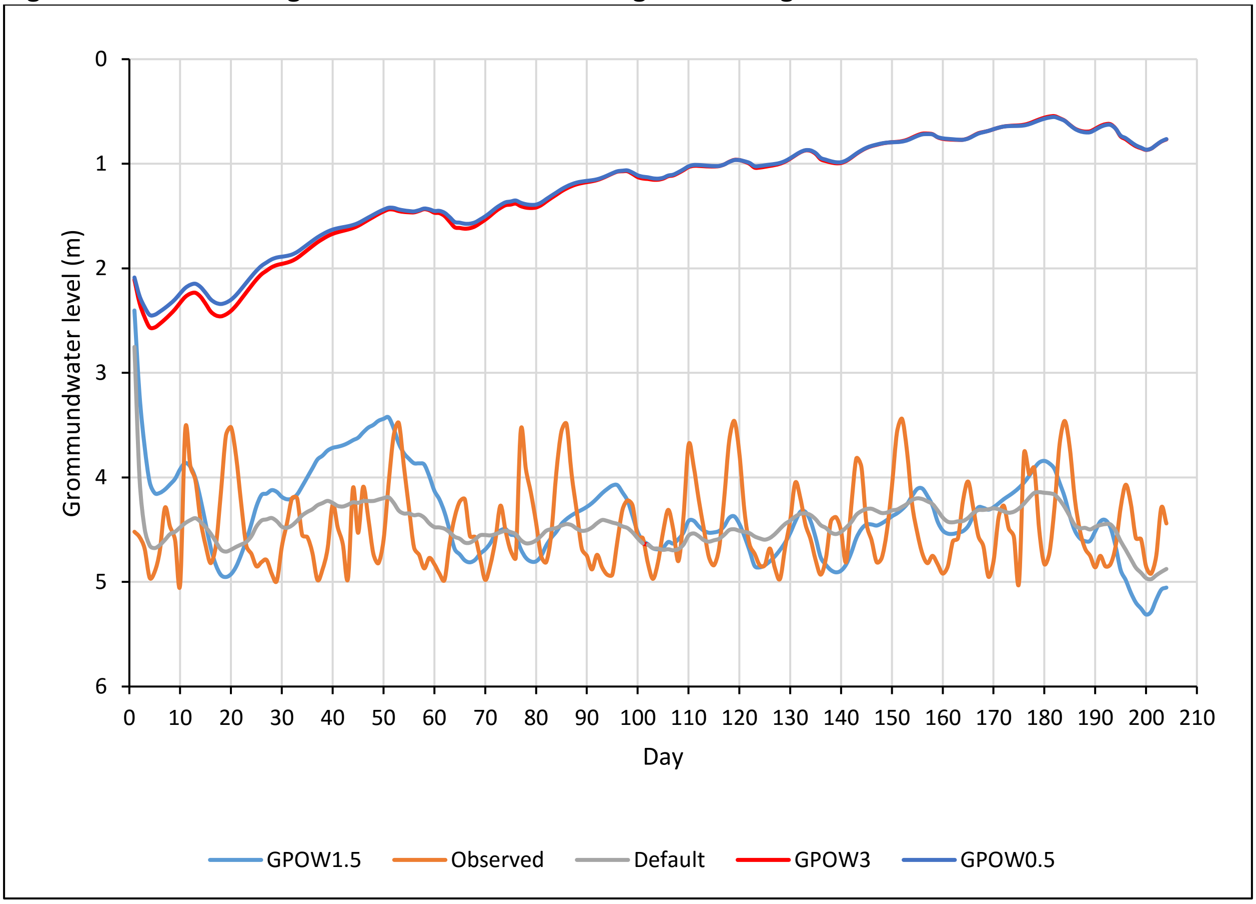


Figure 7.4: Simulated groundwater levels for ranges of GPOW together with observed values

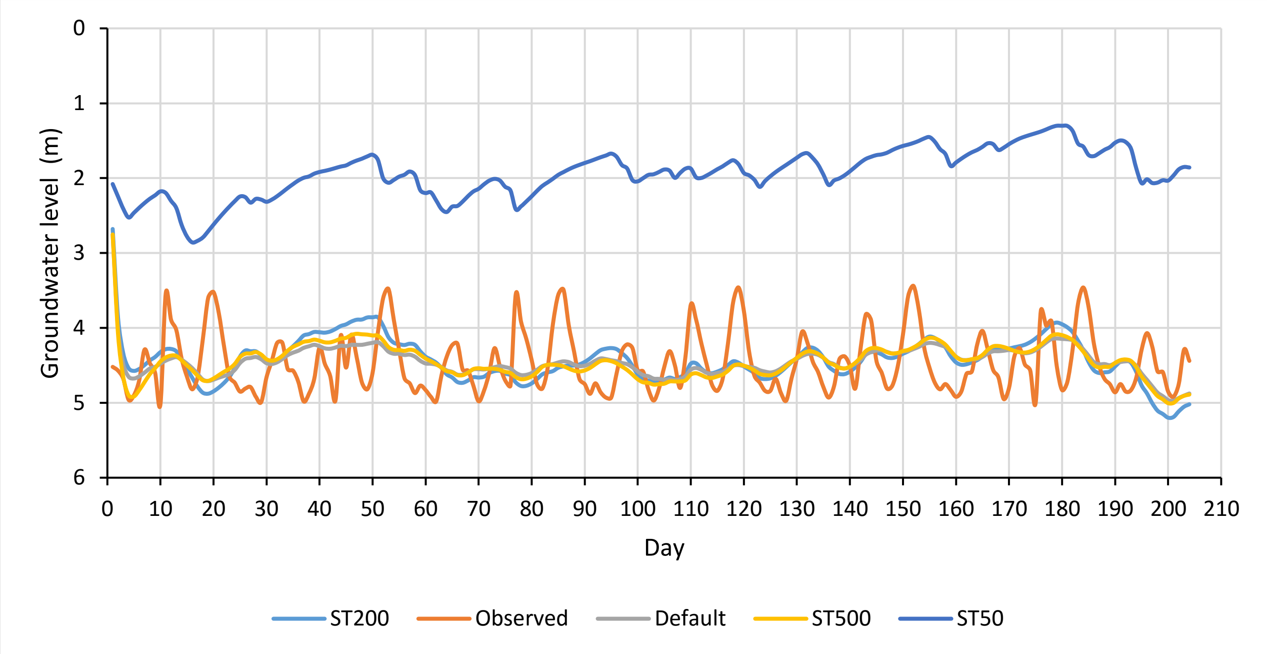


Figure 7.5: Simulated groundwater levels for ranges of ST together with observed values

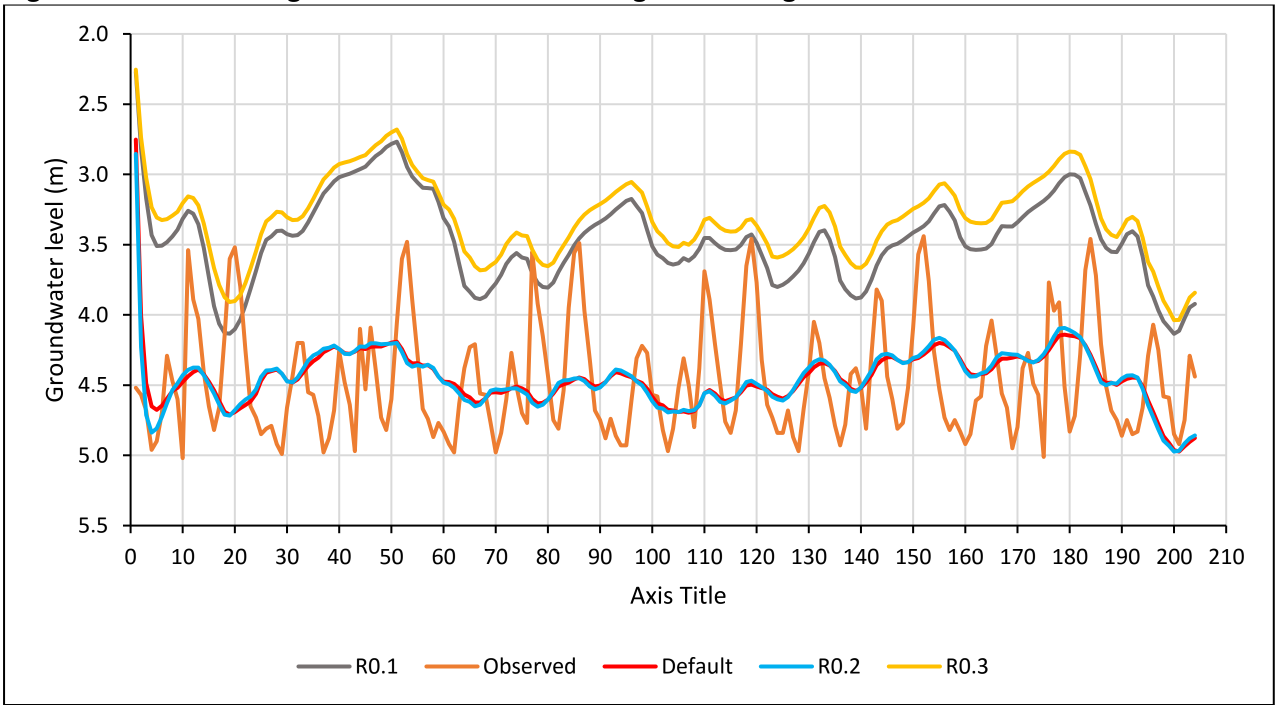


Figure 7.6: Simulated groundwater levels for ranges of R together with observed values

7.3 Calibrated GW-PITMAN model

Ranges of calibrated model parameters for 1000 calibration runs are in Table 7.2. The values of the calibrated parameters are presented in Figures 7.7 and 7.8. Most of the parameters obtained from the 1000 calibration runs were each consistently within the same range of values. This indicated that the values were close to each other and where hence reasonably well defined. ZMAX, POW, FT, ST and R were mostly well defined as they were close to each other for the 10 calibration runs in Ndiritu (2009). Thus, most of the calibrated parameters in this study were also well defined.

Table 7.2: Calibrated GW-PITMAN model parameters

Parameter	Description	Range of lower limit	Range of upper limit	Range for 1000 calibration runs
PI (mm)	Interception storage	2-4	1-8	2.12-3.99
Z_1 (mm)	Minimum catchment absorption rates. Controls surface runoff generation	2-5	1-10	2.06-4.99
Z_3 (mm)	Maximum catchment absorption rates. Controls surface runoff generation	30-50	15-70	30.11-49.99
SL1	As defined in Table 7.1	0.5-1	0-5	0.00-0.01
SL2		0.00-0.5	0.5-50	0.003-0.499
ST (mm)	As defined in Table 7.1	400-500	350-900	400.8-499.8
FT (mm)	As defined in Table 7.1	0.01-0.5	0.5-1.0	0.01-0.5
T_r (m^2/day)	Transmissivity	3-5	5-10	3.02-4.64
STOR	Storativity	0.0015-0.002	0.001-0.005	0.0015-0.002
R	As defined in Table 7.1	0.4-0.6	0-1	0.40-0.50
POW	As defined in Table 7.1	1.0-1.5	5.0-10.0	1.50-1.99
GPOW	As defined in Table 7.1	2.0-3.5	2.5-4.0	2.00-2.24
Pls	Proportion of the catchment with lower side groundwater table	0.3-0.4	0.25-0.6	0.30-0.40
RWL	Rest water level	2-5	5-10	1.50-1.99
GW (mm)	Upper limit of the groundwater recharge rate (at moisture state ST)	5-10	3-15	7.68-9.99
RSW	Width of the riparian strip	1.5-2.0	1.0-5.0	2.00-4.99

Some of the low values of PI, ZMAX and GW, and the high values of GPOW, RSW, transmissivity and storativity, to a limited extent, deviated from the range of most of the values from the 1000 calibration runs (Figures 7.7 and 7.8). This has resulted to wider range of parameter values particularly for PI, ZMAX and GW. Parameters within a wider range of values suggest that they are most likely to be redundant for a particular catchment (Ndiritu, 2009). This would mean that there is some degree of redundancy of the parameters PI, ZMAX, GW, RSW, transmissivity, storativity and GPOW, though this is of limited extent for high values of GPOW, RSW, transmissivity and storativity. It is important to take note of Ndiritu (2009) argument that redundancy of some of the model parameters does not necessarily indicate that the model is overparameterised because the relative importance of parameters varies among catchments depending on the dominant hydrological and hydrogeological processes. The heterogeneous nature of groundwater systems may also result to a wide range of parameters.

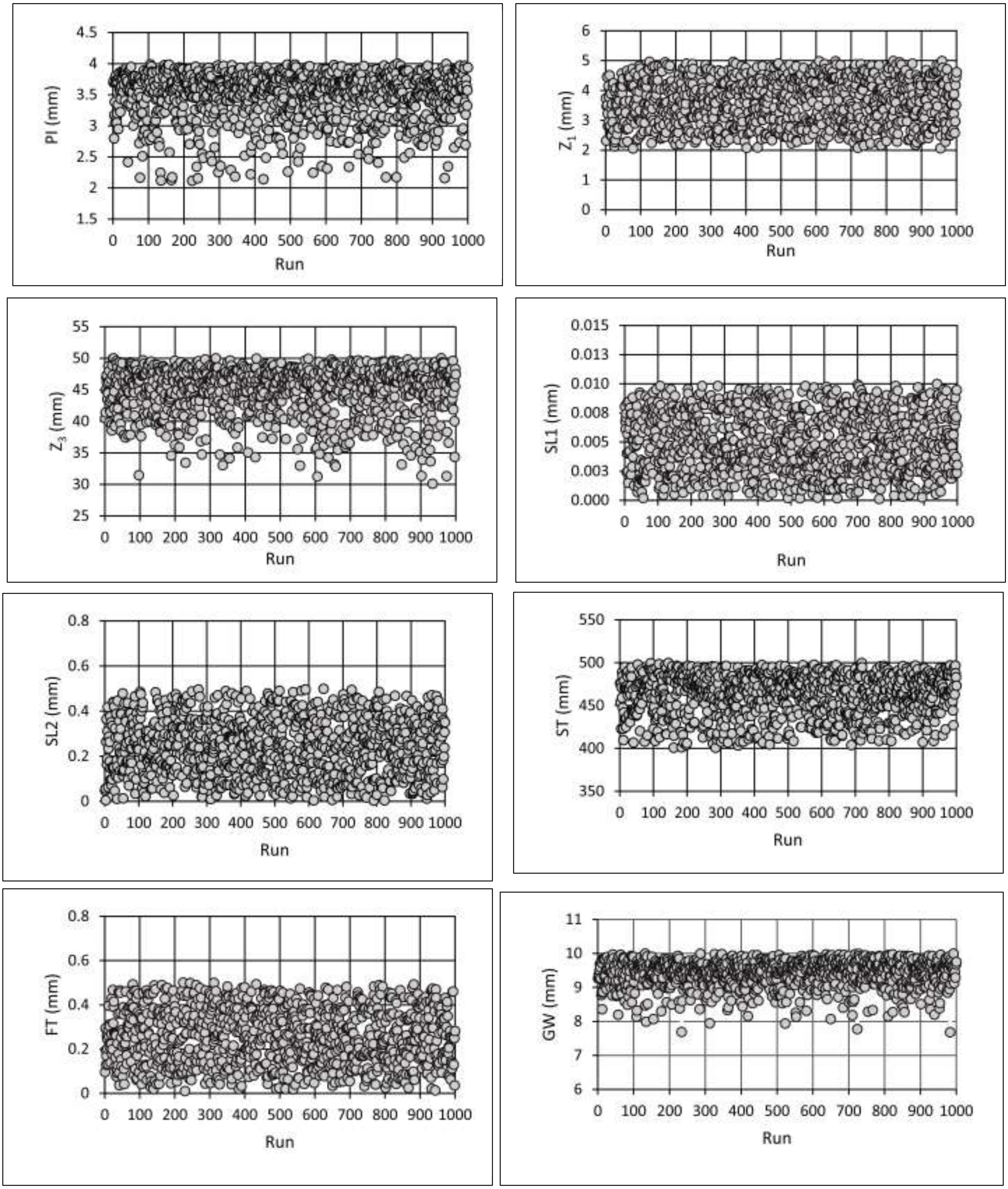


Figure 7.7: Calibrated values for PI, Z1, Z3, SL1, SL2, ST, FT and GW for 1000 calibration runs

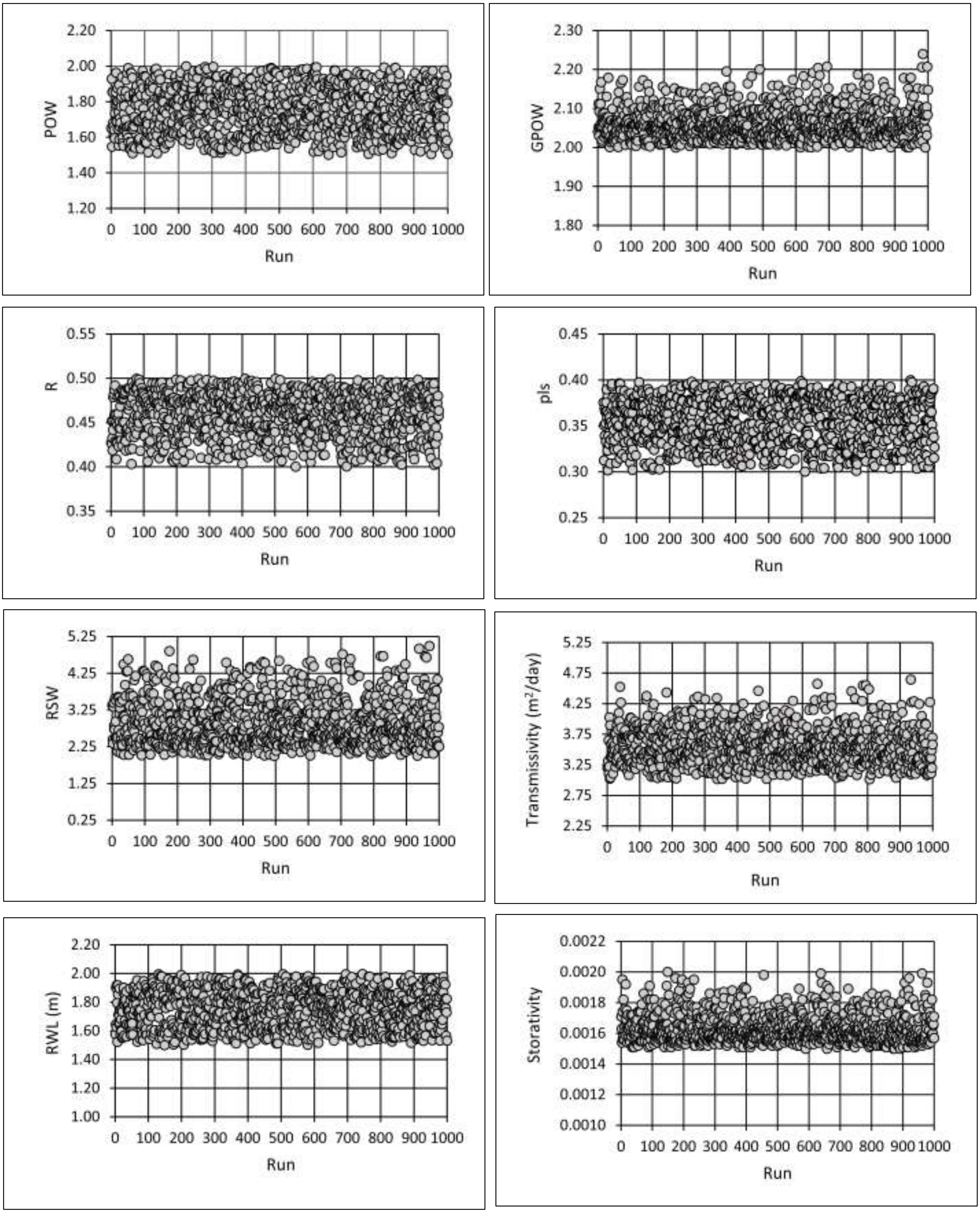


Figure 7.8: Calibrated values for POW and GPOW, R, Pls, RSW, transmissivity, RWL and storativity

The scatter plots of calibrated parameters from 1000 calibration runs and the objective function values (OBF) are presented in Figures 7.9 and 7.10. Most of the points are closely scattered though a few of them are widely scattered particularly after OBF of 95 m. This implies that parameters are not closely related at relatively high OBF values. Table 7.3 shows a comparison of calibrated model parameters with those from physically based methods (following procedures described in Kapangaziwiri (2007) and Kapangaziwiri and Hughes (2008)) and WR2012 (obtained from WRSM/Pitman version 2.9) for quaternary catchment A80A. Calibrated Z_1, Z_3, ST, FT, transmissivity and GPOW values for this study were lower than values estimated from physically based methods and/or WR2012 for A80A. RSW value from WR2012 was within the range of calibrated values. Kapangaziwiri (2007) noted that automatic calibration of Pitman in the Kafue basin in Zambia resulted to very small Z_1 values (less than 10 mm). Calibrated Z_1 values obtained in the current study were also very low and ranged from 2.06-4.00 implying that the study area has low catchment absorption rates.

The highest values of R (0.5) and POW (2) for this study were the same as those of WR2012 study and physically based methods, respectively (Table 7.3). PI, SL_1 and SL_2 are higher than those of WR2012 study. The RSW value of 4.99 m is very close to that of 4.848 m for WR2012 study. The rest of the parameters had values which were lower than those of quaternary catchment A80A from WR2012 study. The difference between WR2012 parameter values and calibrated values maybe due to the fact that groundwater resource unit occupies a small portion of A80A quaternary catchment and would have a limited range of catchment characteristics that influence variations of parameters such as Z_1, Z_3, ST and FT. The groundwater resource unit constitutes 2.5% (7.09 km^2 of 287.4 km^2) of the total area of A80A catchment.

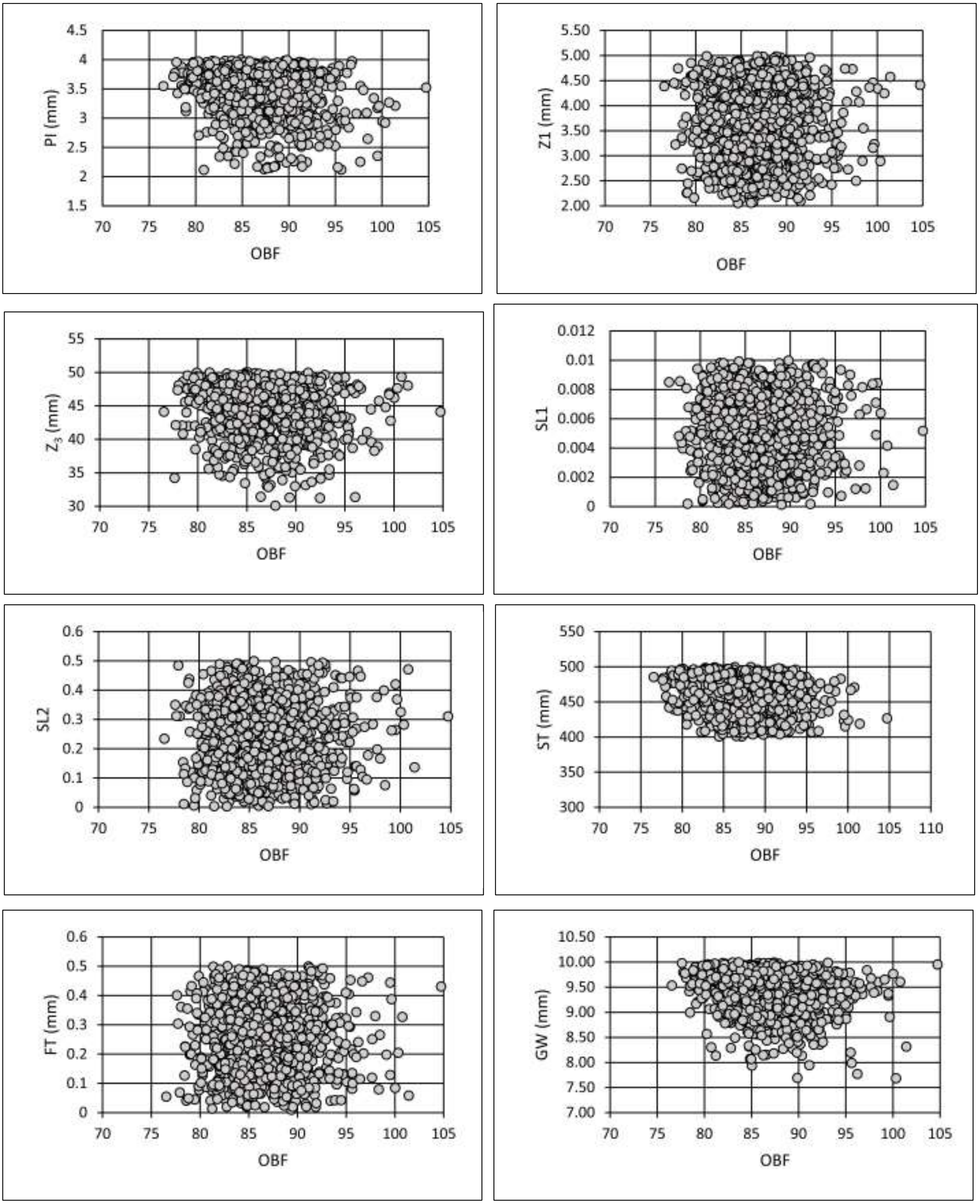


Figure 7.9: Scatter plots of calibrated values for P1, ZMIN, ZMAX, SL1, SL2, ST, FT and GW, and the objective function values

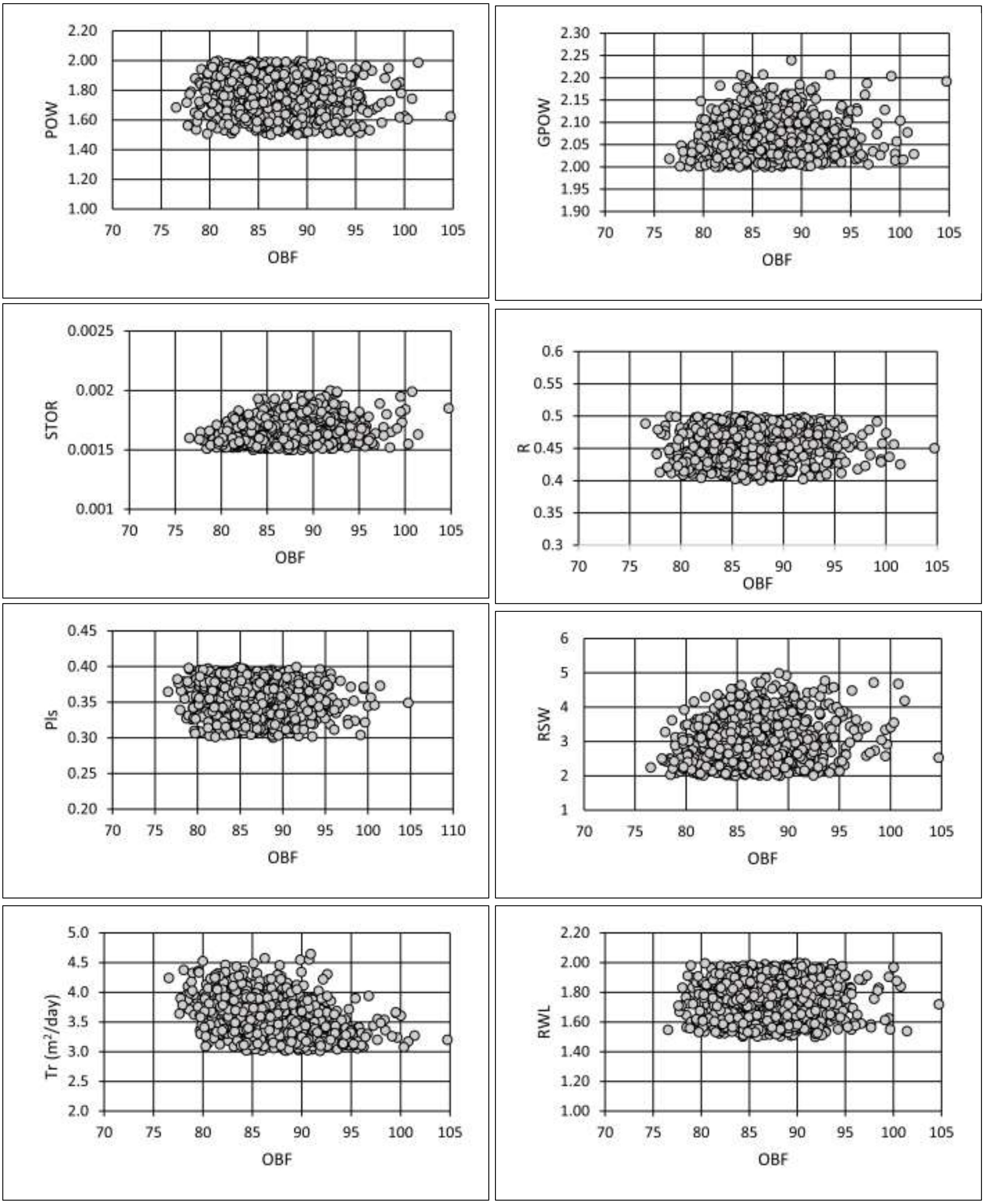


Figure 7.10: Scatter plots of calibrated values for POW, GPOW, R, Pls, RSW, Tr, RWL and storativity, and the objective function values

Table 7.3: Comparison of calibrated model parameters with those based on physical method and WR2012 values for A80A

Parameter	Current study (Range from 1000 calibration runs)	Physically based parameter estimation methods	WR2012 for A80A
1. PI	2.12-3.99		1.5
2. Z_1 (mm)	2.06-4.00	100	50
3. Z_3(mm)	30.10-50.00	500	1200
4. SL_1 5. SL_2	0.00-0.01 0.003-0.50	-	0
6. ST (mm)	400.84-499.80	12	750
7. FT (mm)	0.01-0.50	1.25	20
8. T_r (m^2/day)	3.02-4.64	-	10
9. STOR	0.0015-0.002	0.68	#
10. R	0.40-0.50		0.5
11. POW	1.50-2.00	2	3
12. GPOW	2.00-2.24	-	3
13. Pls	0.30-0.40	-	#
14. RWL	1.50-1.99	-	#
15. RSW	2.00-4.99	-	4.848

- parameter not estimated from physical method; # parameter not available

7.4 Modelled groundwater levels

Figures 7.11 shows observed and estimated groundwater levels for selected calibration runs. Generally, the hydrographs of estimated groundwater levels attempted to mimic the observed one, though the modelled graphical fits are much less variable than the observed. The perceived reason for the smoothing of the modelled groundwater levels is that the model estimates average groundwater levels for the entire groundwater resource unit while observed values are from a single borehole. Thus, model calibration and validation can be improved by using average groundwater levels from a number of boreholes which were not available for this study. As explained in section 4.4.2, calibrating using data from a single borehole is expected to give an indication of the expected behaviour of the system. This also aids in establishing the realistic nature of the simulated groundwater levels.

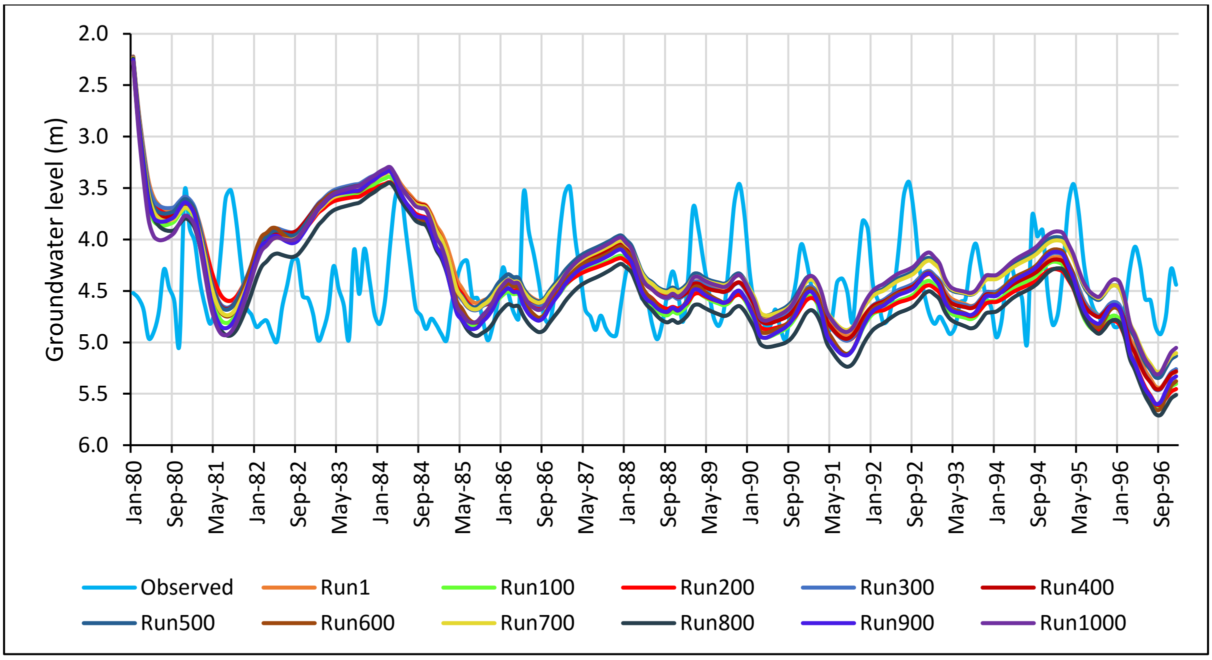


Figure 7.11: Observed and estimated groundwater levels for calibration run

Estimated groundwater levels for the selected calibration runs generally fluctuated with changes in rainfall (Figure 7.12). This behaviour is similar to that observed when groundwater levels simulated by OE-NLHW system identification model presented in Figures 6.6 and 6.7 was compared with rainfall (Figure 6.10). This indicates that the groundwater levels estimated by GW-PITMAN model give an indication of possible ranges of expected average groundwater levels and their fluctuations within the groundwater resource unit, and are hence realistic.

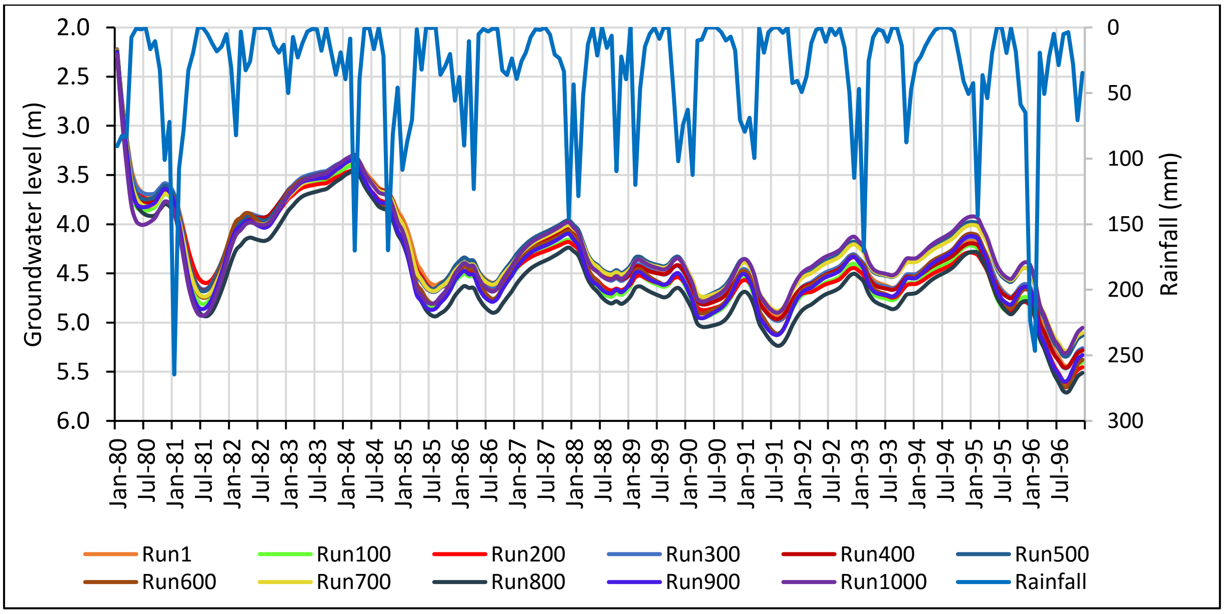


Figure 7.12: Comparison of estimated groundwater levels for calibration run and rainfall

7.5 Modelled streamflow and groundwater recharge

The behaviour of rainfall and streamflow hydrographs for both calibration and validation runs shown in Figures 7.13 and 7.14, respectively, is similar with most of the peak rainfall coinciding with peak streamflow events. This showed that streamflow hydrographs estimated in this study were also realistic. Studies such as Makungo *et al.* (2010) and Odiyo *et al.* (2012) have also used behaviour of rainfall and streamflow hydrographs to indicate realistic estimation of streamflow in ungauged catcments. Makungo *et al.* (2010) noted that similar behaviour of hydrographs and coincidence of peak rainfall and streamflow indicated realistic estimation of streamflow in a delineated sub-quaternary catchment A80A of Nzhelele River Catchment, which was also ungauged. Odiyo *et al.* (2012) also used the same criterion to determine realistic estimation of flows at an ungauged outlet of Latonyanda River quaternary catchment in Luvuvhu River Catchment, South Africa. Mike 11 NAM model was used in both Makungo et al. (2010) and Odiyo et al. (2012). Peak runoff associated with the Feb-2000 flood event was captured by the model (Figure 7.14).

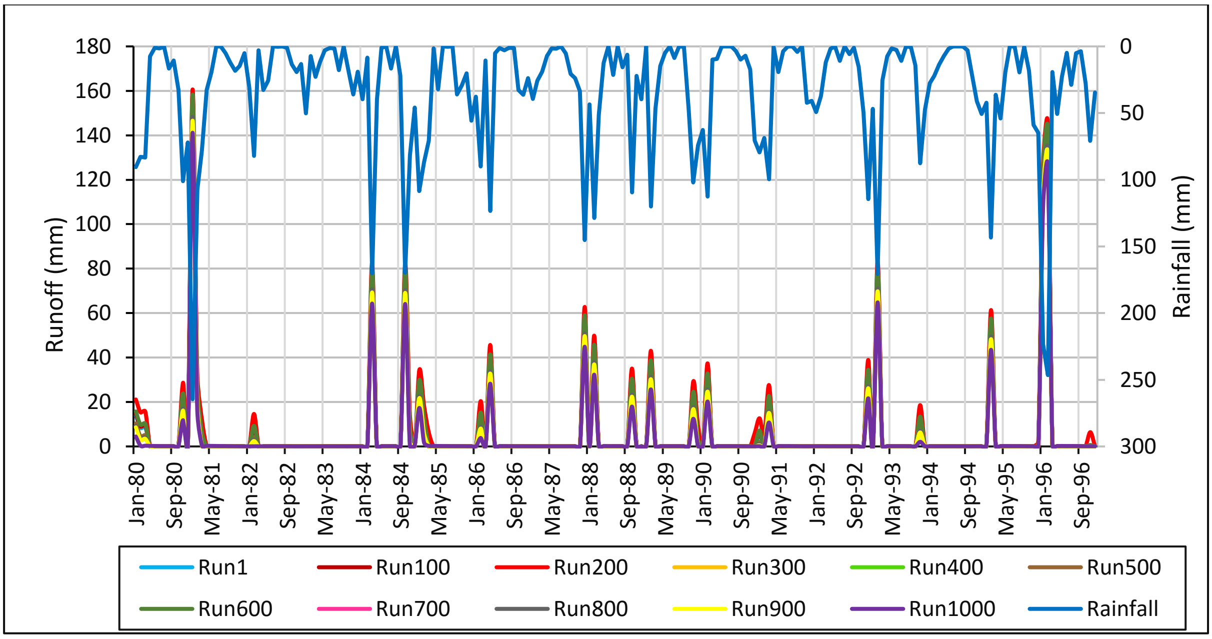


Figure 7.13: Rainfall and modelled runoff for calibration run

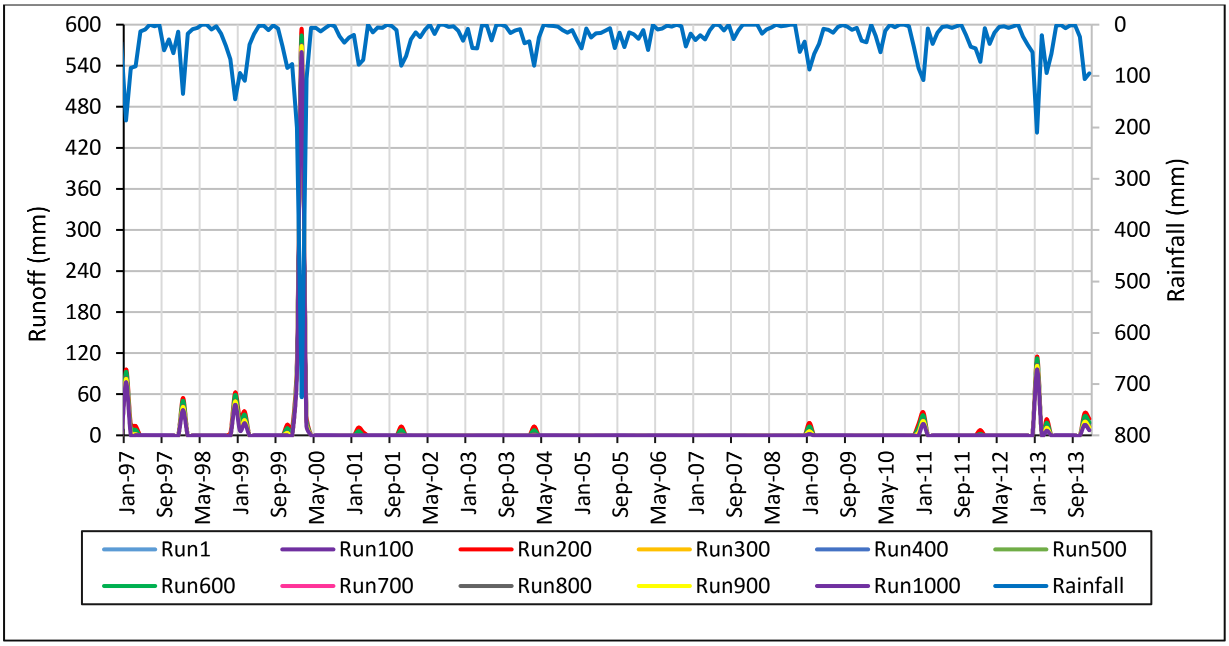


Figure 7.14: Rainfall and modelled runoff for validation run

Figures 7.15 and 7.16 show comparisons of modelled recharge and rainfall for calibration and validation runs, respectively. Recharge values generally increased after rainfall events. Peak recharge events also occurred after peak rainfall events, though there were lags in certain cases. This shows that the estimated recharge values were also realistic. The annual average recharge for these runs obtained from Equation 4.31 ranged from 1.96-12.40 mm. These are lower than the average recharge of 92.94 mm for A80A quaternary catchment from DWAF (2006b). This is expected because Siloam is located in the dry part of the catchment with low rainfall while average recharge value for quaternary catchment A80A incorporates recharge contribution from the upper parts of the catchment with relatively high rainfall.

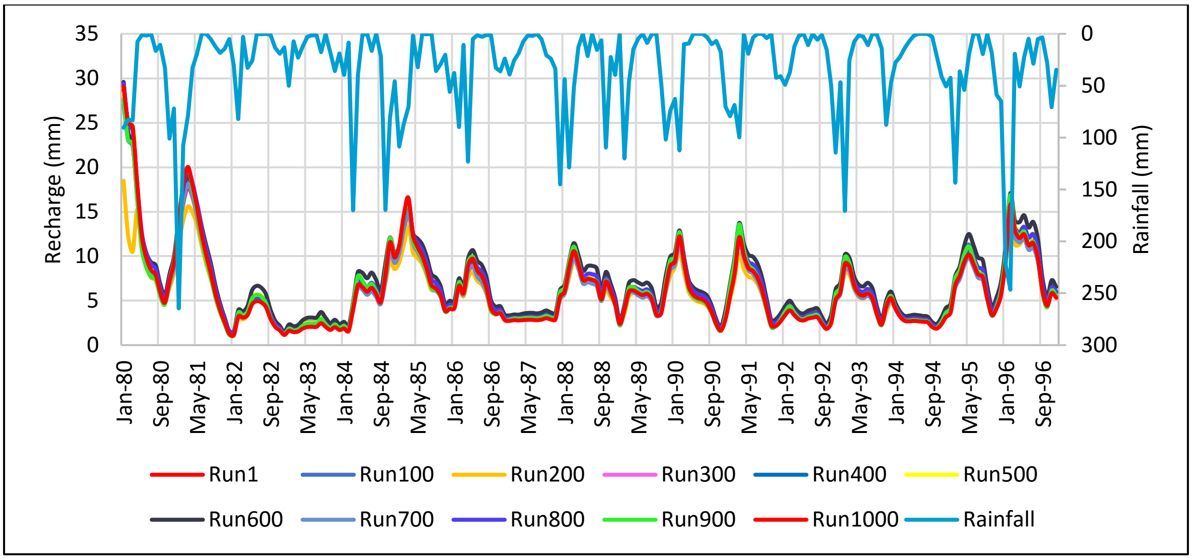


Figure 7.15: Rainfall and recharge for calibration run

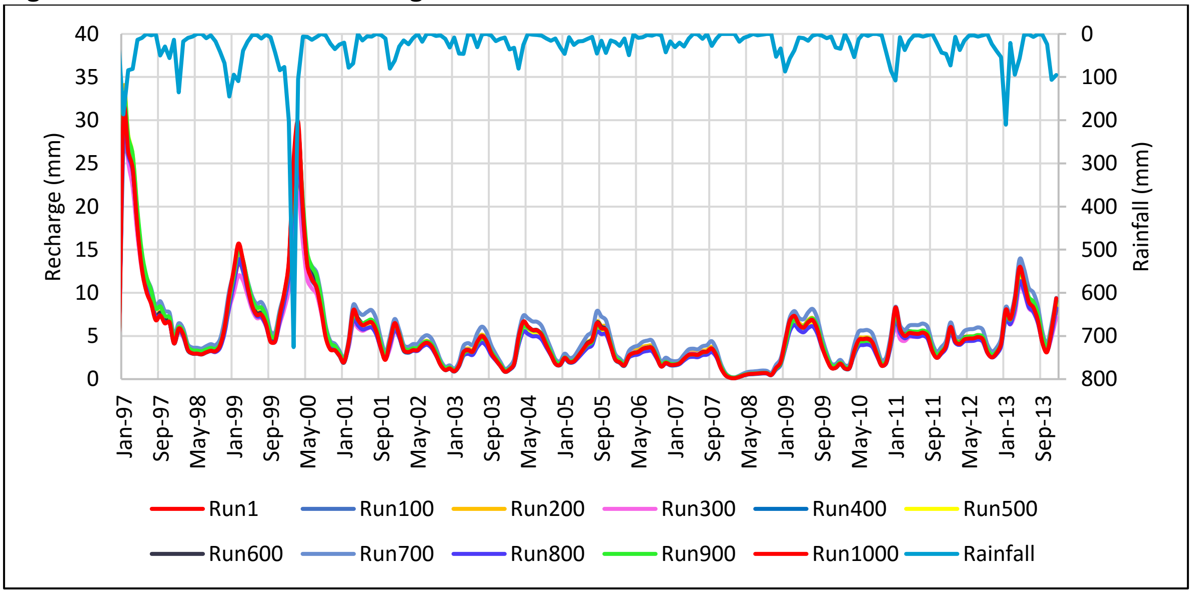


Figure 7.16: Rainfall and recharge for validation run

7.6 Chapter summary and contribution

The results of sensitivity analysis indicated that SL, ST, GPOW and R were sensitive model parameters. FT and POW were not sensitive parameters as they resulted in no significant changes in groundwater levels. Most of the parameters obtained from the 1000 calibration runs were each consistently within the same range of values indicating that values were close to each other. Most points on the scatter plots of each calibrated parameter from 1000 calibration runs and objective function values were closely scattered indicating that they are related. Most of the parameters had values which were lower than those of WR2012 study for A80A quaternary catchment and these differences were not unexpected as the groundwater resource unit occupies a small portion (2.5%) of the quaternary catchment.

Calibrated Z1 values obtained in the current study were also very low and this was in agreement with other studies. Hydrographs of modelled groundwater levels mimicked the mean observed ones reasonably well but had a much lower variability. The low variability of the modelled groundwater levels was likely to be due to the fact that the model estimated average groundwater levels for entire groundwater resource unit while observed values were from a single borehole. Thus, average groundwater levels from a number of boreholes could improve the model fit.

Modelled groundwater levels, streamflow and groundwater recharge for both calibration and validation runs generally fluctuated with changes in rainfall indicating that groundwater levels, streamflow and groundwater recharge were modelled realistically. Thus, the generated groundwater levels were suitable for used in development of simplified groundwater operating rules. This chapter aided in addressing the second specific objective and research question.

CHAPTER 8: GENERATION OF STOCHASTIC RAINFALL, EVAPORATION AND GROUNDWATER LEVELS

8.1 Preamble

This chapter presents the results and assessment of the stochastic generation of rainfall, evaporation and groundwater levels. Box plots of various statistics are used to compare the generated sequences with the historic ones in order to determine if the historic statistics were preserved. Stochastic inputs were required for base yield analysis since the study followed a risk-based approach for development of groundwater operating rules.

8.2 Comparing stochastically generated rainfall, evaporation and groundwater levels with historic data

Box plots comparing statistics of 100 stochastically generated sequences of rainfall, evaporation and groundwater levels with those from historic data are presented in Figures 8.1 to 8.6. The box plots start in the month of July to ensure that the year begins and ends in the driest months as explained in section 3.6. In the box plots, the box indicates the interquartile range (25 to75% quartiles) while lower and upper ends of the whiskers indicate the minimum and maximum values in the stochastically generated sequences, respectively. The lower, middle and upper horizontal lines in the box plots' interquartile range indicate the lower (25%), median (50%) and upper (75%) quartiles, respectively.

8.2.1 The mean, median, 25th and 75th percentiles, lowest, highest, standard deviation and skewness

Comparison of box plots of historic mean, median, 25th and 75th percentiles of rainfall (Figure 8.1). These statistics were within or at the boundary of the interquartile range, except for the historic mean values for February. This means that these statistics generally had good performance following Apipattanavis *et al*. (2007) and Prairie *et al.* (2007). The historic mean rainfall for February was slightly higher than the upper quartile of the box plot (Figure 8.1) while the historic highest rainfall, standard deviation and skewness for February were higher than the upper quartile and closer to the maximum stochastically generated values (Figure 8.2). February is the peak rainfall month in the study area.

The historic standard deviation values for rainfall for the months of September, November and March were below the lower quartile unlike the other months when they were within the interquartile range (Figure 8.2) and were well preserved. The historic skewness values for rainfall for the months of September, November, May and June were below the lower quartile. However, the skewness value for November was equal to the minimum value of the stochastically generated values (Figure 8.2). The rest of the values were within the interquartile range indicating that they were well preserved. In Efstratiadis *et al.* (2014), historic mean and standard deviation were well preserved when Castalia software was used to generate stochastic rainfall. However, there were a few cases where the skewness was not well preserved and it was suggested that improvements of numerical routines of the software would remedy this.

Lowest historic and stochastically generated rainfall values were mostly the same (Figure 8.2). Lowest historic rainfall for March was above zero though it was within the interquartile range. The lowest historic rainfall for November, December and April were also within the interquartile range though they were above the median but below the upper quartile. The lowest historic rainfall for January and February were outside the interquartile range but within the maximum stochastically generated values. This indicated that lowest rainfall was mostly well preserved by the VLB generator.

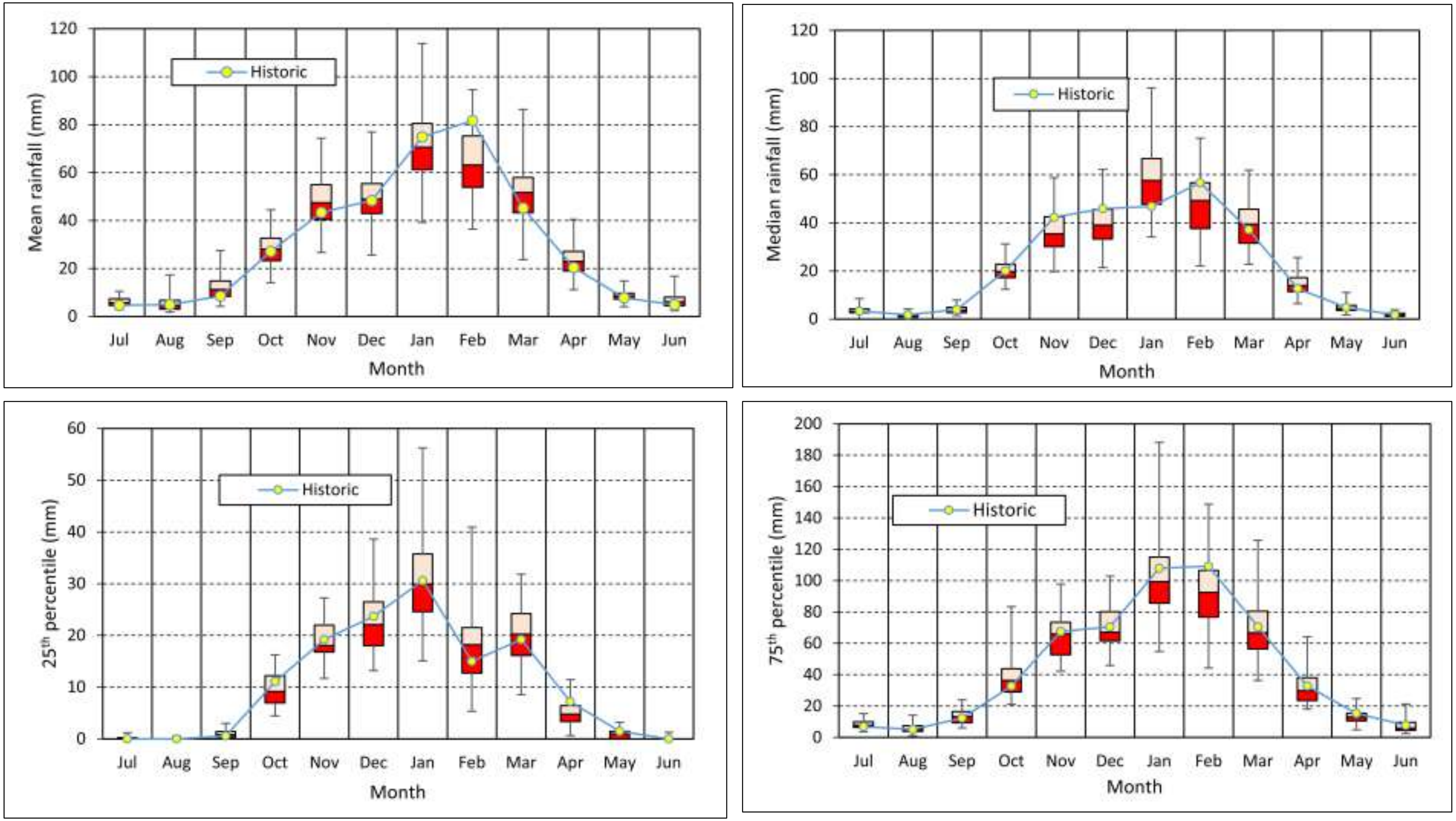


Figure 8.1: Box plots of mean, median, 25th and 75th percentile rainfall compared with historic values

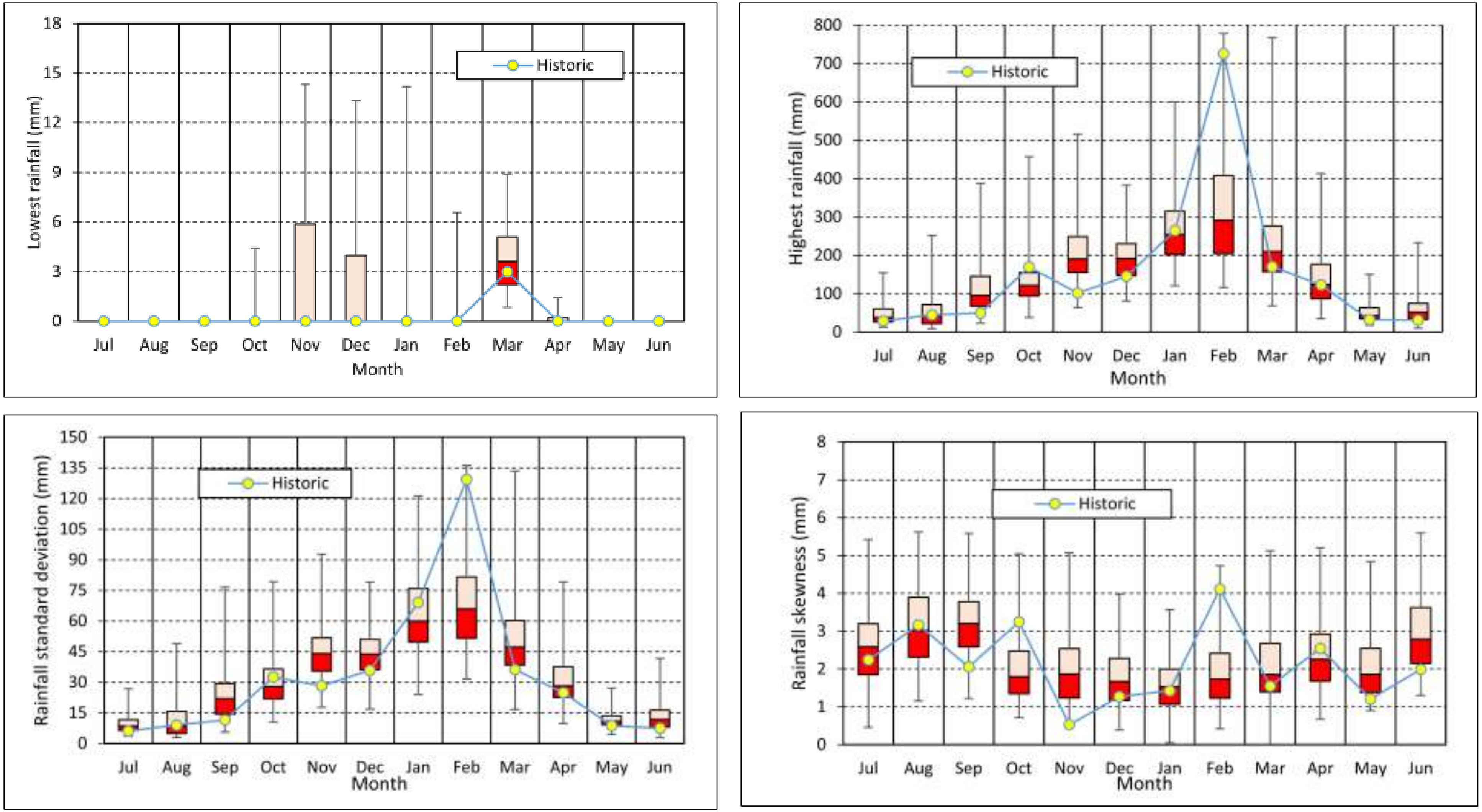


Figure 8.2: Box plots of lowest and highest rainfall, standard deviation and skewness compared with historic values

The historic mean, median, 25th and 75th percentiles of evaporation were mostly within the interquartile ranges of stochastically generated values (Figure 8.3), indicating that they were mostly well preserved. The lowest historic evaporation for November, December, January and March were below the interquartile range but those for the first three of these months were very close to or coincided with the minimum values of the stochastically generated sequences (Figure 8.4). The historic highest evaporation values for July to October and March were slightly below the interquartile range. The historic standard deviation values for evaporation for September, October and March, and November, December and April were slightly below and above interquartile range, respectively. Historic skewness values for all the months were lower than the interquartile range but did not go below the minimum values of stochastically generated sequences (Figure 8.4).

Mean values of evaporation sequences generated using a simple regression model in the northern territory of Australia were similar to historic ones (Chiew and Wang, 1999), though most of the computed values of skewness were lower than historic ones. This was attributed to large uncertainties in the skewness estimated from only 27 years of observed data. Alhassoun *et al.* (1997) also reported that most of the computed values of skewness in one of the stations in Saudi Arabia were lower than the historical ones due to the high variability of the historical data. The findings of Chiew and Wang (1999) and Alhassoun *et al.* (1997) are comparable with those of the current study which indicated that the historic skewness values were mostly lower than the interquartile range and were hence not well preserved. In this study, historical data used in stochastic generation was for a period of 33 years which is comparable to 27 years used in Chiew and Wang (1999). Thus, limited data used in generation of stochastic sequences may have affected the preservation of skewness values for evaporation and groundwater levels.

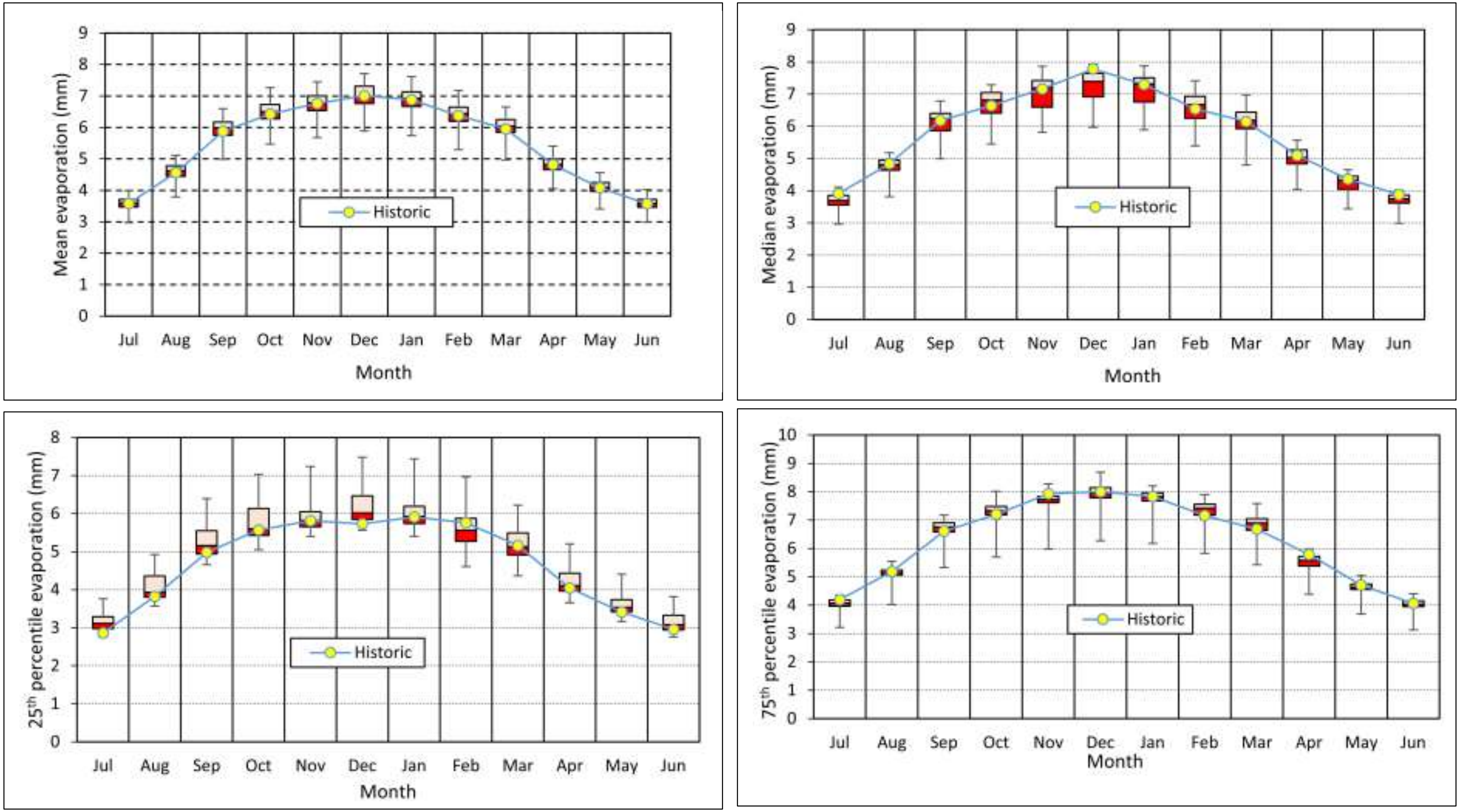


Figure 8.3: Box plots of mean, median, 25th and 75th percentiles evaporation compared with historic values

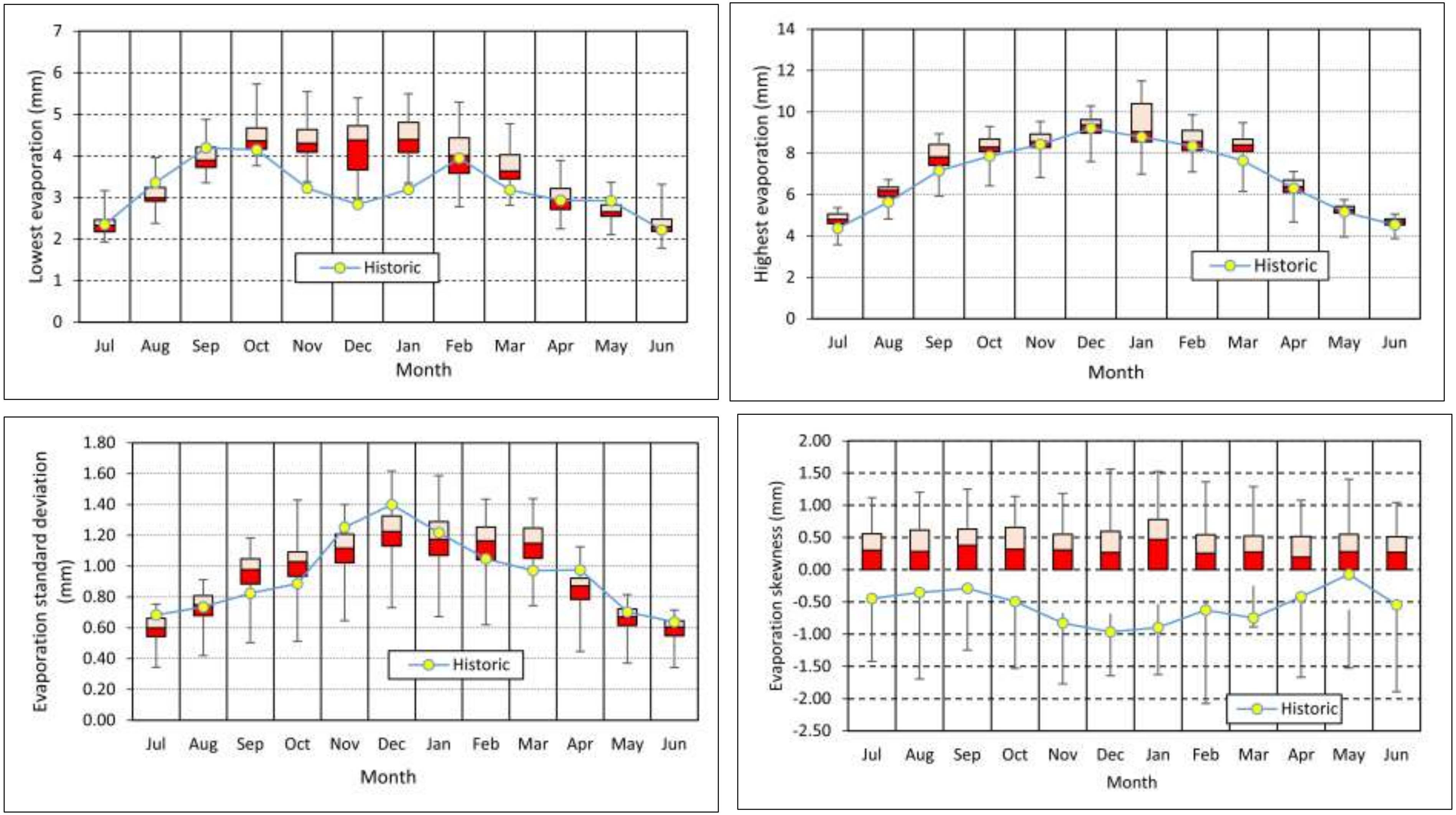


Figure 8.4: Box plots of lowest and highest evaporation, standard deviation and skewness compared with historic values

Mean historic groundwater levels for all months were within interquartile range (Figure 8.5). Median historic groundwater levels for December, February and June, 25th percentile for March and 75th percentile for July were at the upper quartile. Historic 25th percentile value for April was lower than the lower quartile. Historic 75th percentile for August was above the upper quartile. Historic standard deviation values were mostly within the interquartile range (Figure 8.6), indicating that they were mostly well preserved, except for July when it was above interquartile range. Lowest historic groundwater levels for September, March, April and June were above the interquartile range, while the rest were within. All highest historical groundwater levels were higher than the interquartile range though the value for May was closer to the upper quartile. All historic skewness values were below the interquartile range.

de Farias *et al.* (2011) used a neural network based stochastic model to generate 5 synthetic series of daily groundwater levels and obtained mean and standard deviation values which were close to the historic ones. The results are comparable with those of this study. de Farias *et al.* (2011) did not compute other statistics such as highest, lowest, percentiles and skewness which were computed in the current study.

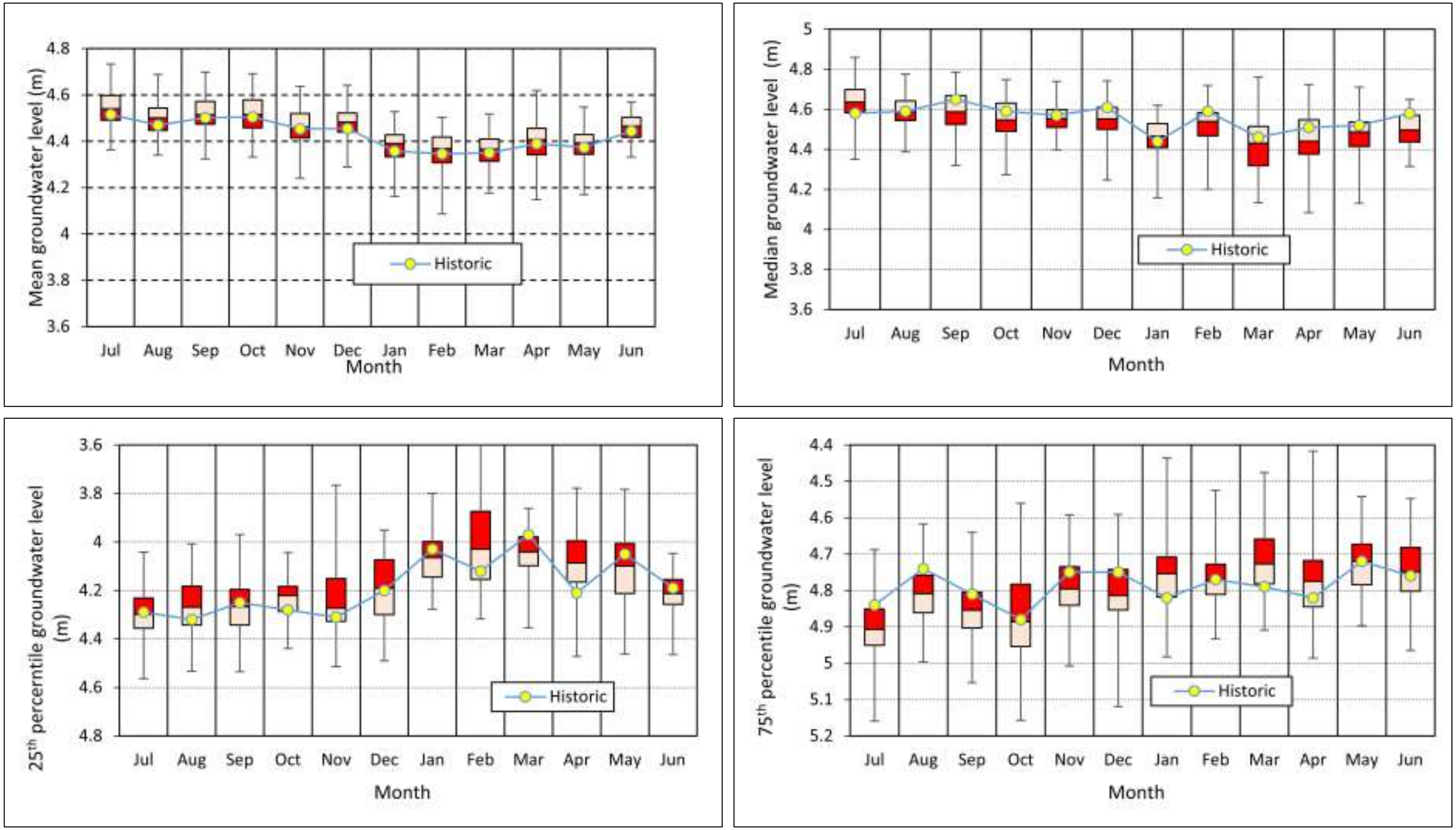


Figure 8.5: Box plots of mean, median, 25th and 75th percentiles of groundwater levels compared with historic values

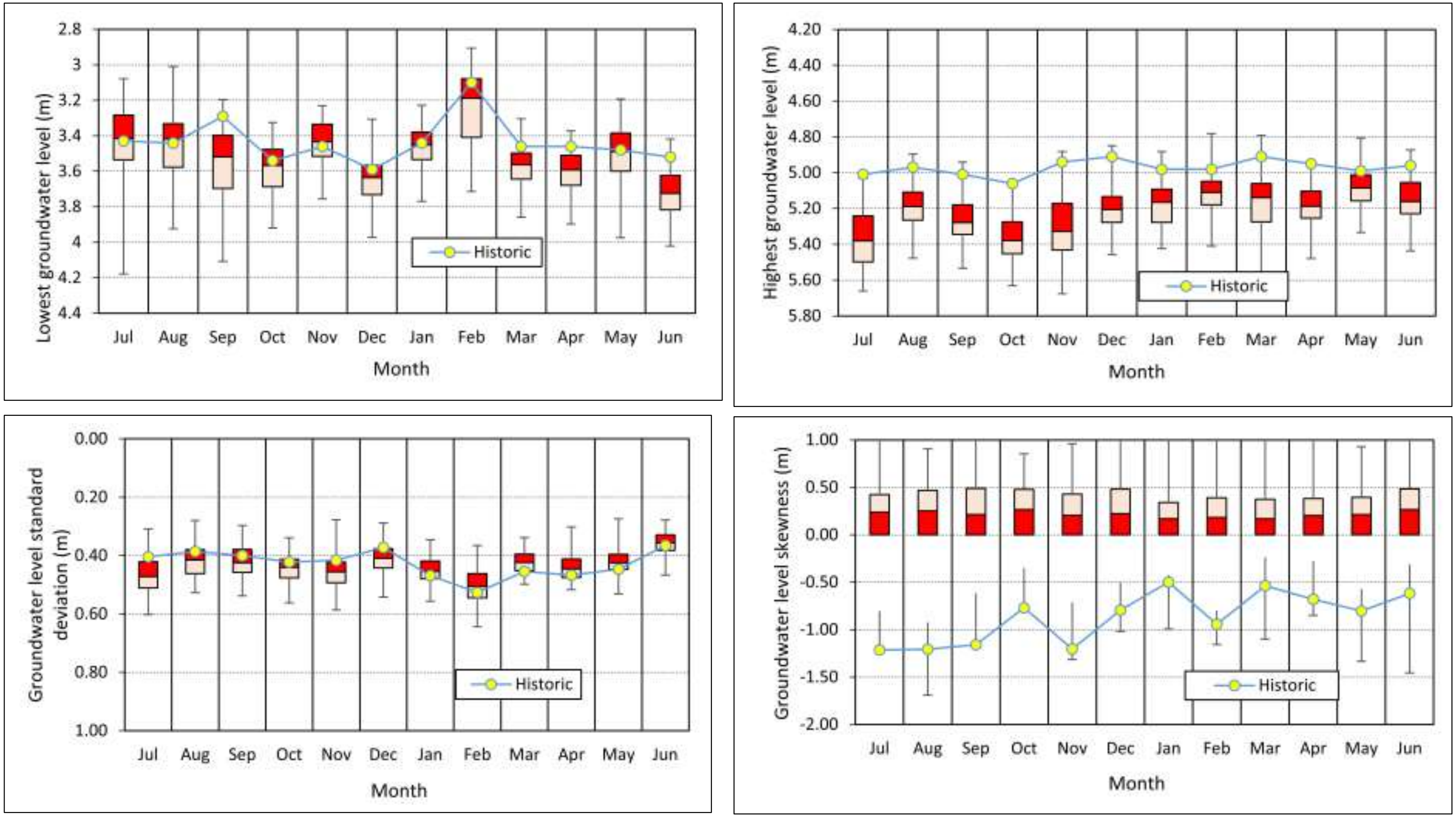


Figure 8.6: Box plots of lowest and highest groundwater level, standard deviation and skewness compared with historic values

Table 8.1 shows the percentage of times that historic statistics were below interquartile range, above interquartile range, and beyond minimum and maximum (BMM) values of the box plots. This is important to further validate the accuracy of VLB in preserving historic statistics. Historic skewness was the statistic with the highest percentage months which were not within the interquartile range for evaporation and groundwater levels. The results of this study indicated that skewness is the statistic that was not well preserved for all variables with values for evaporation and groundwater levels being below interquartile range 100% of the time (12 months). This indicated that the historic skewness for evaporation and groundwater levels were mostly overestimated. Historic highest groundwater levels also indicated that the stochastically generated values were underestimated for 92 percent of the time (11 months).

Table 8.1: Percentage of times that historic statistics were below interquartile range, above interquartile range, and BMM values within a 12 months period

	Rainfall			Evaporation			Groundwater levels		
Statistic	Below interquartile range (%)	Above interquartile range (%)	BMM (%)	Below interquartile range (%)	Above interquartile range (%)	BMM (%)	Below interquartile range (%)	Above interquartile range (%)	BMM (%)
Mean	0	8	0	0	0	0	0	0	0
Median	0	0	0	0	0	0	0	0	0
25th percentile	0	0	0	0	0	0	8	0	0
75th percentile	0	0	0	0	0	0	0	8	0
Lowest	0	0	0	33	17	0	0	33	0
Highest	8	8	0	42	0	0	0	92	0
Standard deviation	8	8	0	25	33	8	0	8	0
Skewness	25	17	0	100	0	0	100	0	0

Obtaining historic statistics below interquartile range (overestimation) is a problem that commonly arises with weather generators (Apipattanavis *et al.*, 2007). Furrer and Katz (2007) also indicated that stochastic generators have the tendency to underestimate variability of weather statistics. The latter study explained that this can be reduced by including additional covariates in stochastic generators that influence atmospheric circulation. A review of stochastic rainfall and streamflow generators provided in Ndiritu and Nyaga (2014) indicated that some of stochastic generators were also unable to reproduce the skewness. This explains why some of the statistics were not well preserved in the current study.

Ndiritu and Nyaga (2014) reported standard deviation and skewness of historic rainfall with more than 10% of their monthly values beyond interquartile range. Nyaga (2014) reported that more than 10% of the monthly values of standard deviation, skewness and highest rainfall were beyond interquartile range. In the current study, historic highest rainfall, standard deviation and skewness were only above interquartile range for 1 or 2 months and were thus better preserved or comparable to those of studies by Ndiritu and Nyaga (2014) and Nyaga (2014). However, this was not the case with evaporation and groundwater levels where some of the lowest, highest, and skewness values were poorly preserved in the current study. Chiew and Wang (1999) and Steinschneider and Brown (2013) attributed poor preservation of skewness to limited historical data used for stochastic generation of weather variables.

8.2.2 Cross and serial correlation coefficients

The historic cross correlation between rainfall and groundwater levels for the months of November, January to April and June were below the lower quartile (Figure 8.7). For the month of September the historical cross correlation between rainfall and groundwater levels was slightly above the upper quartile while for the month of July it was at the upper quartile. The historic cross correlations values for August, October and December were within the interquartile ranges, indicating that they were well preserved. The historic cross correlation value for May was just almost at the lower quartile.

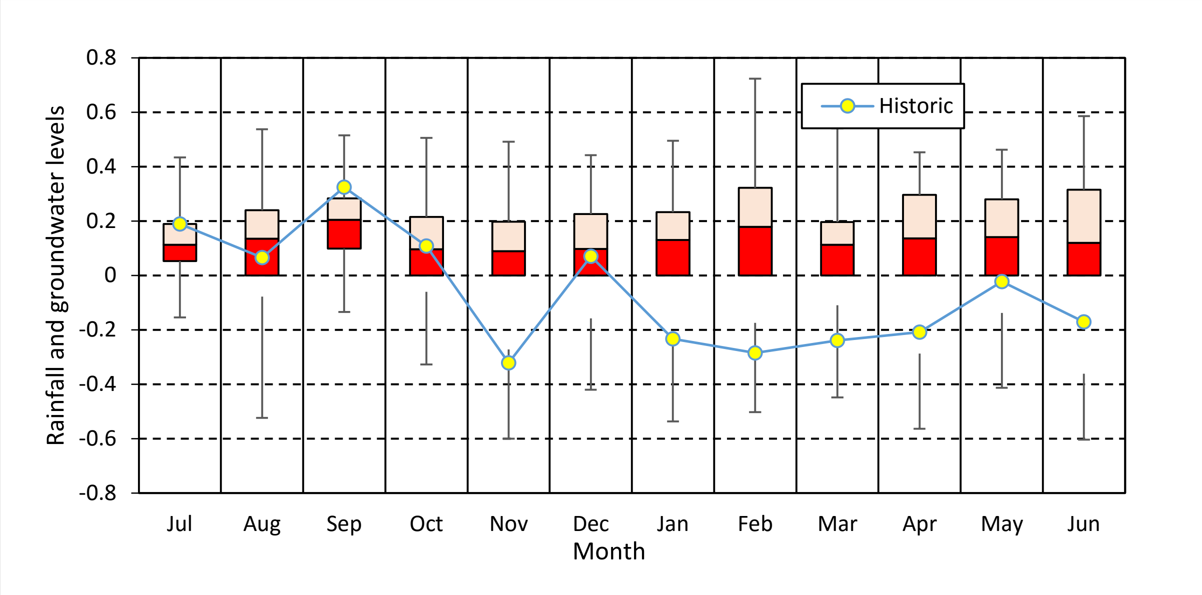


Figure 8.7: Cross correlation of rainfall and groundwater levels

The historic cross correlations of evaporation and groundwater levels were below the lower quartiles for August, September, November and June while that for January was almost at the upper quartile (Figure 8.8). Historic cross correlations of evaporation and groundwater levels for July, October, December and February to May were within interquartile range indicating that they were well preserved. Historic cross correlations of rainfall and evaporation were below the lower quartiles in the months of July, September, February, March, May and June while those for August, January and April were at the lower quartile (Figure 8.9). Historic cross correlation values for October, November and December were within the interquartile range, indicating that they were well preserved.

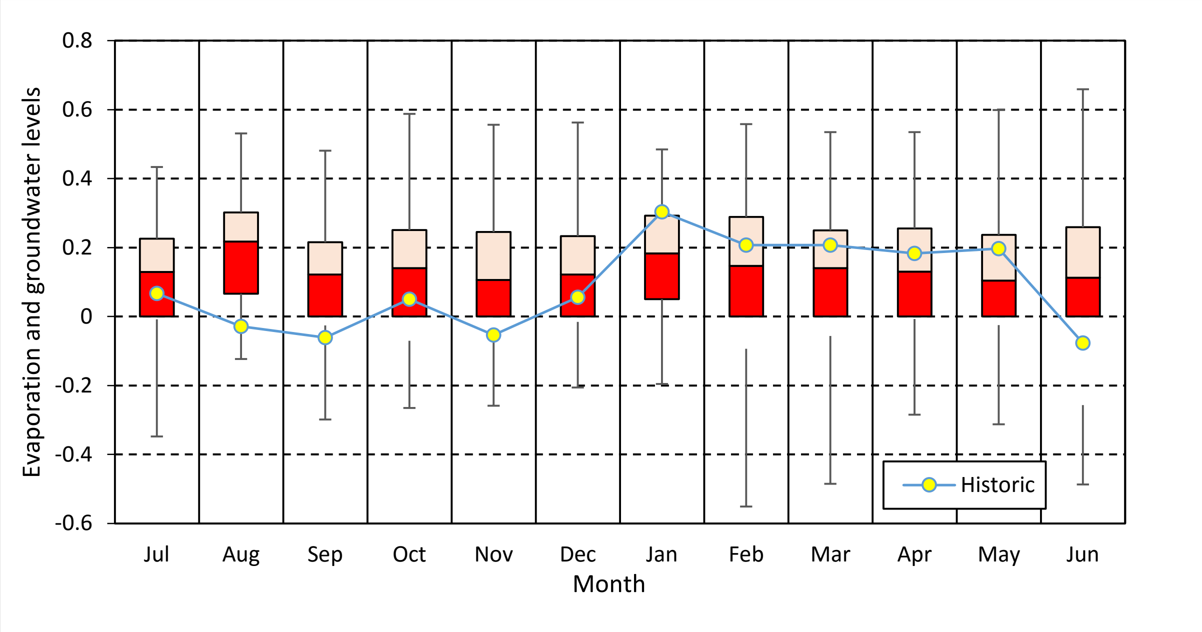


Figure 8.8: Cross correlation of evaporation and groundwater levels

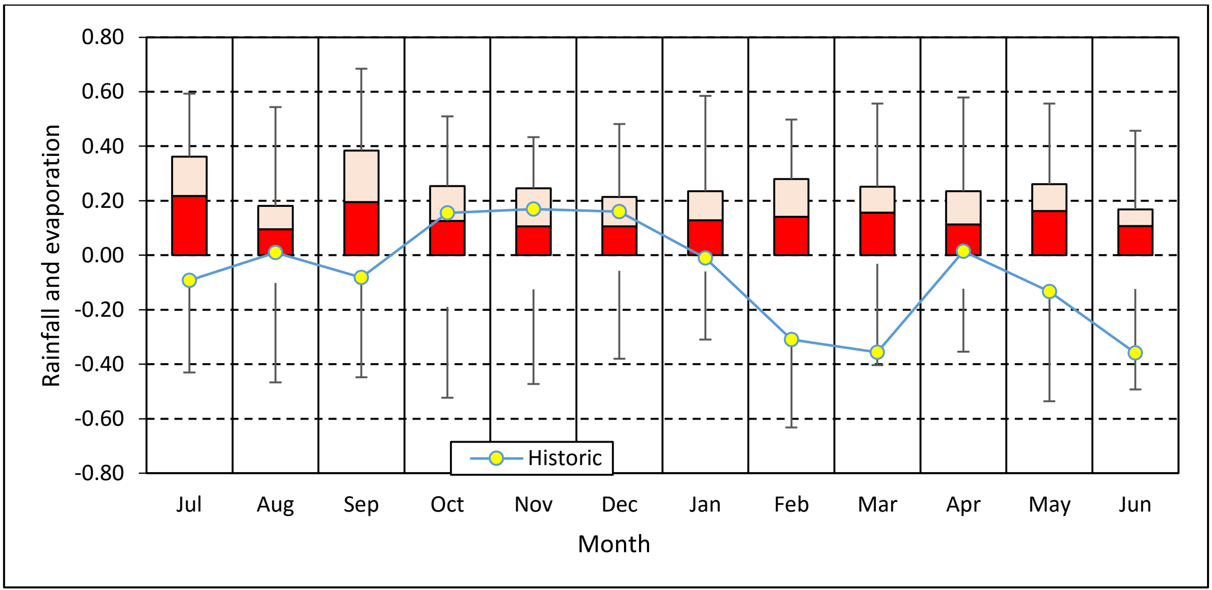


Figure 8.9: Cross correlation of rainfall and evaporation

The historic annual cross correlations between all variables (Figure 8.10) were within the interquartile range indicating that they were well preserved. Preservation of cross correlation of multivariate time series is essential either due to cause-effect relationship of hydrometeorological or common hydroclimatic regime (Efstratiadis *et al.*, 2014).

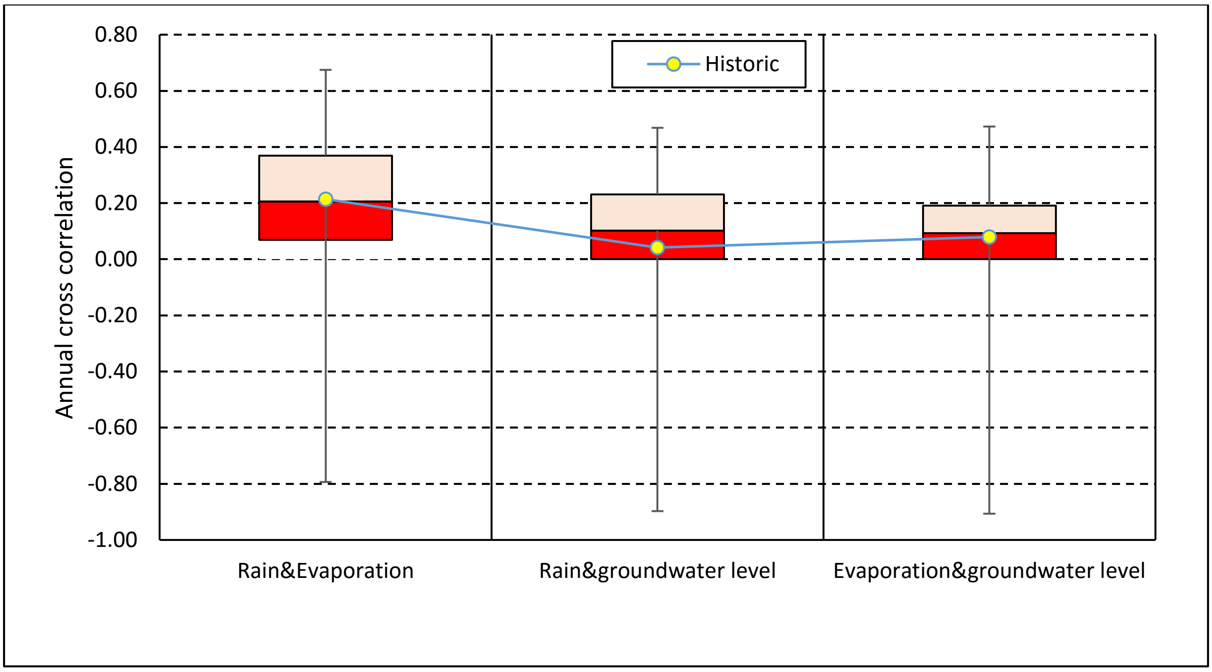


Figure 8.10: Annual cross correlations of rainfall evaporation and groundwater

Figures 8.11 to 8.13 show historic serial correlations for rainfall, evaporation and groundwater levels, respectively. Historic serial correlation of rainfall for September, October and February were well preserved, since they were within the interquartile range. Values for the rest of the months were outside the interquartile range. Historic serial correlation of evaporation for July to September and January to April were within the interquartile range (Figure 8.12), and were thus well preserved. Values for October, November, December and May were at the upper quartile while that of June was outside the box plot limits. Historic serial correlations for groundwater levels in the months of July to October, and December to May were within interquartile range indicating that they were mostly preserved. The value for June is outside the maximum stochastically generated value and thus not preserved while the November value is just below the lower quartile. Ndiritu and Nyaga (2014) reported serial and cross correlations with more than 10% of their monthly values outside interquartile range.

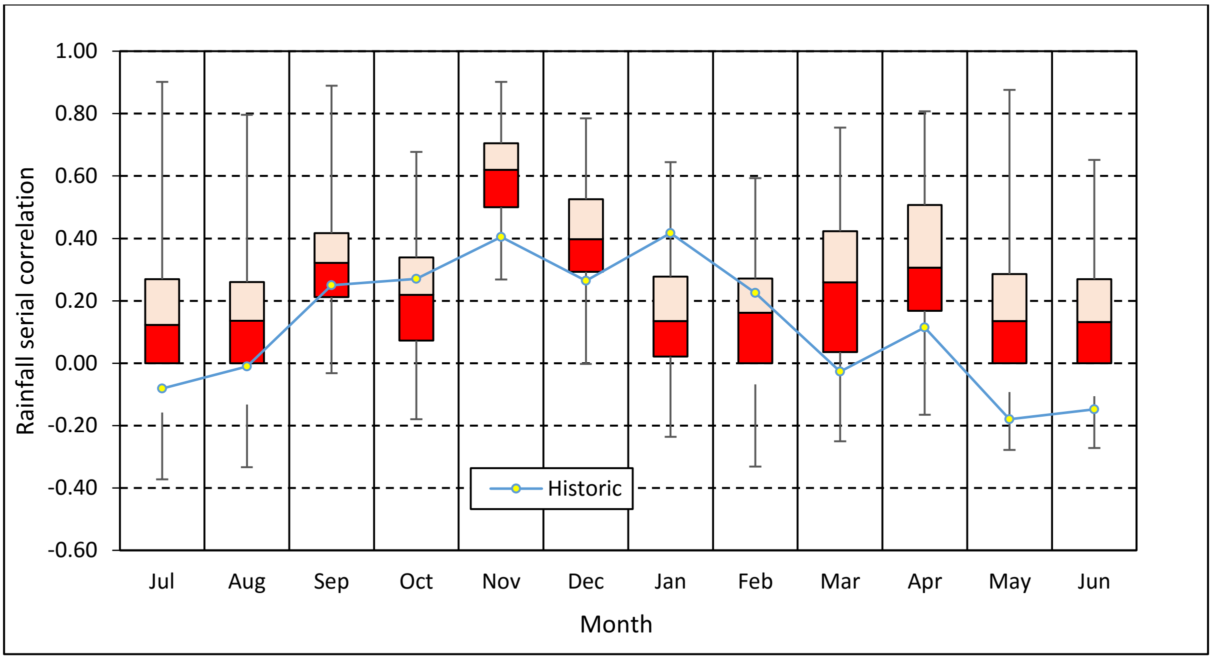


Figure 8.11: Monthly serial correlation of rainfall

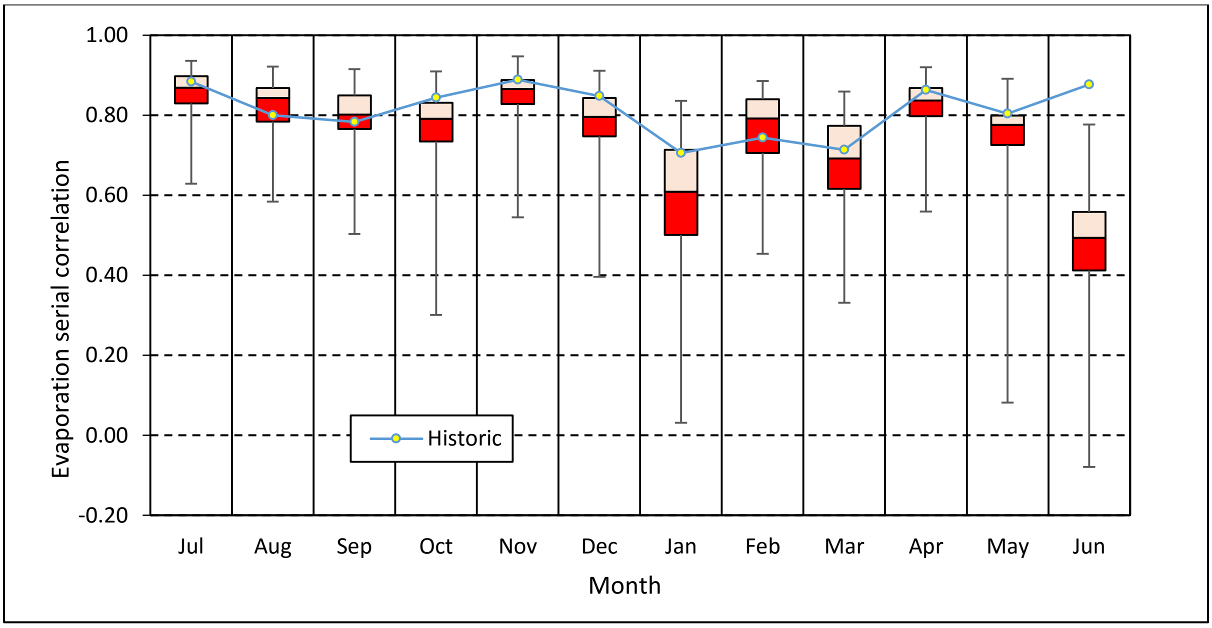


Figure 8.12: Monthly serial correlation of evaporation

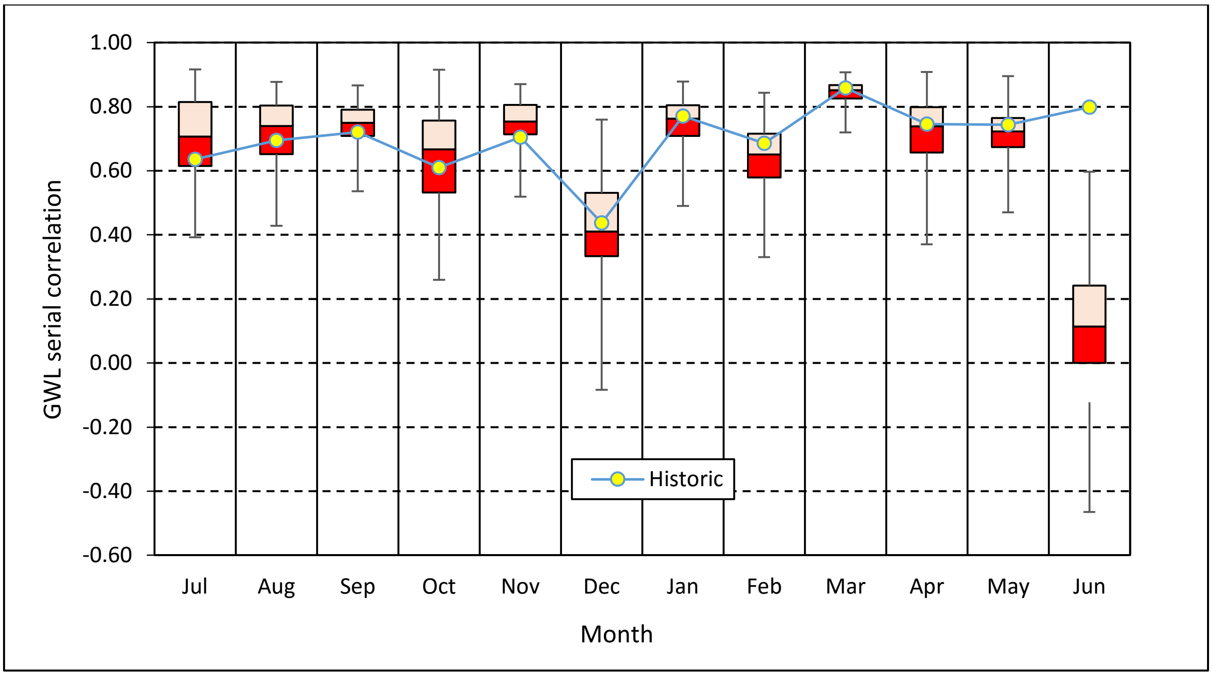


Figure 8.13: Monthly serial correlation of groundwater levels

It is important to note that all the historic statistics for rainfall, evaporation and groundwater levels were within the box plot limits, except for serial correlation of evaporation and groundwater levels for the month of June. The VLB model applied by Ndiritu (2011) included a routine for preserving the serial correlation between the last month of a year and the last month of the following year. This version of VLB was therefore able to preserve 80% of these serial correlations for monthly streamflow generation. The version of the VLB applied for stochastic rainfall generation by Ndiritu and Nyaga (2014) excluded this routine since monthly serial correlations of rainfall are usually negligible. The VLB version used in the current study is that applied by Ndiritu and Nyaga (2014) hence the failure to preserve the evaporation and groundwater level serial correlations for the month of June. For the purposes of generating the groundwater operating rules, the effects of this limitation are considered to be not significant. The annual serial correlations for evaporation was at the maximum box limit while that of groundwater levels was within the box plot limits (Figure 8.14).

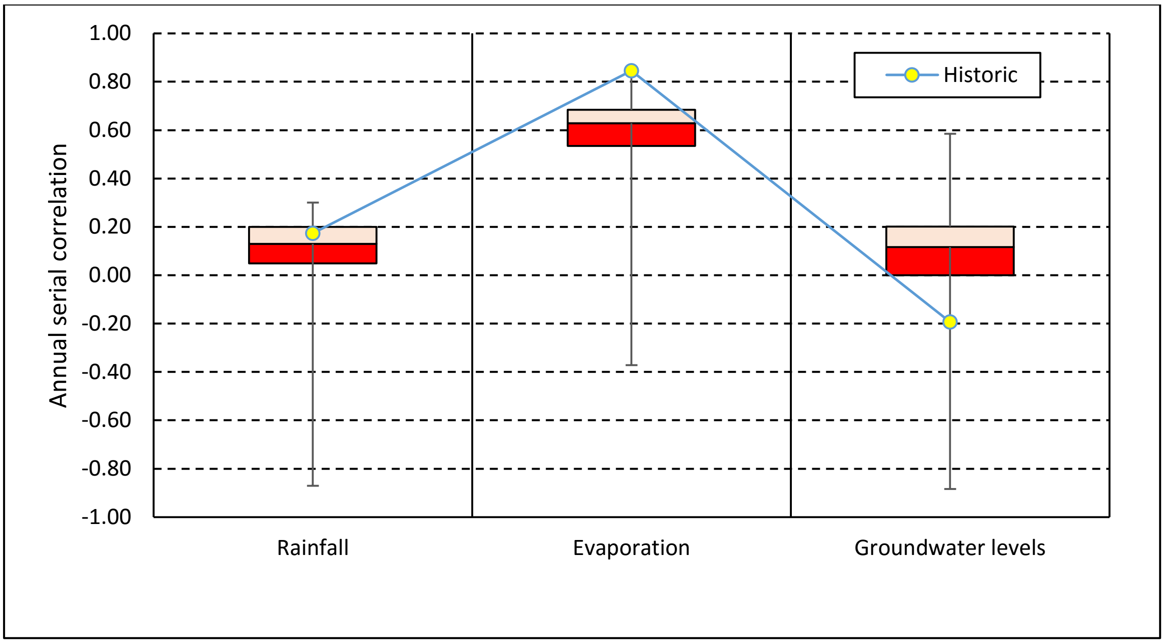


Figure 8.14: Annual serial correlations of rainfall, evaporation and groundwater levels

8.3 Chapter summary and contribution

This results in this chapter aided in addressing the third specific objective and research question which are associated with generation of stochastic inputs for groundwater base yield-recurrence interval analysis. The results of this study indicate that skewness is the statistic that was not well preserved for all variables with historic values for evaporation and groundwater levels being below and above interquartile range for 12 months. Historic highest groundwater levels also indicated that the stochastically generated skewness values were underestimated for 11 months. Historic statistics below interquartile range (overestimation) is a common problem of weather generators which can be reduced by including additional covariates that influence atmospheric circulation. The monthly serial correlations of evaporation and groundwater levels for the month of June were not preserved due to the fact that VLB version used in the study had no routine for preserving the serial correlation between the last month of a year (June in this case considering that the box plots start in July to ensure that the year begins and ends in the driest months). The effects of this limitation were considered to be not significant for the purposes of generating the groundwater operating rules. The incorporation of stochastic rainfall, evaporation and groundwater levels to support generation of groundwater operating rules is a noble approach. Use of stochastically generated data will enable realistic incorporation of reliability of groundwater supply. This is because reservoir operation (as explained in section 2.3) is inherently stochastic due to uncertain nature of its inflows as well as highly variable hydrologic and climatic conditions. Stochastically generated outputs account for these uncertainties and hence aid in realistic incorporation of reliability.

Studies on multivariate and multisite stochastic generation of hydrological variables mostly focus on minimum and maximum temperatures, and rainfall. Stochastic generations of evaporation and groundwater levels for hydrological studies are very limited and the application of the VLB generator in this study aided in assessing its ability in generating multivariate stochastics at multiple sites.

CHAPTER 9: DEVELOPMENT OF STOCHASTIC GROUNDWATER BASE YIELD CURVES AND RISK-BASED OPERATING RULES

9.1 Preamble

This chapter is focused on discussing the results on development of stochastic groundwater base yield-recurrence relationship interval relations analysis and risk-based operating rules. The chapter also informs how the groundwater operating rules can be implemented. The discussion also included generalising the procedures followed in developing risk-based groundwater operating rules for Siloam Village. This was aimed at enabling their application in any delineated groundwater resource unit.

9.2 Groundwater base yield-recurrence interval curves

Figure 9.1 shows stochastic groundwater base yield-recurrence interval relationship curves for given initial groundwater heads from 100 simulations that were run for 5 years stochastically generated sequences and 5 m minimum allowable groundwater head. The curves were generated following the procedure described in section 4.6. Legends of the figure legends indicate the base yield curve associated with a particular initial groundwater head. For example, 10 m refers to a base yield curve generated from a simulation with initial groundwater head of 10 m. Though the basic unit of measurement of groundwater yield is normally liters per second (*l/s*), the generated yield curves have units of cubic meters per annum (m^3/annum) following the standard practice used when generating annual operating rules in surface water systems.

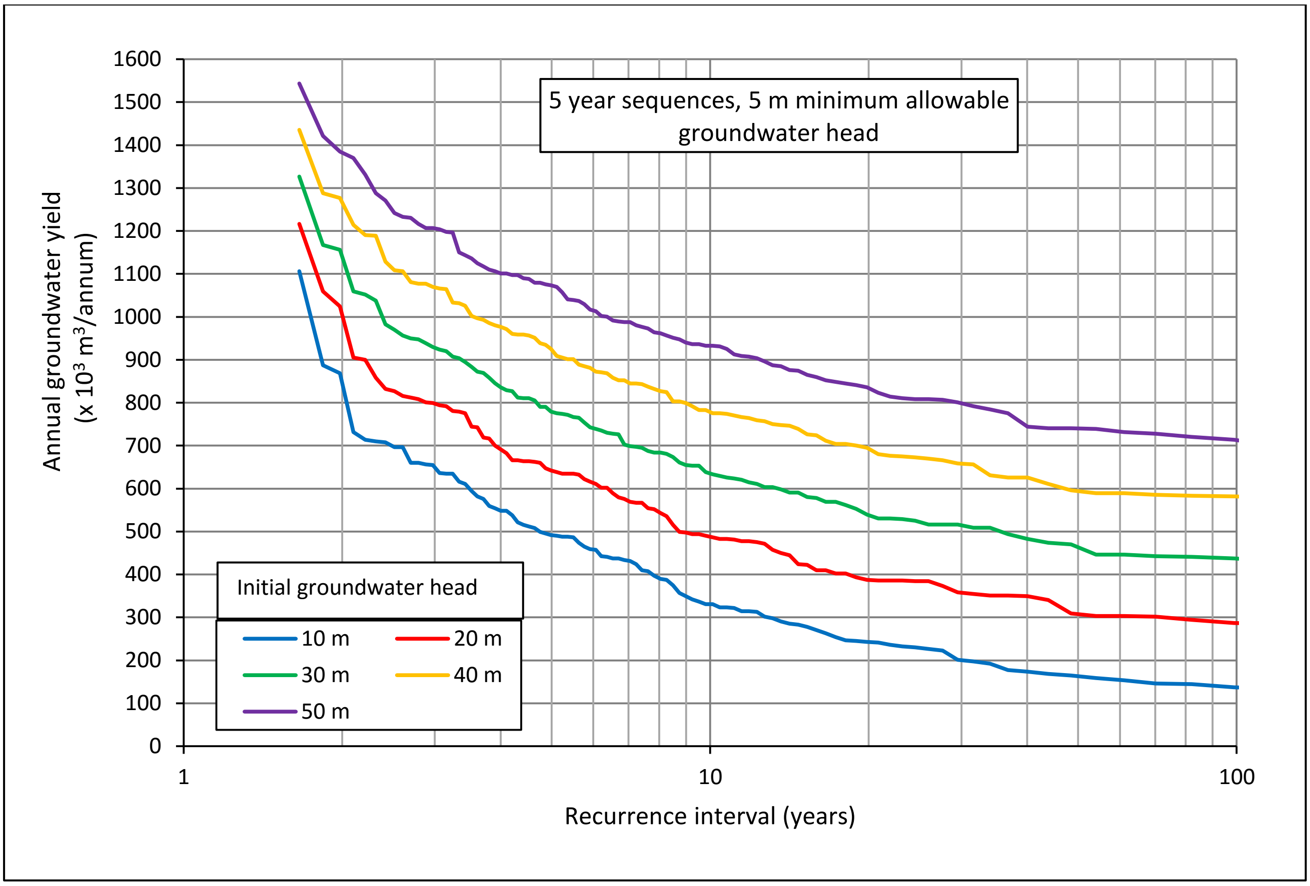


Figure 9.1: Annual groundwater base yield-recurrence interval curves based on 5 year sequences for initial groundwater heads of 10, 20, 30, 40 and 50 m

The curves show that increasing the initial groundwater head increases groundwater yield (Figure 9.1). Results from yield analysis for surface water reservoirs, for example DWAF (2008a) and DWA (2010) indicated that high initial storage levels are associated with relatively high base yield curves. The results in Figures 9.1 indicated the potential to generally obtain high yields at high groundwater heads. Though this may be true at the scale of a groundwater resource unit on which the simulations are based, it is important to note that the heterogeneous nature of the geologic environment in the study area may result to variable yields (i.e. relatively high or low yields) at a point (borehole) scale. The results from groundwater resource unit aquifer characterisation (section 5.4) indicated that 4 boreholes had low potential to store water while 3 others had relatively high storage potential. Thus, indicating variable groundwater storage potential within the groundwater resource unit. This therefore implies that the potential to obtain relatively high yields is also dependent on the ability of the geologic environment to store and transmit water.

9.3 Risk-based groundwater operating rules

The portions of domestic and productive water demands at low (1:10), medium (1:50) and high (1:100) assurance levels which were calculated based on the priority classification in Table 4.4 are in Table 9.1. The total annual water demands (Table 9.1) for domestic and productive uses in Siloam Village were estimated based on per capita uses of 189 and 302.4 liters, respectively, as explained in section 4.6. A high portion of the domestic demand (79.16 x 10^3 m^3/annum) has high assurance level as domestic water use is considered to be a high priority water user (Table 9.1) when compared to productive use. In risk-based analysis, high recurrence intervals indicate low risks of failure to meet the target demand. For example a recurrence interval of 1:100 indicates that there is a possibility that the target draft may, on average, not be met once in a hundred years. Thus, allocating a high portion of domestic water use at 1:100 ensures that most of this demand is met at high level of assurance of supply (low risk of failure).

Table 9.1: Annual water demands for Siloam Village (x 10^3 m^3/annum) based on priority classification

User	**Low**	**Medium**	**High**	
	1:10	**1:50**	**1:100**	**Total**
Domestic Use	15.83 (10%)	63.33 (40%)	79.16 (50%)	158.31
Productive use	75.99 (30%)	126.65 (50%)	50.66 (20%)	253.30
Total	91.82	189.97	129.82	411.61
Cumulative*	411.61	319.79	129.82	

* calculated from the high to low assurance

The base yield curves for groundwater levels of 10, 20, 30, 40 and 50 m and superimposed cumulative demands are indicated in Figure 9.2. The curves in Figure 9.2 are different from those in Figure 9.1 as the base yields are related to groundwater levels instead of groundwater heads. This was done to facilitate development of simplified and practically implementable operating rules as explained in section 4.6. The difference between groundwater head and groundwater level has been explained in Figure 4.6. In Figure 9.2, 10 m in the legend indicates a base yield curve associated with groundwater level of 10 m. Since the groundwater level is a measure of groundwater system's response to variable hydrological conditions, as explained in section 4.6, low and high groundwater levels are also associated with dry and wet hydrological conditions, respectively. For example, the yield curve associated with the groundwater level of 50 m represents low groundwater level which can be associated with dry hydrological conditions while that with an initial groundwater level of 10 m represents high groundwater level due to wet hydrological conditions (Figure 9.2). Thus, operating rules developed based on the approach followed in this study accounts for water availability during variable hydrological periods.

In Figure 9.2, letters **D**, **E** and **F** indicate the cumulative water demands at 1:10, 1:50 and 1:100 assurances levels which are shown in Table 9.1. The base yield at 1:100 assurance level exceeded the demand of 129.82 x 10^3 m^3/annum indicating that the groundwater system can meet the demands at all given initial groundwater levels (Figure 9.2), hence curtailment will not be introduced. Superimposing the cumulative demands (light blue line) on the base yield curves also indicated that the cumulative demand of 319.79 x 10^3 m^3/annum exceeded the base yield when initial groundwater level dropped to 30 m and below (40 and 50 m). The base yields at 1:10, and 1:50 assurance levels for groundwater levels below 40 and 50 m were also exceeded by the cumulative demands (Figure 9.2). This indicated that the system will not have enough water to be allocated at 1:10 and 1:50 levels of assurance of supply once the initial groundwater levels dropped to below 40 m. Thus, curtailment of these demands should be introduced or alternative sources of water supply should be sought once the groundwater system cannot meet the demands. Ndiritu *et al.* (2011a) identified run-of-river and harvested rainwater as alternative sources of water that can be integrated with groundwater to meet the demands.

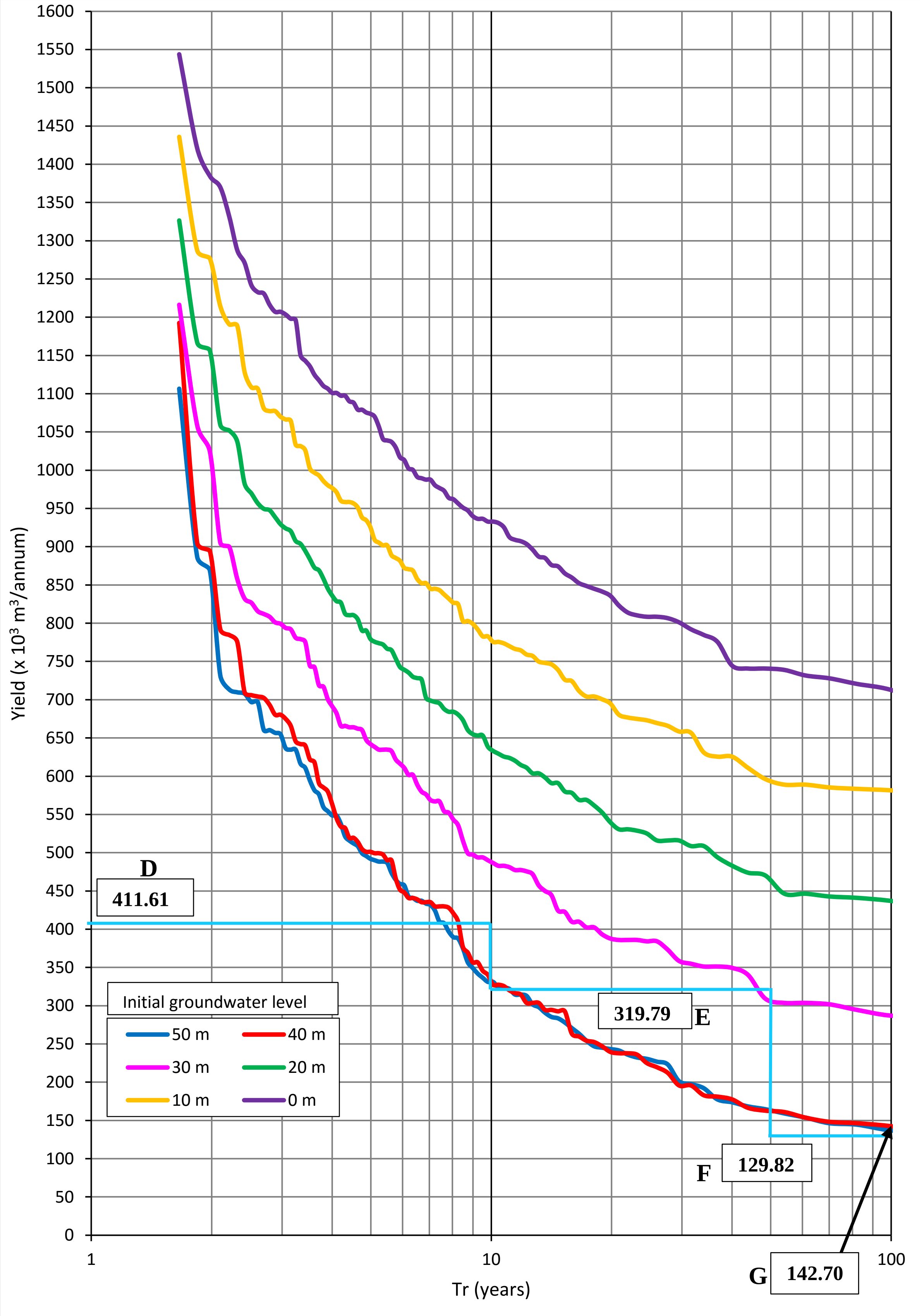


Figure 9.2: Base yields associated with groundwater levels of 10, 20, 30, 40 and 50 m and cumulative demands

The percentages of domestic, productive, and combined domestic and productive water demands that can be supplied at 1:100 assurance level at varying groundwater levels are presented in Figures 9.3, 9.4 and 9.5, respectively. These were obtained by relating the yields at 1:100 assurance level (extracted from Figure 9.2) and the total annual water demands of 158.31 x 10^3 m^3/annum and 253.30 x 10^3 m^3/annum for domestic and productive water uses, respectively, from Table 9.1. This assessment was aimed at determining the ability of the groundwater system to meet all the demands at 1:100 assurance level before curtailments are introduced.

The curves indicate that all users cannot be supplied when the groundwater level is below minimum allowable depth of 5 m measured from the minimum groundwater level of 50 m (indicated by a red line in Figures 9.3-9.5) to prevent overwithdrawal/depletion of groundwater particularly during the dry hydrological years. This level is equivalent to minimum operating level used in surface water reservoirs as explained in section 4.6. Considering that groundwater levels are measured from the ground surface, the minimum allowable groundwater level in this study is 45 m. In cases where the yield exceeded the water demand, the percentage of water to be supplied was considered to be 100% (Figures 9.3-9.5). If the yield was lower than the water demand, the percentage of the demand that can be supplied was computed based on total annual water demand of each user (Table 9.1) and the available yield at 1:100 assurance level at various groundwater levels (0-50 m) (from Figure 9.2) using Equation 4.50 as explained in section 4.6. For example, the percentage of domestic water demand of 90.13% (from Figure 9.3) that could be supplied at 1:100 assurance level when the groundwater level was 40 m was obtained by dividing the yield of 142.70 x 10^3 m^3/annum (denoted by **G** in Figure 9.2) by the domestic water demand of 158.31 x 10^3 m^3/annum and converted to percentage.

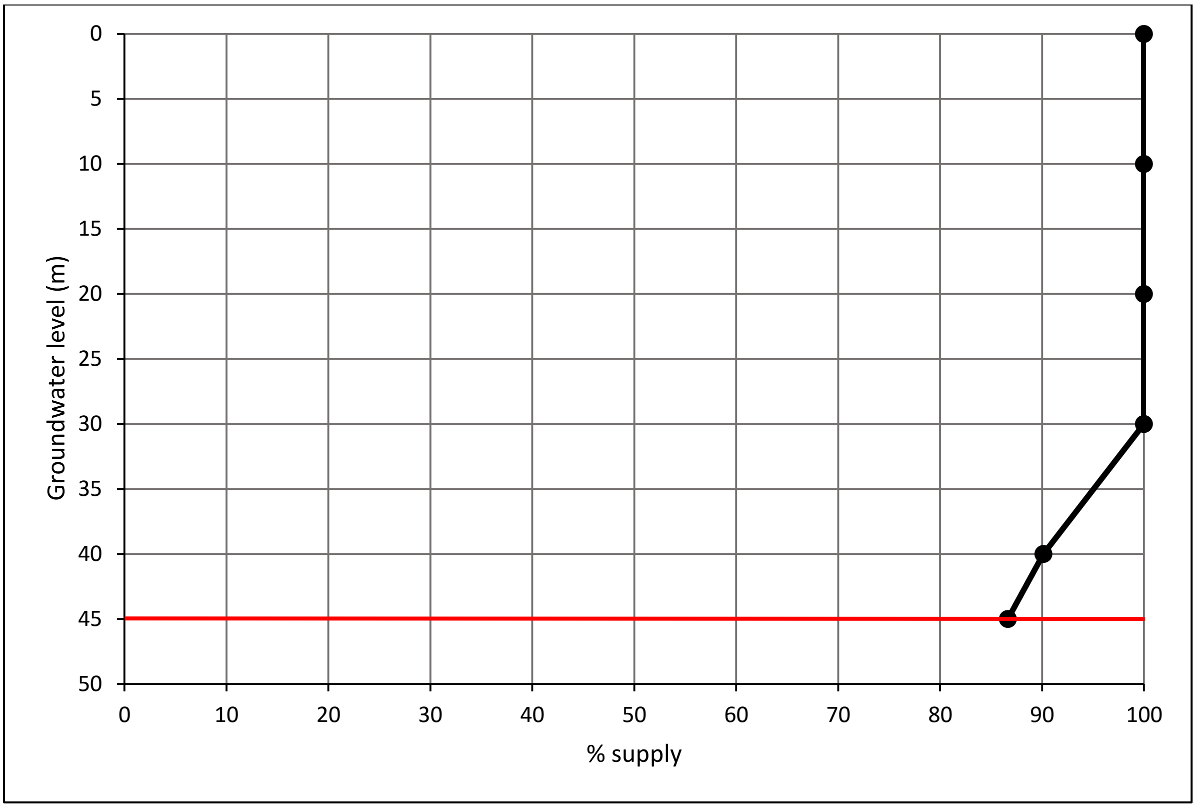


Figure 9.3: Percentages of domestic water use that can be supplied at 1:100 assurance level

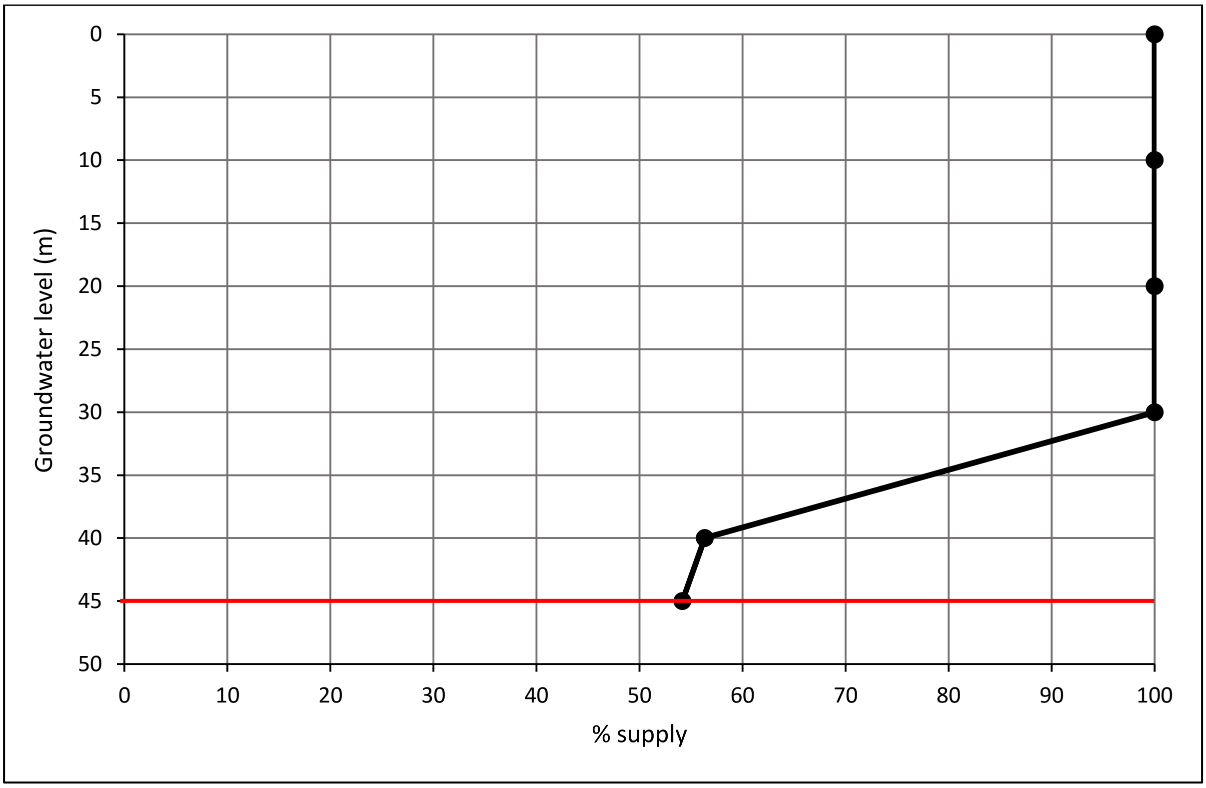


Figure 9.4: Percentages of productive water use that can be supplied at 1:100 assurance level

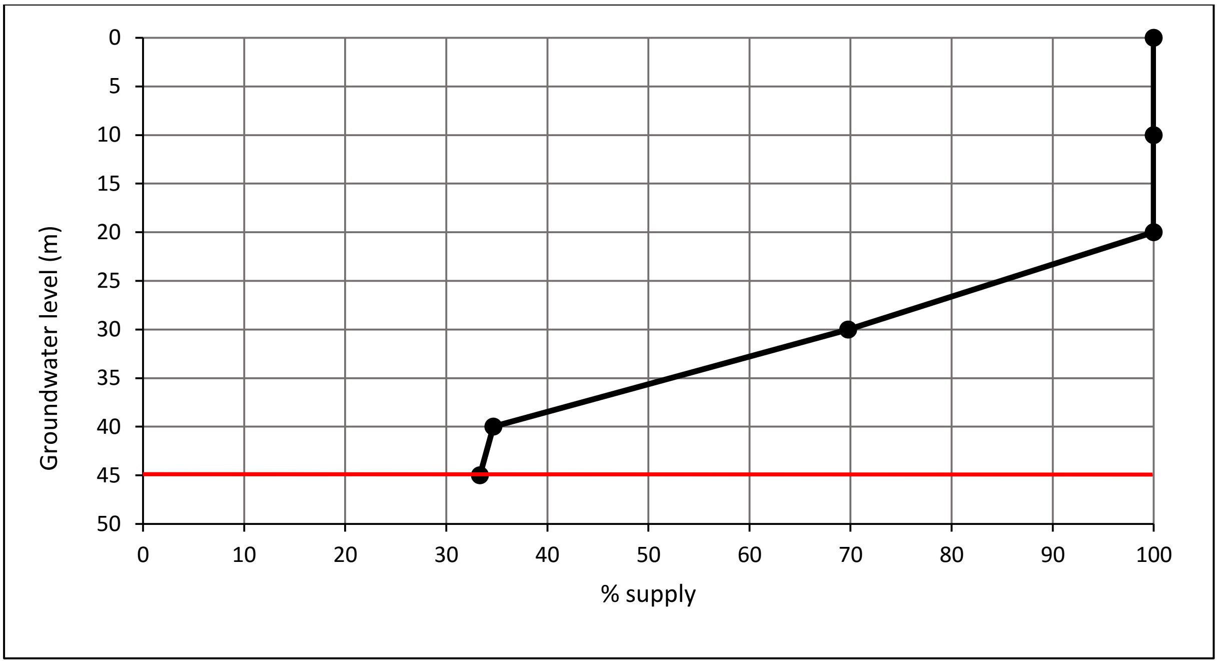


Figure 9.5: Percentages of combined domestic and productive water uses that can be supplied at 1:100 assurance level

The minimum percentages that can be supplied at 1:100 assurance level when the groundwater level was 45 m were 86.66, 54.16 and 33.33 for domestic, productive, and combined domestic and productive uses, respectively (Figures 9.3-9.5). Figures 9.3 and 9.4 indicate that 100% of the individual demands for domestic and productive uses could only be met up to groundwater level of 30 m while the combined domestic and productive water uses (Figure 9.5) could only be met up to groundwater level of 20 m, indicating that groundwater could not meet all water demands at 1:100 assurance level at all times. This shows that without prioritisation, failure to meet combined domestic and productive water uses at 1:100 assurance level would occur earlier (at groundwater level of 20 m) as compared to failure to individually supply domestic and productive water uses. Both domestic and productive water demands need to be curtailed once the groundwater level drops to below 20 m if groundwater is considered the only source of water supply. This, therefore further justifies the use of priority classification to allocate water to different user categories. This will also promote sustainable multipurpose use of water that can enhance rural livelihoods.

Figures 9.6 and 9.7 show the developed operating rule curves for domestic and productive water uses. The operating rule curves indicate the annual water allocation for domestic and productive uses at varying groundwater levels. An example considering the initial groundwater level of 30 m is given here to provide clarification on how the volume of water to be allocated for domestic water use (indicated in the operating rule curve) was obtained. The volume of water to be allocated for domestic water use was calculated based on the yield curve for initial groundwater level of 30 m (Figure 9.2) and priority classification (Table 9.1) starting from high level of assurance of supply as follows:

- The annual yield of 287.18 X 10^3 m^3/annum at 1:100 assurance level can meet the total cumulative demand of 129.82 x 10^3 m^3/annum (Figure 9.2) indicating that 100% of the demands for domestic and productive uses can be supplied without curtailment. The domestic water allocation at 1:100 assurance level (V_{100}) is therefore 79.16 x 10^3 m^3/annum from Table 9.1.
- At 1:50 assurance level, the yield of 309.13 x 10^3 m^3/annum is exceeded by the cumulative demand of 319.79 x 10^3 m^3/annum (Figure 9.2) and can therefore not fully meet the demands. This means that domestic allocation at low (1:10) and medium (1:50) assurance should be calculated using Equation 4.53 following priority classification. The available yield at 1:50 assurance level ($AY_{1:50}$) is obtained as the difference between the yield ($Y_{1:50}$) at 1:50 assurance level (309.13 x 10^3 m^3/annum) and cumulative demand at 1:100 assurance level (D_{T100}) of 129.82 x 10^3 m^3/annum.

$$AY_{1:50} = Y_{1:50} - D_{T_{100}}$$
$$= (309.13 - 129.82) \times 10^3\, m^3 / annum$$
$$= 179.31 \times 10^3\, m^3 / annum$$

The volume of water to be allocated at 1:50 assurance level ($V_{1:50}$) is obtained as the sum of curtailed volumes at 1:10 and 1:50 assurance levels based on the $AY_{1:50}$ as follows:

$$V_{1:50} = \left(\frac{AY_{1:50}}{D_{T50}} \times D_{1:10} \right) + \left(\frac{AY_{1:50}}{D_{T50}} \times D_{1:50} \right)$$
$$= \left(\frac{179.31 \times 10^3}{319.79 \times 10^3} \times 15.83 \times 10^3 \right) + \left(\frac{179.31 \times 10^3}{319.79 \times 10^3} \times 63.32 \times 10^3 \right) m^3 / annum$$
$$= 44.38 \times 10^3\, m^3 / annum$$

D_{T50} is the cumulative demand at 1:50 assurance level while $D_{1:10}$ and $D_{1:50}$ are the domestic demands at 1:10 and 1:50 assurance levels, respectively from Table 9.1.

- The annual yield of 486.50 x 10^3 m^3/annum at 1:10 assurance level can meet the total cumulative demand of 411.61 x 10^3 m^3/annum (Figure 9.2) indicating that 100% of the demands for domestic and productive uses could be supplied without curtailment. The domestic water allocation at 1:10 assurance level (V_{10}) is therefore 15.83 x 10^3 m^3/annum.
- The total allocation for domestic use at a groundwater level of 30 m (V_{30m}) was then obtained as the sum of allocations at 1:100, 1:50 and 1:10 levels of assurance of supply:

  $V_{30m} = V_{100}+V_{50}+V_{10}$

  $= (79.16+44.38+15.83) \times 10^3\ m^3$/annum

  $= 139.37 \times 10^3\ m^3$/annum

If the demands for domestic and productive uses are supplied following the priority classification they can be met up to a maximum water table depth of 25 m for each case (Figures 9.6 and 9.7). This is better as compared to attempting to supply both domestic and productive uses at 1:100 assurance level which increases the risk of failure. Ndiritu *et al.* (2017) noted that implementing the operating rule curve to Hluhluwe Dam in Kwazulu-Natal supply helped to improve the annual water allocation decisions of the dam even during the 2014/2015 drought period. U.S. Army Corps of Engineers (1991) noted that improvements in operating efficiency of reservoir system operations often offer substantial increases in benefits. Implementing the operating rule curve as done in Ndiritu *et al.* (2017) therefore improved the annual water allocation decisions of the dam even during a period of limited water availability. In relation to this study, the developed operating rule curves if adequately implemented, would therefore be expected to improve water supply to both domestic and productive water uses and hence positively impact on livelihoods.

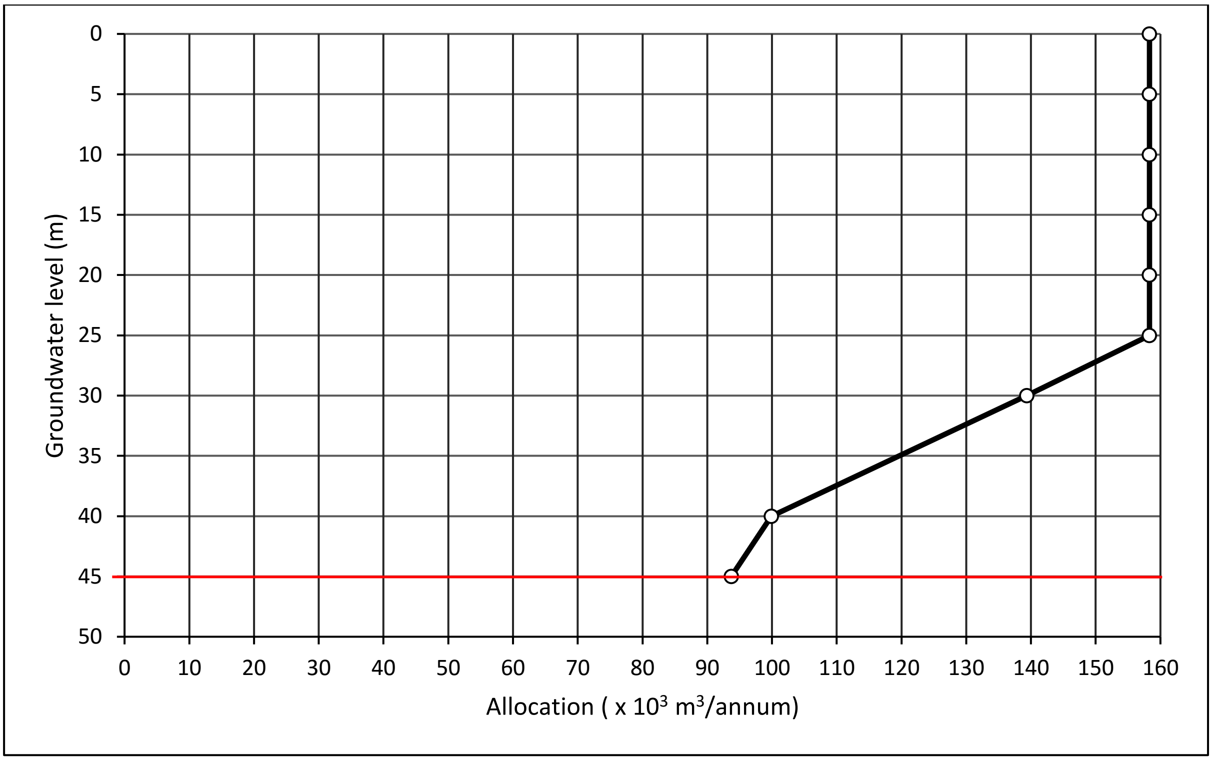


Figure 9.6: Operating rule curve for domestic water use allocation

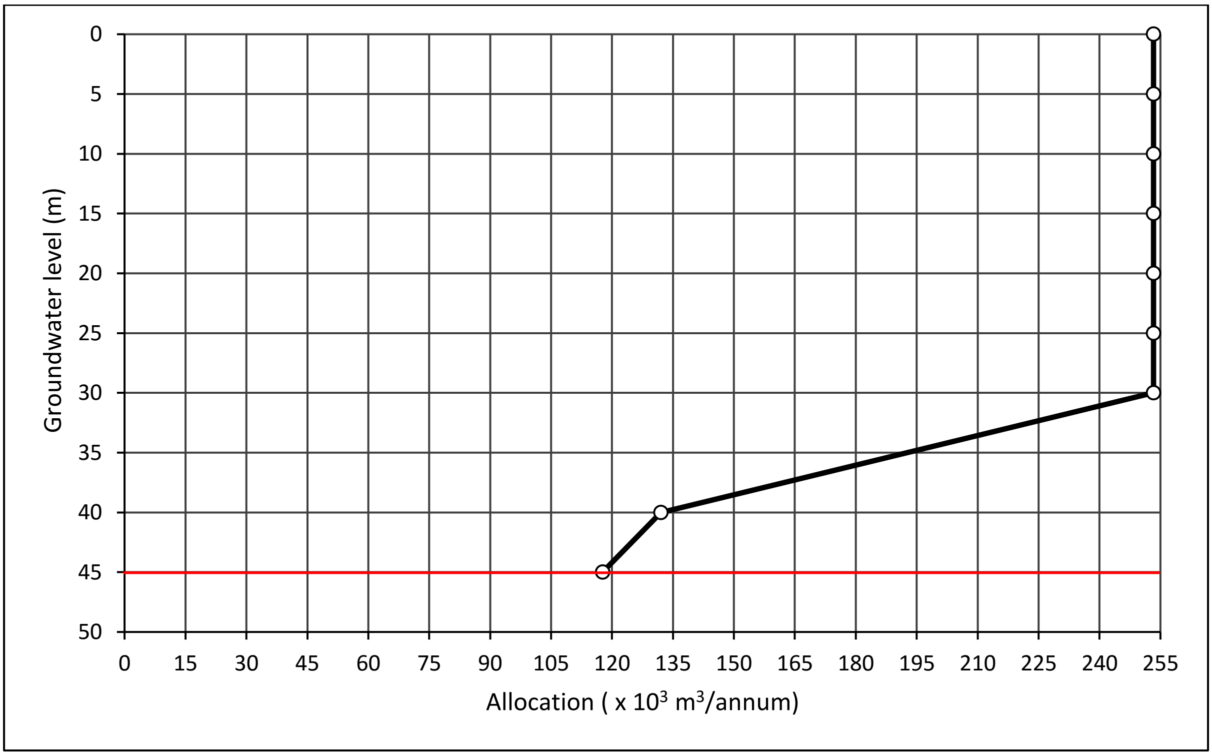


Figure 9.7: Operating rule curve for productive water use allocation

9.4 Proposed procedure for implementing of the operating rule curves

The groundwater storage simulation model estimates the average groundwater levels for the groundwater resource unit. Groundwater levels from a number of boreholes are therefore required to obtain an average value for the groundwater resource unit in the decision month. To facilitate implementation of the operating rules, monitoring boreholes equipped with groundwater level loggers would be required. The proposed procedure for implementing the groundwater operating rules is as follows:

- Obtain groundwater levels from a number of observation boreholes in the groundwater resource unit and compute the average groundwater level for the decision month.
- Read the value of volume of groundwater that can be allocated for the entire year for domestic and productive uses corresponding to the average groundwater level from the operating rule curves in Figures 9.6 and 9.7, respectively.

Implementation of the operating rule curves require monitoring and water supply infrastructure. Available infrastructure in Siloam Village include a number of private and 2 public boreholes. All private and 1 public boreholes are connected to water tanks with a typical size of 2500 liters. The number of private boreholes is expected to increase as more residents strive to meet their water demands from groundwater since surface water supply is limited. This is expected to increase the abstraction of groundwater. Groundwater abstractions should therefore be regularly monitored to prevent over-abstraction. Three monitoring boreholes within the study area are planned to be equipped with groundwater level loggers. These could assist with implementation of the operating rules in addition to improving calibration of the groundwater balance model. The proportion of water to be allocated should be based on expected number of people who are dependent on water from individual boreholes.

9.5 Generalisation of the risk-based groundwater operating rules

The procedures followed in developing risk-based groundwater operating rules for Siloam Village were summarised to assist in their application in any delineated groundwater resource unit. The summary is as follows:

- Determine the existing water uses and their requirements
- Delineate a groundwater resource unit, determine its hydraulic characteristics and develop its hydrogeological conceptual model
- Model and/or extend groundwater levels as well as other inputs required for the water balance such as rainfall and evapotranspiration in case there is no adequate data (for example in data scarce areas) or obtain long-term time series of groundwater levels data for a number of boreholes within a groundwater resource unit and other inputs where available.
- Generate stochastic inputs and derive base yield curves
- Derive risk-based groundwater operating rules based on stochastic yields

9.6 Chapter summary and contribution

The base yield analysis indicated potential for relatively high groundwater yields with increased groundwater heads, though it was noted that heterogenous nature of the geologic environment in the study area may limit the yields even at increased groundwater head. The groundwater base yield curves were generated for varying groundwater levels that indicate the state of the groundwater systems for varying hydrological conditions. Operating rules developed based on the approach followed in this study account for water availability during variable hydrological periods. Superimposing the cumulative demands on the base yield curves indicated that not all demands at assurance levels of 1:50 and 1:10 could be met.

The study established that developed operating rule curves if adequately implemented, would therefore be expected to improve water supply to both domestic and productive water uses and hence improve livelihoods. The procedure for implementation of groundwater operating rules has been described in this chapter. Implementing operating rules would require monitoring and water supply infrastructure. In Siloam Village, private boreholes are connected to water tanks while 3 boreholes are being fitted with groundwater levels loggers to monitor groundwater levels. This would support implementation of operating rules though additional monitoring boreholes would be required to improve the estimation of average groundwater levels.

This study developed operating rules for groundwater supply using a risk-based approach, which is typically used for surface water systems in South Africa. Since the literature review did not find documented studies on stand-alone stochastic based operating rules for groundwater supply (see last paragraph of sub-section 2.3.1), this study therefore, closes the gap on unavailability of risk-based operating rules for groundwater supply. The risk-based operating rules are of crucial importance in areas that are dependent on groundwater and they would specifically aid in water resources management during variable hydrological conditions. With the current climate change predicament in which conditions are expected to worsen in the near future, risk-based management of groundwater resources particularly during drought is essential.

CHAPTER 10: CONCLUSIONS AND RECOMMENDATIONS

10.1 Conclusions

This study aimed to develop risk-based operating rules for groundwater supply using Siloam Village of Limpopo Province in South Africa as a case study. A groundwater resource unit was delineated to provide the basis for generating groundwater levels using the GW-PITMAN model. Its hydrogeological conceptual model was developed and the hydraulic characteristics were estimated. The hydrogeological conceptual model provided an understanding of the nature of the groundwater flow and storage environment that was essential for the development of the groundwater simulation model. The hydrogeological conceptual model indicated presence of faults and diabase dykes which influence preferential flow paths and storage of water in the aquifer (Figures 5.4 and 5.5). It was, however, noted that the knowledge of the extent to which faults and dykes influence groundwater flow and storage is limited since their thicknesses and depths are unknown.

Automatic curve matching was used to identify appropriate aquifer models and test solutions for estimating hydraulic characteristics. The aquifer models fitted were generally good, with mean residual errors ranging from -0.87 to 3.04 m (Table 5.2). This showed that identified aquifer test solutions can aid in accurate estimation of hydraulic characteristics. The results indicated that the study area is dominated by leaky aquifer. Fractured double porosity and single fracture models were also identified. Identification of different aquifer types indicated that the geologic environment where groundwater is stored in the study area is heterogeneous. Storativity, transmissivity and hydraulic conductivity values ranged from 0.0003-0.068, 0.78-12.3 m^2/day and 0.0740 - 0.460 m/day (Table 5.3), respectively, indicating limited aquifer storage with potential for local groundwater supply and for private consumption.

The study area had limited rainfall and groundwater levels data which were some of the major inputs into the GW-PITMAN model. These data therefore needed to be infilled and extended. A non-parametric regression (NPR) model was calibrated and validated for use in extending rainfall data. For extension of groundwater levels, Output Error-Non-linear Hammerstein

Weiner (OE-NLHW) system identification model was used. The statistical performance measures; coefficient of determination (R^2), correlation coefficient (COR), Nash Sutcliffe coefficient of efficiency (NSE), root mean square error (RMSE) and relative error (RE) values were 0.76, 0.87, 0.75, 3.67 mm and 30% for calibration run of NPR model, respectively (Table 6.1). R^2, COR, NSE, RMSE and RE values were 0.7, 0.84, 0.68, 3.03 mm and 29% for validation run of NPR model, respectively (Table 6.1). Thus, NPR modelling had acceptable and satisfactory performance based on the assessed measures of performance indicating that the model can effectively be used for rainfall extension.

The R^2, COR, NSE, RMSE and RE were 0.99 and 0.86, 0.97 and 0.93, 0.99 and 0.84, 0.03 and 0.01 m and 0.08 and 0.11% for calibration and validation, respectively, of the coupled OE-NLHW system identification model (Table 6.3). Most of the scatter points were close to the best fit line showing good agreement of observed and simulated groundwater levels in both calibration and validation runs, and therefore indicate less model errors. The graphical fits, scatter plots and measures of performance generally showed efficient calibration and validation of the model indicating that rainfall and evapotranspiration can be used to simulate groundwater levels based on the coupled OE-NLHW system identification model.

A program for monthly generation of groundwater levels was coded in FORTRAN based on the GW-PITMAN. The GW-PITMAN model has been widely applied in South Africa. A hybrid manual-automatic approach which involved estimation of realistic model parameter ranges based on available hydrogeological data in the study area and their optimisation based on Shuffled Complex Evolution algorithm was used for calibration of the GW-PITMAN model. Sensitivity analysis was done to identify parameters to be used in final model calibration. Model calibration was done using the extended and infilled groundwater levels based on observed data from borehole A8N0508 (Mandala) which is located close to Nzhelele River upstream of the groundwater resource unit. Model validation was achieved by establishing the realistic nature of simulated runoff, groundwater recharge and groundwater levels to give an indication of whether the model is accurately simulating the hydrological processes within the groundwater resource unit. Lower limit of soil moisture below which no groundwater recharge occurs (SL), maximum moisture storage capacity (ST), power of the moisture storage-recharge equation (GPOW) and evaporation-moisture storage relationship (R) were sensitive model parameters while runoff from moisture storage at full capacity (FT) and power of the moisture storage-runoff equation (POW) were not sensitive. This was likely due to the fact that FT and POW do not affect runoff generation in semi-arid areas. Groundwater levels, streamflow and groundwater recharge estimated from the GW-PITMAN model, generally fluctuated with changes in rainfall. This showed that groundwater levels, streamflow and groundwater recharge hydrographs estimated in the current study were realistic.

The development of risk-based groundwater operating rules required the generation of stochastic data. For this, the variable length block (VLB) stochastic generator was applied. The mean, median, 25th and 75th percentiles, standard deviation, highest and lowest values of rainfall, evaporation and groundwater levels were mostly well preserved. Highest historic groundwater levels also indicated that the stochastically generated values were underestimated for 11 months (Table 8.1). Skewness is the only statistic that was mostly not well preserved for all variables with historic values for evaporation and groundwater levels being below interquartile range for 12 months (Table 8.1), indicating that they were mostly overestimated. This has been explained to be a common problem of weather generators which can be reduced by including additional covariates that influence atmospheric circulation.

Analysis of groundwater base yield-recurrence interval relationship and development of risk-based groundwater operating rules followed procedures widely used in South Africa for surface water reservoirs. The base yield simulations at groundwater resource unit scale indicated the potential to generally obtain high yields from relatively high groundwater heads. It was, however, noted that heterogeneous nature of the geologic environment in the study area may limit the yields even at increased groundwater heads. At a groundwater level of 30 m it was only the cumulative demand at 1:50 assurance level which could not be met (Figure 9.2). Superimposing the cumulative demands on the base yield curves also showed that the groundwater system could not meet the water demands at 1:10 and 1:50 assurance levels once groundwater levels dropped to below 40 m. The analysis of percentages of water demands that can be supplied indicated that groundwater cannot individually supply domestic or productive water uses at 1:100 assurance level at all times (Figures 9.3-9.5). In addition, supplying combined domestic and productive uses at 1:100 assurance level is also not feasible as it leads to failure to meet the demands once groundwater levels drop to below 20 m. Thus, to promote sustainable multipurpose use of water that can enhance rural livelihoods, allocating water following priority classification is essential.

Operating rule curves for groundwater supply were derived using a risk-based approach. The operating rule curves indicated that if the demands for domestic and productive uses are supplied following the priority classification they can be met up to a maximum water table depth of 25 m for each case. The developed operating rule curves are therefore expected to improve water supply to both domestic and productive water uses, if they are adequately implemented and hence improve livelihoods.

The preceding paragraphs have provided information that support the hypotheses stated in section 1.5, and indicates that objectives of the study have been achieved. Implementing groundwater operating rules would require monitoring and water supply infrastructure. In Siloam Village, private boreholes are connected to water tanks while 3 boreholes have been planned to be fitted with groundwater levels loggers to monitor groundwater levels. This would support implementation of operating rules though additional monitoring boreholes would be required to improve the estimation of average groundwater levels. The procedure for implementation of the risk-based groundwater operating rules and their generalisation have been described in sections 9.4 and 9.5, respectively.

10.2 Recommendations

Lack of adequate and reliable data required for proper assessment of groundwater resources and derivation of groundwater operating rules is a common problem in rural areas of South Africa. This is despite the fact that most rural areas are dependent on groundwater as they lack adequate and potable water supply from surface water schemes. Similar studies need to be conducted in such areas to promote groundwater resources assessment and sustainable allocation in rural areas. Such studies will also assist in verification of the approach followed in development of risk-based operating rules for groundwater supply.

Problems of lack of adequate and reliable data were also encountered in the current study. Data available for the current study were mostly of short periods and had gaps. This led to the use of models to infill and extend the data to enable groundwater base yield-recurrence interval analysis and generation of operating rules. Application of the modelling approaches for extension of rainfall and groundwater levels in other case studies is essential for further verification and wide application. The groundwater reservoir modelling using the GW-PITMAN model and the stochastic data generation using the VLB model need further testing and possible modification for purposes of deriving risk-based groundwater operating rules. To the knowledge of the author, this is the first time that these models have been applied to derive groundwater operating rules and further testing and verification of the models and the approach is imperative. This would assist in appropriate decision making with respect to groundwater allocation and management.

The characterisation of the groundwater resource unit and development of its hydrogeological conceptual model were also based on limited data. Studies such as geological/stratigraphic, geophysical and geomorphological surveys, examination of borehole core drilling samples are required to improve on characterisation of the groundwater resource unit and development of its hydrogeological conceptual model.

Studies on observation of wellfield performance, identification of groundwater flow dynamics, characterisation of aquifer media at various scales and groundwater flow and transport numerical model are also crucial. It is also essential to develop a hydrostratigraphic subarea model of Siloam aquifer system using fracture network analysis to enhance understanding of influence of fractures on groundwater flow system

It is important to emphasise that this type of study provides opportunities for further studies and identifies areas that require improvement in groundwater resources assessment, development of groundwater operating rules and their implementation. In addition, it provides baseline information that can be used to guide future studies. These may include studies focused on additional and extensive field data collection and monitoring. Undertaking a study of this nature in data-scarce Siloam Village proved to be beneficial as it aided in establishing initiatives such as installation of monitoring networks for rainfall and groundwater levels. The study area can therefore be used as a pilot to showcase practical implementation of groundwater operating rules. This would also encourage participation of local communities in management of their water resources. Piloting of the operating rules in other case studies is recommended to further test their practical application in groundwater systems. This will promote monitoring, evaluation and refinement of risk-based groundwater operating rules.

REFERENCES

Abdullah, A., Akhir J.M. and Abdullah, I. (2010) Automatic mapping of lineaments using shaded relief images derived from digital elevation model (DEMs) in the Maran-Sungi Lembing area, Malaysia, *The Electronic Journal of Geotechnical Engineering*, vol. 15, pp. 949-957.

Abraha, M.G. and Savage, M.J. (2006) Potential impacts of climate change on the grain yield of maize for the midlands of KwaZulu-Natal, South Africa, *Agriculture, Ecosystems and Environment*, vol. 115, pp. 150–160.

Adamowski, K. (1987) Nonparametric techniques for analysis of hydrologic events, *Proceedings of the Rome Symposium*, IAHS Publication no. 164, pp. 56-76.

Adams, S., Titus, R. and Xu, Y. (2004) *Groundwater recharge assessment of the basement aquifers of central Namaqualand*, WRC Report No. 1093/1/04, Water Research Commission, Pretoria, South Africa, 179 pp.

Aguado, E., Remson, I., Pikul, M.F., Thomas, W.A. (1974) Optimal pumping for aquifer dewatering, *Journal of Hydraulics*, vol. 100, pp. 869-877.

Ahmad, M.R. and Al-khazaleh, A.M.H. (2008) Estimation of missing data by using the filtering process in a time series modeling, *Electronic Journal of Statistics*, http://arxiv.org/abs/0811.0659.

Al-alawi, M., Bouferguene, A. and Modamed, Y. (2017) Non-parametric weather generator for modelling construction operations: Comparison with the parametric approach and evaluation of construction-based impacts, *Automation in Construction*, vol. 75, pp. 108-126

Alhassoun, S., Sendil , U., Al-Othman, A.A. and Negm, A.M. (1997) Stochastic generation of annual and monthly evaporation in Saudi Arabia, *Canadian Water Resources Journal*, vol. 22(2), pp. 141-154.

Ali, A. (1998). Nonparametric spatial rainfall characterization using adaptive kernel estimator, *Journal of Geographic Information and Decision Analysis,* vol. 2(2), pp. 34-43.

Allen, R.G., Pereira, L.S., Rae, D. and Smith, M. (1998) Crop evapotranspiration (guidelines for computing crop water requirements*), FAO Irrigation and Drainage Paper No. 56,* 300 pp.

Anderson, M.P. and Woessner, W.W. (1992) *Applied groundwater modelling simulation of flow and advective transpor*t, Academic Press, 381 pp.

Apipattanavis, S., Podestá, G., Rajagopalan, B. and Katz, R.W. (2007) A semiparametric multivariate and multisite weather generator, *Water Resources Research*, vol. 43, W11401, doi:10.1029/2006WR005714.

Ashton, P., Love, D., Mahachi, H., and Dirks, P. (2001) An overview of the impacts of mining and mineral processing operations on water resources and water quality in the Zambezi, Limpopo and Olifants catchments in Southern Africa, *MMSD Southern Africa Research Reports*, Report No. ENV-P-C 2001-042, South Africa, 336 pp.

Aslan, S., Yozgatlıgil, C., İyigün, C., Batmaz, I., Türkeş, M. and Tatli, H. (2010) Comparison of missing value imputation methods for Turkish monthly total precipitation data, *9th International Conference on Computer Data Analysis and Modeling: Complex Stochastic Data and Systems*, Minsk, Belarus, pp. 137–140.

Babikera, M. and Gudmundsson, A. (2004) The effects of dykes and faults on groundwater flow in an arid land: the Red Sea Hills, Sudan, *Journal of Hydrology*, vol. 297, pp. 256–273.

Bailey, A.K. and Pitman, W.V. (2016) Water Resources of South Africa, 2012 Study (WR2012), WRSM/Pitman Theory Manual, WRC Report No. TT 690/16, Water Research Commission, Pretoria, South Africa, 163 pp.

Baiocchi, A., Dragoni, W., Lotti, F. and Piscopo, V. (2014) Sustainable yield of fractured rock aquifers: the case of crystalline rocks of Serre Massif (Calabria, southern Italy), Chapter 5, In: Sharp, J.M. (Ed). *Fractured rock hydrogeology*, CRC Press/Balkema, Netherlands, pp. 79-96.

Barenblatt, G.E., Zheltòv, I.P. and Kochina, I.N. (1960) Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks, *Journal of Applied Mathematics and Mechanics*, vol. 24(5), pp. 1286-1303.

Barker, A.J. (1988) A generalized radial flow model for hydraulic tests in fractured rock, *Water Resources Research*, vol. 24 (10), pp. 1796-1804.

Barker, O., Brandl, G., Callaghan, C.C. and van der Neut, M. (2006) The Soutpansberg and Waterberg Groups and Blouberg formation, Chapter 14, In. Johnson, C. R., Anhaeusser, and R.J Thomas (Eds), *The Geology of South Africa,* Geological Society of South Africa and the Council for Geoscience, pp. 301-318.

Baron, J., Seward, P. and Seymour, A. (1998) *The groundwater harvest potential map of the Republic of South Africa*, Technical Report no. Gh 3917, Department of Water Affairs, Pretoria, South Africa.

Basson, M.S., Allen, R.B., Pegram, G.G.S. and van Rooyen, J.A. (1994) *Probabilistic management of water resource and hydropower systems*, Water Resources Publications, USA, 434 pp.

Basson, M.S., Van Rooyen, J.A. (2001) Practical application of probabilistic approaches to the management of water resource systems, *Journal of Hydrology*, vol. 241, pp. 53–61.

Baumle, R. (2003) *Geohydraulic characterisation of fractured rock flow regimes regional studies in granite (Lindau, Black Forest, Germany) and dolomite (Tsumeb Aquifers, Northern Namibia)*, PhD thesis, University of Karlsruhe, Germany, 149 pp.

Bazgeer, S., Oskuee, E.A., Hagigat, M. and Astane, A.R.D. (2012) Performance of spatial interpolation methods for mapping precipitation data: a case study of Fars Province, Iran, *Trends in Applied Sciences Research*, vol. 7(6), pp. 467-475.

Bear, J. and Levin, O. (1966) Optimal utilization of an aquifer as an element of a water resources system, Technion Research Foundation, Haifa, Israel.

Beauheim R.L., Roberts, R.M. and Avis, J.D. (2004) Well testing in fractured media: flow dimensions and diagnostic plots, *Journal of Hydraulic Research, v*ol. 42. pp. 69-76.

Bejaichund, M., Kijko, A. and Durrheim, R. (2009) Seismotectonic models for South Africa: Synthesis of Geoscientific Information, problems, and the way forward, *Seismological Research Letters*, vol. 80, pp. 65-33.

Bennett, N.D., Croke, B.F.W., Guariso, G.G., Guillaume, J.H.A., Hamilton, H.S., Jakeman, A.J., Marsili-Libelli, S., Newham, L.T.H., Norton, J.P., Perrrin, C., Pierce, S.A., Robson, B., Seppelt, R., Voinov, A.A., Fath, B.D. and Andreassian, V. (2013) Characterising performance of environmental models, *Environmental Modelling and Software*, vol. 40, pp. 1-20.

Bidwell V.J. (2005) Realistic forecasting of groundwater level, based on the eigenstructure of aquifer dynamics, *Mathematics and Computers in Simulation*, vol. 69, pp. 12-20.

Bierkens, M.F.P., Knotters, M. and Hoogland (2010) Space time modelling of water table depth using regionalized time series model and Kalman filter, *Water Resources Research*, vol. 37(5), pp. 1277-1290.

Blake, D., Mlisa, A. and Hartnady, C. (2010) Large scale quantification of aquifer storage and volumes from the Peninsula and Skurweberg Formations in the southwestern Cape, *Water SA,* vol. 36(2), pp. 177-184.

Blersch, C.L. (2014) *Planning for seawater desalination in the context of the Western Cape water supply system,* Master of Engineering, Stellenbosch University, 109 pp.

Borgomeo, E., Hall, J.W., Fung, F., Watts, G., Colquhoun, K. and Lambert, C. (2014) Risk-based water resources planning: Incorporating probabilistic nonstationary climate uncertainties, *Water Resource Research*, vol. 50, pp. 6850-6873.

Boslaugh, S. and Waters, A.W. (2008) *Statistics in a Nutshell*, 1st Edition, O'Reilly Media, Inc., USA, 452 pp.

Braester, C. (2003*) Groundwater flow through fractured rocks*, In: Silveira, G. and Usunoff, E.J. (Eds), *Groundwater*, vol. 2, Encyclopedia of Life Support Sciences, pp. 22-42.

Brandl, G. (1981) The geology of the Messina area, Explanation sheet, Geological Survey South Africa, 2230 (Messina), 35 pp.

Brandl, G. (2003) Geology, In: Macdonald I.A.W., Gaigher, I. and Gaigher, R. (Eds), A first synthesis of the environmental, biological and cultural assets of the Soutpansberg, Lajuma Synthesis workshop, http://www.soutpansberg.com/workshop/pdf_files/geology.pdf, accessed on 01 July 2015.

Brassington, F.C. and Younger, P.L. (2010) A proposed framework for hydrogeological conceptual modelling, *Water and Environment Journal*, vol. 24, pp. 261-273.

Braune, E. and Xu, Y. (2008) Groundwater management issues in Southern Africa-An IWRM perspective, *Water SA*, vol. 34 (6), pp. 699-706.

Breinl, K., Turkington, T. and Stowasser, M. (2013) Stochastic generation of multi-site daily precipitation for applications in risk management, *Journal of Hydrology*, vol. 498, pp. 23–35.

Buchanan, R. and Buddemeier, R.W. (2005), Kansas Ground Water, Kansas Geological Survey, Educational Series 10, http://www.kgs.ku.edu/Publications/Bulletins/ED10/07_manage.html, accessed on 31 December 2018.

Bumby, A.J., Eriksson, P.G., van der Merwe, R. and Steyn, G.L. (2002) A half-graben setting for the Proterozoic Soutpansberg Group (South Africa): evidence from the Blouberg area, *Sedimentary Geology*, vol. 147 (1-2), pp. 37-56.

Buskirk, T.D., Willoughby, L.M. and Tomazic, T.J. (2013) Nonparametric statistical techniques, In: Little T.D., *The Oxford handbook of quantitative methods*, vol. 2, statistical analysis, Oxford University Press, United States of America.

Calow, R., McDonald, A., Nicol, A., Robins, N. and Kebede, S. (2002) *The struggle for water: Drought, water security and rural livelihoods*, British Geological Survey Commissioned Report CR/02/226N, UK, 67 pp.

Cao, Y., Poh, K.L. and Cui, W. J. (2008) A non-parametric regression approach for missing value imputation in microarray, *Intelligent Information Systems*, pp. 25-34.

Celeste, A.B. and Billib, B. (2009) Evaluation of stochastic reservoir operation optimization models, *Advances in Water Resources*, vol. 32, pp. 1429-1443.

Celeste, A.B., Curi, W.F., and Curi, R.C. (2009) Implicit stochastic optimization for deriving reservoir operating rules in semiarid Brazil, *Pesquisa Operacional*, vol.29 (1), pp. 223-234.

Chandler, R. and Scott, M. (2011) *Statistical methods for trend detection and analysis in the environmental sciences*, John Wiley and Sons, 388 pp.

Chang, Y.-C., Yeh, H.-D., Liang, K.-F. and Kuo, M.-C. T. (2011) Scale dependency of fractional flow dimension in a fractured formation, *Hydrology Earth System Sci*ences, vol. 15, pp. 2165-2178.

Chiew, F.H.S. and Wang, Q.J. (1999) *Hydrological analysis relevant to surface water storage at Jabiluka*, Supervising Scientist Report 142, Supervising Scientist, Canberra, Australia, 25 pp.

Chilton, P.J. and Foster, S.S.D. (1995) Hydrological characterization and water-supply potential of crystalline aquifers in tropical Africa, *Hydrogeology Journal,* vol. 3, pp. 36-49.

Chinoda, G., Matura, N., Moyce, W. and Owen, R. 2009. Baseline Report on the Geology of the Limpopo Basin Area, a contribution to the Challenge Program on Water and Food Project 17 "Integrated Water Resource Management for Improved Rural Livelihoods: Managing risk, mitigating drought and improving water productivity in the water scarce Limpopo Basin". WaterNet Working Paper 7. WaterNet, Harare, 44 pp.

Chu, J., Zhang, C., Fu, G., Li, Y. and Zhou, H., (2015) Improving multi-objective reservoir operation optimization with sensitivity-informed dimension reduction, *Hydrology and Earth System Sciences*, vol. 19, pp. 3557-3570.

Cleveland, W. S. and Devlin S. J. (1988) Locally weighted regression: an approach to regression analysis by local fitting, *Journal of the American Statistical Association*, vol. 83 (403), pp. 596-610.

Cleveland, W.S. (1979) Robust locally weighted regression and smoothing, *Journal of the American Statistical Association*, vol. 74(368), pp. 829-836.

Cleveland, W.S. and Grosse, E. (1991) Computational methods for local regression, *Statistics and Computing,* vol. 1, pp. 47-62.

Cleveland, W.S. and Loader, C. (1996) Smoothing by local regression: principles and methods, Chapter 2, In: Hardie, W. and Schimek, M.G. (Eds) *Statistical theory and computational aspects of smoothing*, Springer publishers, pp. 10-49.

Cobbing, J., Witthuser, K., Pietersen K., Fourie, F., Nyabeze, W. and Esterheizen, D. (2009) Integrating groundwater into national water resources planning, *Proceedings of the Groundwater Conference*, Somerset West, Cape Town, South Africa.

Cohen, R.A. (1999) An introduction to PROC LOESS for local regression, Statistics, data analysis, and modeling, *Proceedings of the 24th SAS Users Group Conference*, Florida, USA.

Conrad, J. and van der Voort, I. (2000) Classification of groundwater resources under the National Water Act of South Africa, In: Sililo *et al.* (Eds) *Groundwater: Past achievements and Future Challenges*, Balkema Publishers, Netherlands, pp. 901-905.

Conrad, J.E. and van der Voort, I. (1999) *A GIS-based experimental methodology to determine the utilisable potential of South African aquifer*, WRC Report No. 840/1/99, Water Research Commission, Pretoria.

Cooper, H.H. and Jacob, C.E. (1946) A generalized graphical method for evaluating formation constants and summarising well field history, *American Geophysical Union Transactions*, vol. 27, pp. 526-534.

Daly, C., Gibson, W. P., Taylor, G. H., Johnson, G. L., Pasteris, P. (2002) A knowledge-based approach to the statistical mapping of climate, *Climate Research*, vol. (22), pp. 99-113.

Daniels, B.K. (2014) *Hydrologic response to climate change in California: observation and modelling studies*, PhD in Earth Sciences, University of California, Santa Cruz, 160 pp.

Das Gupta, A. and Onta, P.R. (1994) Groundwater management models for Asian developing countries, *Water Resources Development*, vol. 10(4), pp. 457-474.

Das Gupta, A., Nobi, N. and Paudyai, G.N. (1992) Modelling a multi-aquifer system for groundwater management, In: Gayer, J., Starosolszky, O. and Maksimovic, O. (Eds) *Hydrocomp '92*, Water Resources Research Center (VITUKI), Hungary, pp. 41-48.

Das Gupta, A., Nobi, N. and Paudyai, G.N. (1996) Ground-water management model for an extensive multiaquifer and an application, *Groundwater*, vol. 34(2), pp. 349-357.

Das, A. and Datta, B. (2001) Application of optimisation techniques in groundwater quantity and quality management, *Sadhana*, vol. 26(4), pp. 293-316.

Datta, B. and Hakrishna, V. (2005) Optimization applications in water resources systems engineering, directions, *Research Journal of IIT Kanpur*, vol. 6(3), pp. 57-64.

de Farias, C.A.S., Kadota, A., Suzuki, K. and Shiguematsu, K. (2011) Stochastic generation of daily groundwater levels by artificial neural network, *Journal of Japan Society of Civil Engineers*, vol. 67(4), pp. I-55-I-60.

de Mendiguren J.C. and Mabelane, M. (2001) Economics of productive uses for domestic water in rural areas: a case study from Bushbuckridge South Africa, AWARD Research Report.

De Silva, R.P., Dayawansa, N.D.K. and Ratnasir, M.D. (2007) A comparison of methods used in estimating missing rainfall data, *Journal of Agricultural Sciences*, vol.3(2), pp. 101-108.

DeAngelis, A.M., Broccoli, A.J. and Decker, S.G. (2013) A comparison of CMIP3 simulations of precipitation over North America with observations: daily statistics and circulation features accompanying extreme events, *Journal of Climate*, vol. 26, pp. 3209-3230.

Dennis S.R. (2007) *Development of a decision tool for groundwater management*, PhD thesis, Faculty of Natural and Agricultural Sciences, Institute for Groundwater Studies, University of the Free State, Bloemfontein, 198 pp.

Dhungel, R. and Fiedler, F. (2016) Water balance to recharge calculation: implications for watershed management using systems dynamics approach, *Hydrology*, vol. 3(13), doi:10.3390/hydrology3010013.

Di Piazza, A., Lo Conti, F., Noto, L.V., Viola, F. and La Loggia, G. (2011) Comparative analysis of different techniques for spatial interpolation of rainfall data to create a serially complete monthly time series of precipitation for Sicily, Italy*, International Journal of Applied Earth Observation Geoinformation,* vol.13, pp. 396-408.

Di Piazza, A., Lo Conti, F., Viola, F., La Loggia, G., Eccel, E. and Noto, L.V. (2015) Comparative analysis of spatial interpolation methods in the Mediterranean Area: application to temperature in Sicily, *Water*, vol. 7, pp. 1866-1888.

Dippenaar, M.A., Witthüser, K.T. and Van Rooy, J.L. (2009) Groundwater occurrence in basement aquifers in Limpopo Province, South Africa: model-setting-scenario approach, *Environmental Earth Sciences*, vol. 59, pp. 459-464.

Dracup, J., and Dale, L. (2011) *Conjunctive use of aquifers and reservoirs: tradeoffs between energy generation and water supply*, California Energy Commission, PIER Energy-Related Environmental Research Program. CEC-500-2010-023, 35 pp.

Draper, A.J. (2001) *Implicit stochastic optimization with limited foresight for reservoir systems*, PhD Dissertation, University of California Davis, 164 pp.

Dreizin, Y.C. and Haimes, Y.Y. (1977) A hierachy of response functions for groundwater management, *Water Resources Research*, vol. 13(1), pp.78-86.

du Toit W.H, Mulin, H. and Jonck, F. (2002) 1:500 000 Hydrogeological map series of the Republic of South Africa, Messina 2230, 1st Edition, *DWAF*, Pretoria.

du Toit, W., Holland, M., Weidemann, R. and Botha, F. (2011) Can groundwater be successfully implemented as a bulk water resource within rural Limpopo Province? Analysis based on GRIP datasets, *Water SA*, vol. 38(3), pp. 391-398.

Duan, Q.Y., Sorooshian, S., Gupta, V. (1992) Effective and efficient global optimization for conceptual rainfall–runoff models, *Water Resources Research*, vol. 28 (4), pp. 1015–1031.
Duffield, G.M. (2007) *Aqtesolv user guide*, Version 4.5, HydroSOLVE, Inc, 529 pp.

DWA (2010) Orange River System: Annual Operating Analysis 2008-2010, Report no: P RSA C000/00/12210, Department of Water Affairs, Pretoria, South Africa.

DWA (2011) Development of a reconciliation strategy for all towns in the Northern Region, First Order Reconciliation Strategy for Nzhelele Regional Water Supply Scheme, Draft 1.2, DWA Contract WP 971, Pretoria, South Africa, 12 pp.

DWA, (2013) *Feasibility study for augmentation of the Lusikisiki Regional Water Supply Scheme: assessment of augmentation from groundwater*, DWA Report no. PWMA 12/T60/00/3811, Department of Water Affairs, Pretoria, South Africa.

DWAF (2008a) Vaal River System: feasibility study for utilisation of Taung Dam Water: water demand and use, Report no: P WMA 10/C31/00/0608, Department of Water Affairs, Pretoria, South Africa.

DWAF (2008b) Vaal River System: Feasibility study for utilisation of Taung Dam water: yield and system analysis, DWAF Report Number: P WMA 10/C31/00/0708, Department of Water Affairs and Forestry, Pretoria, South Africa.

DWAF, (2004) *Approaches to quantifying groundwater*, Department of Water Affairs and Forestry, Pretoria, 43 pp.

DWAF, (2006a) *Groundwater resource assessment II: Task 1D-groundwater quantificatio*n, Department of Water Affairs and Forestry, Pretoria, South Africa, 106 pp.

DWAF, (2006b) *Groundwater resource assessment II: Task 3aE-recharge*, Department of Water Affairs and Forestry, Pretoria, South Africa, 129 pp.

DWS, (2016) Drought status report no.4, *Department of Water and Sanitation*, Pretoria, South Africa, 27 pp.

Efstratiadis, A., Dialynas, Y.G., Kozanis, S. and Koutsoyiannis, D. (2014) A multivariate stochastic model for the generation of synthetic time series at multiple time scales reproducing long-term persistence, *Environmental Modelling and Software*, vol. 62, pp. 139-152.

Ehlig-Economldes, C. (1988) Use of the pressure derivative for diagnosing pressure-transient behavior*, Journal of Petroleum Technology*, pp. 1280-1282.

Elshorbagy, A. A., Panu, U. S. and Simonovic, S. P. (2000). Group-based estimation of missing hydrological data: I. Approach and general methodology, *Hydrological Sciences Journal*, vol. 45(6), pp. 849-866.

Fallah-Mehdipour, E., Bozorg Haddad, O. and Mariño, M.A. (2013) Prediction and simulation of monthly groundwater levels by genetic programming, *Journal of Hydro-environment Research*, vol. 7(4), pp. 253-260.

FAO, (2003) *Groundwater management-the search for practical approaches*, Water Report 25, Rome, Italy.

FAO, (2004) *Drought impact mitigation and prevention in the Limpopo River Basin: A Situational Analysis*, Land and Water Discussion Paper Vol.4, Rome, Italy, 160 pp.

Feng, S., Kang, S., Huo, Z., Chen, S. and Mao, X. (2008) Neural networks to simulate regional groundwater levels affected by human activities, *Groundwater,* vol. 46, (1), pp. 80-90.

Fox, J. and Weisberg, S. (2011) Nonparametric regression in R, An appendix to an R companion to applied regression, 2nd Edition, Sage Publishers, United States of America.

Fox, J. (2002) Nonparametric regression, *http://cran.r-project.org/doc/contrib/Fox-Companion/appendix-nonparametric-regression.pdf*, accessed on 27 February 2014.

Fung, D.S.C. (2006) *Methods for the estimation of missing values in time series*, Master of Science, Edith Cowan University, Perth, Western Australia, 202 pp.

Furrer, E.W. and Katz, R.W. (2007) Generalized linear modeling approach to stochastic weather generators, *Climate Research*, vol. 34, pp. 129–144.

Gallagher, M. and Leach, L. (2010) The Groundwater Operational Management Package (GWOMP), Groundwater 2010, Canberra, *www.groundwater2010.com/documents/GallagherMark.pdf*.

Geier, J.E., Doe, T.W., Benabderrahman, A. and Hassler, L. (1996) Generalized radial flow interpretation of well tests for the SITE-94 Project, SKI Report 96:4, 171 pp.

GEOSS, (2006) *Groundwater assessment of the North-West Sandveld and Saldanha Peninsula as an integral component of the Cape Fine-Scale Biodiversity Planning Project*, Component 5.1, Fine-scale biodiversity planning, GEOSS Consultancy Report, C.A.P.E. Fine-scale Biodiversity Planning Project, Cape Town.

Gernand, J.D. and Heidtman, J.P. (1997) Detailed pumping test to characterize a fractured bedrock aquifer, *Groundwater*, vol. 35(4), pp. 632-637.

Gorelick, S.M. (1983). A review of distributed groundwater management modeling method, *Water Resources Research*, vol. 19(2), pp. 305-319.

Greene, A.M., Hellmuth, M. and Lumsden, T. (2012) Stochastic decadal climate simulations for the Berg and Breede Water Management Areas, Western Cape Province, South Africa, Water Resources, vol.48, W06504, doi:10.1029/2011WR011152.

Griffioen, J. and Kruseman, G.P. (2004) Determining hydrodynamic and contaminant transfer parameters of groundwater flow, Chapter 8, In: Kovalevsky, V. S., Kruseman, G.P. and Rushton K.R. (Eds), *Groundwater studies, An international guide for hydrogeological investigations*, IHP-VI, Series on Groundwater No. 3, UNESCO, pp. 217-237.

Gringarten, A.C. and Witherspoon, P.A. (1972) A method of analyzing pump test data from fractured aquifers, *Int. Soc. Rock Mechanics and Int. Ass. Eng. Geol., Proc. Symp.* Rock Mechanics, Stuttgart, vol. 3-B pp. 1-9.

Guggino, E., Rossi, G. and Hendricks, D.W. (2012) *Operation of complex water systems: operation, planning and analysis of already developed water systems*, Springer Publishers, 532 pp.

Haddad, O.B. and Mariño, M.A. (2010) Optimum operation of wells in coastal aquifers, *Proceedings of the ICE - Water Management*, vol. 164(3), pp. 135-146.

Halford, K.J. and Kuniansky, E.L. (2002) *Documentation of spreadsheets for the analysis of aquifer-test and slug-test data*, Open-File Report 02-197, U.S. Geological Survey, Carson City, Nevada, 51 pp.

Hall, P. and Chen, J. (1996) *Water well and aquifer test analysis*, Water Resources Publication, 412 pp.

Hallaji, K. and Yazicigil, H. (1996) Optimal management of a coastal aquifer in southern Turkey, *Journal Water Resources Planning Management,* vol. 122, pp. 233-244.

Hammond, P.A. and Field, M.S. (2014) A reinterpretation of historic aquifer tests of two hydraulically fractured wells by application of inverse analysis, derivative analysis, and diagnostic plots, *Journal of Water Resource and Protection*, vol. 6, pp. 481-506.

Hantush, M.S. and Jacob, C.E. (1955) Non-steady radial flow in an infinite leaky aquifer, *Transactions of the American Geophysical Union*, vol. 36, no. 1, pp. 95-100.

Harboe, R. and Ratnayake, U. (1993) Simulation of a reservoir with standard operating rule, Extreme hydrological events: Precipitation, floods and droughts*, Proceedings of the Yokohama Symposium*, IAHS Publication no. 213, pp. 421-428.

Härdle, W. (1989) *Applied nonparametric regression*, http://ft-sipil.unila.ac.id/dbooks/applied%20nonparametric%20regression.pdf, 378 pp.

Harpaz, Y. and Schwrz, J. (1967) Operating a limestone aquifer as a reservoir for a water supply system, *Hydrological Sciences Journal*, vol. 12 (1), pp. 78-90.

Harrold, T.I. (2002). *Stochastic generation of daily rainfall for catchment water management studies*, PhD Thesis, University of New South Wales, Sydney, Australia, 198 pp.

Harrold, T.I., Sharma*,* A. and Sheather S.J. (2003). A nonparametric model for stochastic generation of daily rainfall amounts*, Water Resources Research*, vol. 39*,* pp. 1-12.

Hasan, M.M. and Croke, B.F.W. (2013) Filling gaps in daily rainfall data: a statistical approach 20th International Congress on Modelling and Simulation, Adelaide, Australia, www.mssanz.org.au/modsim2013.

Hemker, K. and Randall, J. (2013) Modelling with MLU, applying the multilayer approach to aquifer test analysis, Amsterdam – Seattle, 66 pp.

Henderson, D.B., Ferril, D.A. and Clarke, K.C. (1996) Mapping geological faults using image processing techniques applied to hill-shaded digital elevation models, Proceeding of the IEEE South West Symposium on Image Analysis and Interpretation, San Antonio, Texas, pp. 240-245.

Herrera, M., Natarajana, S., Coleya, D.A., Kershawa, T., Ramallo-Gonzálezb, A.P., Eamesc, M., Fosasa, D. and Wood, M. (2017) A review of current and future weather data for building simulation, *Building Services Engineering Research and Technology*, vol. 38 (5), pp. 602-627.

Hofstra, N., Malcolm, H., New, M., Jones, P. and Frei, P. (2008) Comparison of six methods for the interpolation of daily, European climate data, *Journal of Geophysical Research*, vol. 113, D21110, doi:10.1029/2008JD010100.

Holland, M. (2011) *Hydrogeological characterisation of crystalline basement aquifers within the Limpopo Province*, South Africa, University of Pretoria, Pretoria, South Africa, 163 pp.

Holland, M. (2012) Evaluation of factors influencing transmissivity in fractured hard-rock aquifers of the Limpopo Province, *Water SA*, vol. 38 (3), pp. 379-390.

Houston, J.F.T. (1983) Ground-water systems simulation by time-series techniques, *Groundwater*, vol. 21(3), pp. 301-310.

Hughes, D.A. (2004) Incorporating groundwater recharge and discharge functions into an existing monthly rainfall–runoff model, *Hydrological Sciences, vol.* 49(2), pp. 297-311.

Hughes, D.A. (2013) A review of 40 years of hydrological science and practice in southern Africa using the Pitman rainfall-runoff model, *Journal of Hydrology*, vol. 501, pp. 111-124.

Hughes, D.A., Kapangaziwiri, E. and Sawunyama, T. (2010) Hydrological model uncertainty assessment in southern Africa, *Journal of Hydrology*, vol. 387, pp. 221-232.

Hunt, B. (2012) *Groundwater analysis using function.xls*, University of Canterbury, http://www.civil.canterbury.ac.nz/staff/hunt/Groundwater%20Analysis%20Using%20Function.pdf, accessed on 21 April 2016.

Illinois State University (2013) Field geology guidebook and notes, http://geo.illinoisstate.edu/downloads/Field%20Guide%202013.pdf, accessed on 10 June 2015.

Ilunga, M. (2010) Infilling annual rainfall data using feedforward back-propagation Artificial Neural Networks (ANN): Application of the standard and generalised back propagation techniques, *Journal of South African Institution of Civil Engineering*, vol. 52(1), pp. 2–10.

Ilunga, M. and Stephenson, D. (2005) Infilling streamflow data using feed-forward back-propagation (BP) artificial neural networks: Application of standard BP and pseudo Mac Laurin power series BP techniques, *Water SA*, vol. 31(2), pp. 171-176.

Institute of Soil, Water and Climate (2003) Land types of the maps 2228 Alldays and 2230 Messina. *Memoirs agric. nat. Resour. S. Afr. No.* 37. ARC-Institute for Soil, Climate & Water, Pretoria.

Ishaku, J.M., Kwada, I.A. and Adekeye, J.I.D. (2009) Hydrogeological characterization and water supply potential of basement aquifers in Taraba State, N.E. Nigeria, *Nature and Science*, vol. 7(3), pp. 75-83.

Izady, A., Davary, K., Alizadeh, A., Nia, A.M. (2013) Application of NN-ARX model to predict groundwater levels in the Neishaboor Plain, Iran, *Water Resources Management*, vol. 13(27), pp. 4773-4794.

Jackson, J.M. (2007) *Hydrogeology and groundwater flow model, Central catchment of Bribie Island, South of Queensland*, MSc Thesis, Queensland University of Technology, Queensland, Australia.

Javan, K., Lialestan, M.R.F. and Nejadhossein, M. (2015) A comparison of ANN and HSPF models for runoff simulation in Gharehsoo River watershed, Iran Model, *Modeling Earth Systems and Environment,* vol. 1(4), pp. 1-13.

Jayawardena, U.S. and Sarathchhandra, M.J. (1995) Land subsidence and other environmental impacts due to groundwater extraction from fractured hard rocks in Sri Lanka, Land Subsidence *Proceedings of the Fifth International Symposium on Land Subsidence*, IAHS Publ. no. 234, pp. 439-444.

Jeanne, P. (2012) Architectural, petrophysical and hydromechanical properties of fault zones in fractured-porous rocks: compared studies of a moderate and mature fault zones (France), PhD in Earth Sciences, University of Nice-Sophia Antipolis, 222 pp.

Johnson, L.E. (1993) Water supply systems adjustment to drought, In: The Colorado Internet Center for Environmental Problem Solving (Ed.), *Improving the environmental problem-solving process: Lessons from the 1988-?? California drought*, CU Conflict Resolution Consortium, Boulder.

Joodavi, A., Zare, M. and Mahootchi, M. (2015) Development and application of a stochastic optimization model for groundwater management: crop pattern and conjunctive use consideration, *Stochastic Environmental Research and Risk Assessment*, vol. 29, pp. 1637–1648.

Juárez-Torres, M., Richardson J.W. and Vedenov, D. (2013) Semiparametric copula-based stochastic weather generator, Banco de Mexico, Working Paper no. 2013-09, 40 pp.

Kabanda, T.A. (2004) Climatology of long-term drought in the Northern region of the Limpopo Province of South Africa, Unpublished PhD, Thesis, University of Venda, South Africa, 199 pp.

Kalra, A. and Ahmad, S. (2011) Evaluating changes and estimating seasonal precipitation for Colorado River Basin using stochastic nonparametric disaggregation technique, *Water Resources Research,* vol. 47, pp. 1-26.

Kalra, A. and Ahmad, S. (2011) Evaluating changes and estimating seasonal precipitation for Colorado River Basin using stochastic nonparametric disaggregation technique, *Water Resources Research,* vol. 47, pp. 1-26.

Kangrang, A., Prasanchum, H. and Hormwichian, R., (2018) Development of future rule curves for multipurpose reservoir operation using conditional genetic and tabu search algorithms, *Advances in Civil Engineering*, vol. 2018, Article ID 6474870, 10 pp.

Kapangaziwiri, E. (2007) *Revised parameter estimation methods for the Pitman monthly rainfall-runoff model*, Master of Science, Rhodes University, South Africa, 165 pp.

Kapangaziwiri, E. and Hughes, D.A. (2008) Towards revised physically based parameter estimation methods for the Pitman monthly rainfall-runoff model, *Water SA*, vol. 34(2), pp. 183-192.

Kapangaziwiri, E., Hughes, D.A. and Wagener, T., (2012) Incorporating uncertainty in hydrological predictions for gauged and ungauged basins in southern Africa, *Hydrological Sciences Journal*, vol. 57(5), pp. 1000-1019.

Karamouz, M. and Kerachian, R., (2004) Optimal operation of reservoir systems considering the water quality: application of stochastic sequential genetic algorithms, *ASCE World Water and Environmental Resources Congress*, Salt Lake City, Utah.

Karay, G. (2013) Evaluating methods of pumping tests to analysing flow properties in fractured rocks. In: Józsa János, Németh Róbert, Lovas Tamás (Eds), *Proceedings of the Second Conference of Junior Researchers in Civil Engineering*, Budapest, Hungary, pp. 235-242.

Kebede, S. (2013) *Groundwater in Ethiopia, Number and opportunities*, Springer Hydrogeology, 283 pp.

Kenabatho, P.K., McIntyre, N.R., Chandler, R.E. and Wheater, H.S. (2012) Stochastic simulation of rainfall in the semi-arid Limpopo basin, Botswana, *International Journal of Climatology*, vol. 32, pp. 1113-1127.

Kerachian, R., and M. Karamouz (2006) Optimal reservoir operation considering the water quality issues: A stochastic conflict resolution approach, *Water Resources Research*, vol. 42, W12401, doi:10.1029/2005WR004575.

Khalema, L. (2010) *Hydrological flow modelling using geographic information systems (GIS): the case study of Phuthiatsane Catchment, Lesotho*, Master of Environmental Management Dissertation, University of KwaZulu-Natal, 81 pp.

Khan, L.R. and Mawdsley, J.A. (1988) Reliable yield of unconfined aquifers, *Hydrological Sciences Journal*, vol. 33 (2), pp. 151-171.

Kigobe, M., McIntyre, N., Wheater, H. and Chandler, R. (2011) Multi-site stochastic modelling of daily rainfall in Uganda, *Hydrological Sciences Journal*, vol. 56 (1), pp. 17-33.

King, L.M. (2012) *Application of a K-Nearest Neighbour weather generator for simulation of historical and future climate variables in the Upper Thames River basin*, Master of Engineering Science, University of Western Ontario, 138 pp.

Knapp, K.C. and Olson, L.J. (1995) The economics of conjunctive groundwater management and stochastic surface supplies, *Journal of Environmental Economics and Management*, vol. 28, pp. 340-356.

Knotters, M. and Bierkens, M.F.P. (2001) Predicting water table depths in space and time using a regionalised time series model, *Geoderma*, vol. 103, pp. 51-77.

Knotters, M. and van Walsum, P.E.V. (1997) Estimating fluctuation quantities from time series of water-table depths using models with a stochastic component*, Journal of Hydrology*, vol. 197, pp. 25-46.

Knüppe, K. (2011) The challenges facing sustainable and adaptive groundwater management in South Africa, *Water SA*, vol. 37(1), pp. 68-79.

Konecny, G. (2012) Keynote paper: Global perspective and multilateral cooperation in earth observation, *Geospatial World Forum 2012*, Amsterdam, The Netherlands.

Kraemer, S.R., and Haitjema**,** H.M. (1989) A modeling approach to regional fracture flow systems, *International Conference on solving groundwater problems with models*, Indianapolis, Indiana.

Krasny, J. (1993) Classification of transmissivity magnitude and variation, *Groundwater*, vol. 31(2), pp. 230–236.

Kruseman, G.P. and de Ridder, N.A. (2000) Analysis and evaluation of pumping test data, 2nd Ed, *International Institute for Land Reclamation and Improvement*, Publication no. 47, pp. 372.

Kumar, C.P. (2004) Groundwater assessment methodology, *Proceedings of the XXIII annual convention of AHI and National Seminar, Water Resources Management and People's participation*, Baroda.

Kuniansky, E.L. and Bellino, J.C. (2012) *Tabulated transmissivity and storage properties of the Floridan aquifer system in Florida and parts of Georgia, South Carolina, and Alabama*, U.S. Geological Survey Data Series 669, U.S. Department of the Interior and U.S. Geological Survey, 37 pp.

Kuusela-Lahtinen, A., Niemi, A. and Luukkonen, A. (2003) Flow dimension as an indicator of hydraulic behaviour in fractured rock, *Groundwater*, vol. 41(3), pp. 333-341.

Labadie, J.W. (2004) Optimal operation of multireservoir systems: state-of-the-art review, *Journal of Water Resources Planning and Management*, vol. 130 (2), pp. 93-111.

Ladki, M.; Seshoka, J.; Faysse, N.; Lévite, H.; Van Koppen, B. (2004) Possible impacts of the transformation of water infrastructure on the productive water uses: The case of the Seokodibeng village in South Africa. Working Paper 74. Colombo, Sri Lanka: International Water Management Institute.

Lall, U. (1995) Nonparametric function estimation: recent hydrologic applications, Reviews of Geophysics, *US National Report 1991-1994*, pp. 1093-1102.

Lall, U., Rajagopalan, B. and Tarboton, D.G. (1996) A nonparametric wet/dry spell model for resampling daily precipitation, *Water Resources Research*, vol. 32(9), pp. 2803-2823.

Lasher, M. (2011) Application of fluid electrical conductivity logging for fractured rock aquifer characterisation at the University of the Western Cape's Franschhoek and Rawsonville Research Sites, Master of Science, University of Western Cape, Cape Town, South Africa, 119 pp.

Le Borgne, T., Bour, O., de Dreuzy, J.R., Davy, P. and Touchard, F. (2009) Equivalent mean flow models for fractured aquifers: Insights from a pumping tests scaling interpretation, *Water Resources Research*, vol. 40, W03512, doi:10.1029/2003WR002436.

Lee, D., An, H., Lee, Y., Lee, J., Lee, H-S. and Oh, H-S. (2010) Improved multisite stochastic weather generation with applications to historical data in South Korea, *Asia-Pacific Journal of Atmospheric* Sciences, vo. 46(4), pp. 497-504.

Lee, H. and Kang, K. (2015) Interpolation of missing precipitation data using kernel estimations for hydrologic modeling, *Advances in Meteorology*, Article ID 935868, 12 pp.

Lee, J-H. and Labadie, J.W. (2007) Stochastic optimization of multi reservoir systems via reinforcement learning, *Water Resources Research*, vol. 43, W11408, doi:10.1029/2006WR005627.

Legates, D.R., and McCabe, G.J. (1999) Evaluating the use of "goodness-of-fit" measures in hydrologic and hydroclimatic model validation, *Water Resources Res*earch, vol. 35(1), pp. 233-241.

Leveinen, J., Rönkä, E., Tikkanen, J. and Karro, E. (1998) Fractional flow dimensions and hydraulic properties of a fracture-zone aquifer, Leppävirta, Finland., *Hydrogeology Journal*, vol. 6, pp. 327–340.

Levy, J. (2011) Groundwater management and groundwater/surface-water interaction in the context of South African water policy, *Hydrogeology Journal*, vol. 20, pp. 205-226.

Lin, S.-C., WU, R.-S. and Wang M.-H. (2003). A hybrid model coupling analytic simulation with optimization for rule-curve based operation in tandem multireservoir systems, *Transactions on Ecology and the Environment*, vol. 61, pp. 75-83.

Lindner, W., Lindner, K., and Karadi, G. (1988). Optimal groundwater management of large scale aquifers, *Water Resources Bulletin*, vol. 24 (1), pp. 27-33.

Liu, B., Wang, H., Lei, X., Liu, Z. and Quan, J., (2018) Emergency operation rules for water-supply reservoirs under uncertainty and risk in dry seasons, *Water Science and Technology: Water Supply*, vol. 18(5), pp. 1682-1695.

Liu, P., Zhao, J., Li, L. and Shen, Y. (2012) Optimal reservoir operation using stochastic dynamic programming, *Journal of Water Resource and Protection*, vol. 4, pp. 342–345.

Ljung, L. (1998) *System identification theory for the user*, 2nd Ed., Linköping University, Sweden.

Ljung, L. (2010) Perspectives on system identification, *Annual Reviews in Control*, vol. 34, pp. 1–12.

Louck, D.P. and van Beek, E. (2017) *Water resources systems planning and management, An introduction to methods, models and applications*, Springer Publishers, Switzerland, 624 pp.

Lund, J.R. and Ferreira, I. (1996) Operating rule optimization for Misssouri River reservoir system, *Jounal of Water Resources Planning and Management*, vol. 22(4), pp. 287-295.

Ly, S., Charles, C. and Degré, A. (2013) Different methods for spatial interpolation of rainfall data for operational hydrology and hydrological modeling at watershed scale, A review, *Biotechnology Agronomy Society and the Environ*ment, vol. 17(2), pp. 392-406.

Lynch, S.D. (2003) *Development of a raster database of annual, monthly and daily rainfall for Southern Africa*, WRC Report No. 1156/1/04, Water Research Commission, Pretoria, South Africa, 78 pp.

MacDonald, A.M. and Calow, R. (1996) *The effect of drought on the availability of groundwater: towards an analytical framework*, British Geological Survey Technical Report *No. WD/96/3,* UK, 12 pp.

Macian-Sorribes, H. (2017) *Design of optimal reservoir operating rules in large water resources systems combining stochastic programming, fuzzy logic and expert criteria*, PhD Thesis, Politechnical University of Valencia, 252 pp.

Makhuvha, T., Pegram, G., Sparks, R. and Zucchini, D. (1997b) Patching rainfall data using regression methods: 2, Comparisons of accuracy, bias and efficiency, *Journal of Hydrology.*, vol. 198, pp. 308–318.

Makhuvha, T., Pegram, G., Sparks, R. and Zucchini, W. (1997a) Patching rainfall data using regression methods: 1, Best subset selection, EM and pseudo-EM methods: Theory, *Journal of Hydrology*, vol. 198, 289-307.

Makungo, R. (2009) *An operating strategy for run-of-river abstractions for typical rural water supply schemes using Siloam Village as a case study*, MSc Thesis, University of Venda, 146 pp.

Makungo, R. and Odiyo, J.O. (In Press) Application of non-parametric regression in estimating missing daily rainfall data, International Journal of Hydrology Science and Technology, Manuscript no. IJHST-153438.

Makungo, R. and Odiyo, J.O. (2017) Estimating groundwater levels using system identification models in Nzhelele and Luvuvhu areas, Limpopo Province, South Africa, *Physics and Chemistry of the Earth*, vol. 100, pp. 44-50.

Makungo, R., Odiyo, J.O., Ndiritu, J.G. and Mwaka, B. (2010) Rainfall runoff modelling approach for ungauged catchments: a case study of Nzhelele River Sub-Quaternary Catchment, *Physics and Chemistry of the Earth*, vol. 35, pp. 596-607.

Makungo, T.E. (2008) The adequacy of water supply to meet the demand in Siloam Village of Limpopo Province, South Africa, unpublished Honours dissertation, Department of Hydrology and Water Resource, University of Venda, South Africa, 91 pp.

Mallory, S., Pashkin, J. and Jonker, V. (2017) Water resources management during droughts in South Africa, *IAHS2017-206 IAHS Scientific Assembly 2017*, Port Elizabeth, South Africa.

Manzione, R.L. , Knotters, M., Heuvelink, G.B.M., Von Asmuth, J. R. and Câmara, G. (2009) Predictive risk mapping of water table depths in a Brazilian Cerrado area, Chapter 7, In: Stein, A., Shi, W., and Bijker, W. (Eds), *Quality aspects in spatial data mining*, CRC Press, pp. 73-89.

Maré, H.G., Mwaka, B. and Sinha, P. (2007) Development of simplified drought operating rules. URL: http://www.miya-water.com/data-and-research/articles-by-miyas-experts/water-resource-management, accessed on 01 April 2012.

Maréchal, J-C, Dewandel, B., Subrahmanyam, K. and Torri, R. (2003) Special methods for the evaluation of hydraulic properties in fractured hard-rock aquifers, *Current Science*, vol. 85 (4), pp.511-516.

Marques, G.F., Lund, J.R. and Howitt, R.E. (2010) Modeling conjunctive use operations and farm decisions with two-stage stochastic quadratic programming, *Journal of Water Resources Planning and Management*, vol. 136(3), pp. 386-394.

Mason, R. (1973) The Limpopo Mobile Belt, Southern Africa, *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences,* vol. 273, pp. 463-485.

McCuen, R.H. (1998) *Hydrologic analysis and design*, 2nd Ed., Prentice-Hall, Englewood Cliffs, New Jersey, 814 pp.

McDowell, R.J. (2010) Assessing sustainable yield of aquifers as part of the Georgia comprehensive statewide water management plan, GSA Denver Annual Meeting, *Geological Society of America Abstracts with Programs*, vol. 42(5), pp. 490.

McMahon, T.A. and Adeloye, A.J. (2005) *Water resources yield*, Water Resources Publications, 218 pp.

McPhee, J. and Yeh, W.W-G. (2004) Multiobjective optimization for sustainable groundwater management in semiarid regions, *Journal of Water Resources Planning and Management*, vol. 130(6), pp. 490-497.

Mehrotra, R. and Sharma, A. (2006) A nonparametric stochastic downscaling framework for daily rainfall at multiple locations, *Journal of Geophysical Research*, vol. 111, D15101, DOI: 10.1029/2005JD006637.

Meyer, R. (2002) *Guidelines for the monitoring and management of groundwater resources in rural water supply schemes*, WRC Report Number 861/1/02a, Water Research Commission, Pretoria, South Africa, 55 pp.

Middleton, B.J. and Bailey, A.K. (2009) Water Resources of South Africa, 2005 Study (WR2005), WRC Report Number TT380/08, Water Research Commission, Pretoria.

Milne-Home, W.A. (1988) Interpretation of aquifer tests in fractured aquifers: from theory to routine field analysis, *Proceedings of IV Canadian/American Conference on Hydrology: Fluid Flow, Heat Transfer and Mass Transport in Fractured Rocks (Banff, Alberta, Canada)*, pp. 177-184.

Misstear, B.D.R. and Beeson, D. (2000) Using operational data to estimate the reliable yields of water-supply wells, *Hydrogeology Journal*, vol. 8, pp. 177-187.

Moench, A.F. (1985) Transient flow to a large-diameter well in an aquifer with storative semi-confining layers, *Water Resources Research,* vol. 21(8), pp. 1121-1131.

Mohamed, L., Mohamed, S., Mohamed, A., Zaki, A., Sauck, W., Soilman, F., Yan, E., Elkadir, R. and Abouelmagd, A. (2015) Structural controls on groundwater flow in basement terrains: geophysical, remote sensing, and field investigations in Sinai, *Survey in Geophysics*, DOI 10.1007/s10712-015-9331-5.

Monirul, I.M. and Kanungoe, P. (2005) Natural recharge to sustainable yield in the Barind aquifer a tool in preparing effective management plan of groundwater resources, *Water Science and Technology*, vol. 52 (12), pp. 251-258.

Morel, E.H. and Wikramaratna, R.S. (1982) Numerical modelling of groundwater flow in regional aquifers dissected by dykes, *Hydrological Sciences Journal*, vol. 27(1), pp. 63-77.

Moriasi, D. N., Arnold, J. G., van Liew, M. W., Bingner, R. L., Harmel, R. D., and Veith, T. (2007) Model evaluation guidelines for systematic quantification of accuracy in watershed simulations, *Transactions of the American Society of Agricultural and Biological Engineers,* vol. 50(3), 885-900.

Mott, P., Sammis, T.W. and Southward, G.M. (1994). Climate data estimation using information from surrounding climate stations, *Applied Engineering in Agriculture,* vol. 10(1), pp. 41-44.

Mousavi, S.J., Ponnambalam, K. and Karray, F., (2007) Inferring operating rules for reservoir operations using fuzzy regression and ANFIS, *Fuzzy Sets and Systems*, vol. 158(10), pp.1064-1082.

Moustafa, M.H. (2011) Application of derivative analysis technique for pumping test interpreation, coastal aquifer, Sharm El Sheikh Area, Egypt, *JPME*, 14(2), pp. 53-65.

Murray, R., Baker, K., Ravenscroft, P., Musekiwa, C. and Dennis R. (2012). A groundwater-planning toolkit for the main Karoo basin: Identifying and quantifying groundwater-development options incorporating the concept of well field yields and aquifer firm yields, *Water SA*, vol. 38(3), pp. 407-416.

Mwelwa, E.M. (2004) *The application of the monthly time step Pitman rainfall-runoff model to the Kafue River Basin of Zambia*, Master of Science, Rhodes University, Grahamstown, South Africa, 182 pp.

National Groundwater Strategy (2010) *Planning for water resources-groundwater*, Newsletter no. 3, http://www.dwa.gov.za/Groundwater/Documents/NGS%20Newsletter%20-%20April%202010.pdf, accessed on 07/03/2011.

Ndiritu J. (2011) A variable-length block bootstrap method for multisite synthetic streamflow generation, *Hydrological Sciences Journal*, vol. 56(3), pp. 362-379.

Ndiritu, J. (2009) A comparison of automatic and manual calibration using the Pitman model *Physics and Chemistry of the Earth*, vol. 34, pp. 729–740.

Ndiritu, J. and Nyaga, J. (2014) *A non-parametric multi-site stochastic rainfall model with applications to climate change*, WRC Report No. 2148/1/13, Water Research Commission, Pretoria, South Africa, 107 pp.

Ndiritu, J.G. (2003) Reservoir system optimisation using a penalty approach and a multi-population genetic algorithm, *Water SA*, vol. (29)3, pp. 273-280.

Ndiritu, J.G. (2005) Maximizing water supply system yield subject to multiple reliability constraints via simulation-optimisation, *Water SA*, vol. 31(4), pp. 423-434.

Ndiritu, J.G., Odiyo, J.O., Makungo, R., Ntuli, C. and Mwaka, B. (2011a) Yield–reliability analysis for rural domestic water supply from combined rainwater harvesting and run-of-river abstraction, *Hydrological Sciences Journal*, vol. 56(2), pp. 238-248.

Ndiritu, J.G., Odiyo, J.O., Makungo, R., Ntuli, C. and Mwaka, B. (2011b) Incorporating hydrological reliability in rural rainwater harvesting and run-of-river supply, *Risk in Water Resources Management,* IAHS Publication no 347.

Ndiritu, J.G., Odiyo, J.O., Makungo, R., Ntuli, C., Mwaka, B. and Mthethwa, N. (2017) Development of probabilistic operating rules for Hluhluwe Dam, South Africa, *Physics and Chemistry of the Earth*, vol. 100, pp. 343-352.

Ndzabandzaba, C. and Hughes, D.A. (2017) Regional water resources assessments using an uncertain modelling approach: The example of Swaziland, *Journal of Hydrology: Regional Studies*, vol. 10, pp. 47-60.

Ngo, L.L. (2007) Optimising reservoir operation: A case study of the Hoa Binh reservoir, Vietnam. Kgs. Lyngby, DTU Environment, 39 pp.

Nikoo, M.R., Karimi, A., Kerachian, R., Poorsepahy-Samian, H. and Daneshmand, F., (2013) Rules for optimal operation of reservoir-river-groundwater systems considering water quality targets: application of M5P model, *Water Resources Management*, vol. 27(8), pp. 2771-2784.

Nkuna, T.R. and Odiyo, J.O. (2011) Filling of missing rainfall data in Luvuvhu River Catchment using artificial neural networks, *Physics and Chemistry of the Earth,* vol. 36, pp. 830-835.

Nourani, V. and Mano, A. (2007) Semi-distributed flood runoff model at the sub continental scale for southwestern Iran, *Hydrological Processes,* vol. 21, pp. 3173-3180.

Nyabeze, P.K, Venter, J.S., Olivier, J. and Motlakeng T.R. (2010) Characterization of the thermal aquifer associated with the Siloam hot spring in Limpopo, South Africa, *Proceedings of the 3rd IASTED African Conference*, Water Resource Management, Gaborone, Botswana.

Nyabeze, P.K., Gwavava, O., Olivier, J. Chirenje, E. and Sekiba, M. (2011b) Geophysical characteristics of aquifers associated with thermal springs in Limpopo, South Africa. *International Groundwater Conference*, Pretoria, South Africa, 123 pp.

Nyabeze, P.K., Gwavava, O., Olivier, J., Chirenje E. and Sekiba, M. (2011a) Geophysical characteristics of a shallow aquifer associated with a hot spring in the Limpopo Province, South Africa: results for the Siloam hot spring, *Geosynthesis 2011 Conference and Exhibition*, Cape Town International Convention Centre, South Africa.

Nyaga, J.M. (2014) *The use of empirical mode decomposition (EMD) and variable length bootstrap (VLB) for stochastic rainfall generation*, Masters of Science in Engineering, University of the Witwatersrand, South Africa, 67 pp.

Nyende, J., van Tonder, G. and Vermeulen, J. (2016) Characterisation of weathered and granitised fractured bedrock aquifer based on WISH and FC methods, *International Journal for Research in Agricultural Research*, vol. 2(1), pp. 8-23.

Obiero, J.P.O., Hassan, M. A., Gumbe, L.O. M. (2011) Modelling of streamflow of a catchment in Kenya, *Journal of Water and Protection,* vol. 3, pp. 667-677.

Ochoa, F.D.C and Reinoso, J.C.M. (1997) Model of long-term water-table dynamics at Doñana National Park, *Water Resources Research*, vol. 31(10), pp. 2586-2596.

Odiyo, J.O. Phangisa, J. and Makungo R. (2012) Rainfall-runoff modelling for estimating Latonyanda River flow contributions to Luvuvhu River downstream of Albasini Dam, *Physics and Chemistry of the Earth*, vol. 50–52, pp. 5-13.

Odiyo, J.O., Makungo, R., Ndiritu, J., Mwaka, B. and Ntuli, C. (2015) Yield-reliability analysis and operating rules for run-of-river abstractions for typical rural water supply: Siloam Village case study, *Water SA*, vol. 41, pp. 375-381.

Odling, N.E., West, L.J., Hartmann, S. and Kilpatrick, A. (2013) Fractional flow in fractured chalk, a flow and tracer test revisited, *Journal of Contaminant Hydrology*, vol. 147, pp. 96 - 111.

Ökten, S. and Yacigizil, H. (2005). Investigation of safe and sustainable yields for the Sandy Complex Aquifer System in the Ergene River Basin, Thrace Region, Turkey Turkish, *Journal of Earth Sciences*, vol. 14, pp. 209-226.

Oliveira, M.T. (2001) Modeling water content of a vineyard soil in the Douro Region, Portugal, *Plant and Soil,* vol. 233, pp. 213-221.

Oliveira, R. and Loucks, D.P., (1997) Operating rules for multireservoir systems, *Water Resources Research*, vol. 33(4), pp. 839-852.

Panigrahi, D.P. and Mujumdar, P.P., (2000) Reservoir operation modelling with fuzzy logic, *Water Resources Management*, vol. 14(2), pp. 89-109.

Paredes, J. and Lund, J.R., (2006) Refill and drawdown rules for parallel reservoirs: quantity and quality, *Water Resources Management*, vol. 20(3), pp. 359-376.

Parsons, R. (2004) *Surface Water: groundwater interactions in South African context– a geohydrological perspective,* WRC Report No. TT 218/03, Water Research Commission, Pretoria, South Africa.

PCI Geomatica (2001) PCI Geomatica user's guide version 9.1, Ontario. Canada: Richmond Hill.
Pegram, G. (1997) Patching rainfall data using regression methods: 3, Grouping, patching and outlier detection, *Journal of Hydrology*, vol. 198, pp. 319-334.

Peralta, R.C., Azarmnia, H. and Takahashi, S. (1991) Embedding response matrix techniques for maximising steady-state groundwater extraction, *Groundwater*, vol. 29(3), pp. 357-364
Philbrick, C.R. and Kitanidis, P.K. (1998) Optimal conjunctive-use operations and plans, *Water Resources Research*, vol. 34(5), pp. 1307–1316.

Piesse, M. (2016) *South Africa: drought threatens food, energy and water security*, http://www.futuredirections.org.au/wp-content/uploads/2016/03/South-Africa-Drought-Threatens-Food-Energy-and-Water-Security.pdf, accessed 29 April 2017.

Pietersen, K., Titus, R. and Cobbing, J. (2009) *Effective groundwater management in Namaqualand: sustaining supplies*, WRC Report No. TT 418/09, Water Research Commission, Pretoria, South Africa, 31 pp.

Pietersen, K.C. (2004) *A decision-making framework for groundwater management in arid zones (with a case study in Namaqualand)*, PhD Thesis, University of the Western Cape, 294 pp.

Pietersen, K.C. (2006). Multiple criteria decision analysis (MCDA): A tool to support sustainable management of groundwater resources in South Africa, *Water SA*, vol. 32(2), pp.119-128.

Pitman, W.V. (1973) *A mathematical model for generating monthly river flows from meteorological data in South Africa*, Report No. 2/73, Hydrological Research Unit, University of the Witwatersrand, Johannesburg, South Africa.

Prairie, J., Rajagopalan, B., Lall, U. and Fulp, T. (2007) A stochastic nonparametric technique for space-time disaggregation of streamflows, *Water Resources Research*, vol. 43, W03432, doi:10.1029/2005WR004721, 2007.

Qian, B., Corte-Real, J. and Xu, H. (2002) Multisite stochastic weather models for impact studies, *International Journal of Climatology*, vol. 22, pp. 1377–1397.

Rajagopalan, B. and Lall, U. (1999) A k-nearest-neighbor simulator for daily precipitation and other weather variables, *Water Resources Research*, vol. 35(10), pp. 3089-3101.

Rajagopalan, B., Salas, J.D. and Lall, U. (2010) Stochastic methods for modeling precipitation and streamflow, Chapter 2, In: Sivakumar, B. and Berndtsson, R. (Eds) Advances in data-based approaches for hydrologic modeling and forecasting, World Scientific Publishing, London, UK, pp. 17-52.

Ramirez, H.D.R. (2004) *Flood control reservoir operations for conditions of limited storage capacity*, PhD thesis, Texas A&M University, 198 pp.

Rathjens, H. and Oppelt, N. (2012) SWAT model calibration of a grid-based setup, *Advances in Geosciences*, vol. 32, pp. 55-61.

Ratnayka, D.D., Brandt, M.J. and Johnson, M. (2009) *Water Supply*, 6th Ed, Butterworth-Heinemann Publishers, Britain, 711 pp.

Rayner, D., Achberger, C. and Chen, D. (2016) A multi-state weather generator for daily precipitation for the Torne River basin, northern Sweden/western Finland, *Advances in Climate Change Research*, vol. 7, pp. 70-81.

Reilly, T.E. (2001) *System and boundary conceptualization in groundwater flow simulation*, Chapter B8, Techniques of water resources investigations of the United States Geological

Survey, Book 3, Applications of Hydraulics, U.S. Department of the Interior and U.S. Geological Survey, 29 pp.

Renard, P., Glenz, D. and Mejias, M. (2009) Understanding diagnostic plots for well-test interpretation, *Hydrogeology Journal*, vol. 17, pp. 589-600.

Richman, M.B., Trafalis, T.B. and Adrianto, I. (2009) Multiple imputation through machine learning algorithms, Chapter 7, In: Haupt, S.E., Pasini, A. and Marzban, C., *Artificial intelligence methods in the environment sciences*, Springer, pp. 153-169.

Riemann, K. and Blake, D. (2010) *Groundwater reserve determination for current and potential wellfield development of TMG Aquifers*, WRC Report No. KV 236/10, Water Research Commission, Pretoria, South Africa, 42 pp.

Ritter, A. and Muñoz-Carpena, R. (2013) Performance evaluation of hydrological models: Statistical significance for reducing subjectivity in goodness-of-fit assessments, *Journal of Hydrology*, vol. 480, pp. 33-45.

Roques, C., Bour, O., Aquilina, L., Dewandel, B., Leray, S., Schroetter, J.M., Longievergne, L., Le Borgne, T., Hochreutener, R., Labasque, T., Lavenant, N., Vergnaude-Ayraud, V. and Mougin, B. (2014) Hydrological behaviour of a deep sub-vertical fault in crystalline basement and relationship with surrounding reservoirs, *Journal of Hydrology*, vol. 509, pp. 42-54.

Russell, S.O. and Campbell, P.F., (1996) Reservoir operating rules with fuzzy programming, *Journal of Water Resources Planning and Management*, vol. 122(3), pp.165-170.

Sargent, D.M. (1979) Reservoir operating rules for drought conditions, *Hydrological Sciences Bulletin*, vol. 24(1), pp. 83-94.

Sasireka, K. and Neelakantan, T.R., (2017) Assessing performance of multipurpose reservoir system using two-point linear hedging rule, *IOP Conference Series: Earth and Environmental Science*, vol. 80 (1), IOP Publishing.

Schulze, R.E. and Maharaj, M. (2003) Development of a database of gridded daily temperatures for Southern Africa, *ACRUcons Report 41*, University of Natal, South Africa, 81pp.

Scottish Environment Protection Agency, (2013) *Regulatory method (WAT-RM-26), Determination of aquifer properties*, Version: v4.0, 13 pp.

Sechi, G.M., and Sulis, A. (2009). Water system management through a mixed optimizations simulation approach, *Journal of Water Resources Planning and Management*, vol. 135(3), pp. 160-170.

Sefelnasr, A.M. (2007) Development of groundwater flow model for water resources management in the development areas of the Western Desert, Egypt, *PhD dissertation*, Martin Luther University Halle-Wittenberg, 171 pp.

Sen, Z. (2009) *Fuzzy logic and hydrological modelling*, CRC Press, New York, 348 pp.

Seo, Y., Kim, S. and Singh, V.P. (2015) Estimating spatial precipitation using regression kriging and artificial neural network residual kriging (RKNNRK) hybrid approach, *Water Resources Management*, vol. 29, pp. 2189-2204.

Shamir, U. and Bear, J. (1984). Optimal annual operation of a coastal aquifer, *Water Resources Research*, vol. 20 (4), pp. 435-444.

Shamsudin, S. and Hashim, N., (2002) Rainfall runoff simulation using Mike11 NAM, *Journal of Civil Engineering*, vol.5(2), pp. 1-13.

Sharif, M. and Burn, D. (2006) Simulating climate change scenarios using an improved K-nearest neighbour model, *Journal of Hydrology*, vol. 325, pp. 179–196.

Sharma, A. and Lall, U. (1999) A nonparametric approach to daily rainfall simulation, *Mathematics and Computers in Simulation*, vol. 48, pp. 367-371.

Sharma, S.P. and Baranwal, V.C. (2005) Delineation of groundwater-bearing fracture zones in a hard rock area integrating very low frequency electromagnetic and resistivity data, *Journal of Applied Geophysics*, vol. 57, pp. 155-166.

Sheets, R.A. and Simmons, L.A. (2006) *Compilation of regional ground-water divides for principal aquifers corresponding to the Great Lakes Basin, United States,* U.S. Geological Survey Scientific Investigations Report 2006–5102, 23 pp.

Shiri, J., Kisi, O., Yoon, H., Lee, K-K. and Nazemi, A.H. (2013) Predicting groundwater level fluctuations with meteorological effect implications-A comparative study among soft computing techniques, *Computers and Geosciences*, vol. 56, pp. 32-44.
Shourian, M., Mousavi, S.J., Menhaj, M.B. and Jabbari, E., (2008) Neural-network-based simulation-optimization model for water allocation planning at basin scale, *Journal of Hydroinformatics*, vol. 10(4), pp. 331-343.

Simolo, C., Brunetti, M., Maugerie, M. and Nanni, T. (2010) Improving estimation of missing values in daily precipitation series by a probability density function-preserving approach, *International Journal of Climatology*, vol. 30, pp. 1564-1576.

Simonovic, S.P. (1992) Reservoir systems analysis: closing the gap between theory and practice, *Journal of Water Resources Planning and Management*, vol. 118, pp. 262-280.

Singh, J., Knapp, H. V., and Demissie, M. (2004) *Hydrologic modeling of the Iroquois River watershed using HSPF and SWAT*, Illinois State Water Survey Contract Report 2004-08, Illinois Department of Natural Resources and Illinois State Geological Survey, Champaign, Illinois.

Singh, M., Kijko, A. and Durrheim, A. (2009) Seismotectonic models for South Africa: Synthesis of geoscientific information, problems, and the way forward, *Seismological Research Letter*, vol. 80(1), pp. 71-80.

Singhal, B.B.S. and Gupta R.P. (2010) *Applied hydrogeology of fractured rocks*, 2nd Ed, Springer, 408 pp.

Solomatine, D., See, L.M. and Abrahart, (2008) Data-driven modelling: concepts, approaches and experiences, Chapter 2: In: Abrahart, R.J., See, L.M. and Solomatine, D. (Eds.), *Practical hydroinformatics, computational developments in water applications,* Water Science and Technology Library 68, Springer-Verlag Berlin Heidelberg, pp. 17-30.

Srikanthan, R., Harrold, T.I., Sharma, A. and McMahon T.A. (2005) A comparison of two approaches for generation of daily rainfall data, *Stochastic Environmental Research and Risk Assessment*, vol. 19 (3), 215-226.

Steinschneider, S., and Brown, C. (2013) A semiparametric multivariate, multisite weather generator with low-frequency variability for use in climate risk assessments, *Water Resources Research*, vol. 49, pp. 7205–7220.

Suhaila, J., Sayang, M.D. and Jemain, A.A. (2008). Revised spatial weighting methods for estimation of missing rainfall data, *Asia-Pacific Journal of Atmospheric Sciences*, vol. 44(2), pp. 93-104.

Sujay, R.N. and Paresh, C.D. (2015) Forecasting monthly groundwater level fluctuations in coastal aquifers using hybrid wavelet packet–support vector regression, *Cogent Engineering,* vol. 2, https://doi.org/10.1080/23311916.2014.999414.

Sulis, A. (2014) *Improved implicit stochastic optimisation technique for multireservoir water systems under drought conditions*, 7th International Congress on Environmental Modelling and Software, San Diego, California, USA.

Sun, Y., Wendi, D., Kim, D.E. and Liong, S-Y. (2015) Application of artificial neural networks in groundwater table forecasting-a case study in Singapore swamp forest, *Hydrology Earth System Sciences Discussions*, vol. 12, pp. 9317-9336.

Tanner, J.L. and Hughes, D.A. (2015) *Understanding and modelling surface water-groundwater interactions*, WRC Report No. 2056/2/14, Water Research Commission, Pretoria, South Africa, 92 pp.

Tospornsampan, J., Kita, I., Ishii, M. and Kitamura, Y. (2004) Operating Rule Curves for Multiple Reservoir System: A Case Study in Mae Klong River Basin, Thailand. *Journal of Rainwater Catchment Systems*, vol. 9(2), pp.11-19.

Teegavarapu, R.S.V (2012) Spatial interpolation using nonlinear mathematical programming models for estimation of missing precipitation records, *Hydrological Sciences Journal*, vol. 57(3), pp. 383-406.

Teegavarapu, R.S.V. and Chandramouli, V. (2005) Improved weighting methods, deterministic and stochastic data-driven models for estimation of missing precipitation records, *Journal of Hydrology*, vol. 312, pp. 191-206.

Terzi, Ö. (2012) Monthly rainfall estimation using data-mining process, *Applied Computer Intelligent Soft Computing*, 10.1155/2012/698071.

Thankachan, A. and Anitha, A.B., (2015) Systems approach for optimal operation of water resources schemes in Kuttiadi River Basin, Kerala, *Aquatic Procedia*, vol. 4, pp. 593-600.

Theis, C.V. (1935) The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, *Transactions of American Geophysical Union*, vol. 16, pp. 519-524.

Trinchero, P., Sanchez-Vila, X., Copty, N. and Findikakis, A. (2008) A new method for the interpretation of pumping tests in leaky aquifers, *Groundwater*, vol. 46(1), pp. 133-143.

Tshimanga, R.M. and Hughes, D.A. (2012) Climate change and impacts on the hydrology of the Congo Basin: The case of the northern sub-basins of the Oubangui and Sangha River, *Physics and Chemistry of the Earth*, vol. 50-52, pp. 72–83.

Tshimanga, R.M., and Hughes, D.A. (2014), Basin-scale performance of a semi distributed rainfall-runoff model for hydrological predictions and water resources assessment of large rivers: The Congo River, *Water Resources Research*, vol. 50, pp. 1174-1188.

Tumbo, M. and Hughes, D.A. (2015) Uncertain hydrological modelling: application of the Pitman model in the Great Ruaha River basin, Tanzania, *Hydrological Sciences Journal*, vol. 60(11), pp. 2047-2061.

U.S. Army Corps of Engineers (1991) Optimization of multiple-purpose reservoir system operations: a review of modeling and analysis approaches, Research Document No. 34. Davis, CA, 83 pp.

Uddameri, V. and Honnungar, V. (2007) Interpreting sustainable yield of an aquifer using a fuzzy framework, *Environmental Geology*, vol. 51, pp. 911-919.

Umadevi. P.P, James. E.J., Jegathambal, P. (2014) Evolving reservoir operation rules using fuzzy logic inference system for irrigation management in a sub-basin scale, *International Journal of Engineering Research and Technology* , vol. 3(3), pp. 221-227.

Vadillo, D.C. (2014) *System engineering applied to Fuenmayor Karst Aquifer (San Julián de Banzo, Huesca) and Collins Glacier (King George Island, Antarctica)*, PhD Dissertation, University of Zaragoza, 202 pp.

van Liew, M.W., Garbrecht, J.D., and Arnold, J.G. (2003) Simulation of the impacts of flood retarding structures on streamflow for a watershed in southwestern Oklahoma under dry, average, and wet climatic conditions, *Journal of Soil Water Conservation,* vol. 58(6), pp. 340–348.

van Tonder G., Bardenhagen, I., Riemann, K., van Bosch, J, Dzanga, P. and Xu, Y. (2002) *Manual on pumping test analysis in fractured rock aquifers*, WRC Report No III6/1/02, Water Research Commission, Pretoria South Africa, 228 pp.

van Tonder, G.J., Botha, J.F., Chiang, W.-H., Kunstmann, H. and Xu, Y. (2001) Estimation of the sustainable yields of boreholes in fractured rock formations, *Journal of Hydrology*, vol. 241, pp. 70-90.

van Tonder, G.T., Kunstmann, H., Xu, Y. and Fourie, F. (2000) Estimation of the sustainable yield of a borehole including boundary information, drawdown derivatives and uncertainty propagation, *Calibration and Reliability in Groundwater Modelling,* Proceedings of the ModelCARE 99 Conference held at Zurich, Switzerland, IAHS Publ. no. 265, pp. 367-373.

van Wyk, E., van Tonder, G.J., and Vermeulen, D. (2012) Characteristics of local groundwater recharge cycles in South African semi-arid hard rock terrains: Rainfall–groundwater interaction, *Water SA*, vol. 38(5), pp. 747-754.

Venkatesan, V. and Rajesh, P.P. (2015) Simulating groundwater flow using numerical model and artificial neural network - A case study, *International Journal of Civil and Structural Engineering Research*, vol. 3(1), pp. 190-214.

Verweiji H.J.M. and Barker J.A. (1999) Well hydraulic and yield analysis, Chapter 3, In: Lloyd J.W. (Ed), *Water Resources in hard rock aquifers in arid and semi-arid zones*, UNESCO Publishing, Paris, France, pp. 128-158.

Villazón, M. F. and Willems, P. (2010). Filling gaps and daily disaccumulation of precipitation data for rainfall-runoff model. Morell, M., Popovska, C., Morell, O. and Stojov, V. (Eds) *Proceeding of 4th International Scientific Conference on Water Observation and Information Systems for Decision Support,* BALWOIS, pp. 1-9.

von Asmuth (2012) *Groundwater system identification through time series analysis*, PhD Thesis, Delft University, Netherlands, 221 pp.

von Asmuth, J.R. and Knotters, M. (2004) Characterising groundwater dynamics based on a system identification approach, *Journal of Hydrology*, vol. 296, pp. 118-134.

Walker, D.D. and Roberts, R.M. (2003) Flow dimensions corresponding to hydrogeologic conditions, *Water Resources Research*, vol. 39(12), doi:10.1029/2002WR001511.

Walton, W.C. (1960) *Leaky artesian aquifer conditions in Illinois*, Report of investigation 39, Illinois State Water Survey, 27 pp.

Wanakule, N., Mays, L.W. and Lasdon, L.S. (1986). Optimal management of large-scale aquifers: methodology and applications, *Water Resources Research,* vol. 22, pp. 447-465.

Western Cape Government (2017) Informing the Western Cape agricultural sector on the 2015-2017 drought, A Drought Fact Sheet, http://www.elsenburg.com/sites/default/files/services-at-a-glance-forms/2017-12-13/drought-fact-sheet-final.pdf, accessed on 29 May 2018.

Wilks, D.S. and Wilby, R.L. (1999) The weather generation game: a review of stochastic weather models, *Progress in Physical Geography*, vol. 23(3), pp. 329-357.

Willis, R. (1983) A unified approach to regional groundwater management, In: Rosenshein, J.S. and Bennett, G.D. (Eds), *Groundwater hydraulics*, Water Resource Monograph Series*, American Geophysical Union*, Washington, 416 pp.

Willis, R. and Liu, P. (1984) Optimisation model for groundwater planning, *Journal of Water Resources Planning and Management,* vol. 110, pp. 333-347.

Wilson, A.S. (2005) Hydrogeology, conceptual model and groundwater flow within alluvial aquifers of the Tenthill and Ma Ma Catchments, Lokyer Valley, Queensland, *MSc Thesis*, Queensland University of Technology, Queensland, Australia, 141 pp.

Wilson, S. and Davidson, P. (2011) *Groundwaters of Marlborough*, Marlborough District Council, 302 pp.

Witthüser, K., Cobbing, J., Rian, T., Pietersen, K. and Fourie, F. (2009b) A framework for groundwater resources assessment (GRA) methodologies and a proposal for GRA3, *Proceedings of the Groundwater Conference, Somersetwest*, Cape Town, South Africa.

Witthüser, K., Cobbing, J., Titus, R., Holand, M. and Pietersen, K. (2009a). *Strategy and guideline development for national groundwater planning requirements, A proposed GRA3 methodology*, DWA Report Number PRSA 000/00/11609/7, Department of Water Affairs, Pretoria, South Africa.

WMO (2009) Guide to hydrological practices, Volume II: Management of water resources and application of hydrological practices, WMO Report no. 168, 6th Ed, Geneva, Switzerland.

Wright, E. P. (1992) *The hydrogeology of crystalline basement aquifers in Africa*, Wright, E.P. and Burgess, W.G. (Eds), Hydrogeology of Crystalline Basement Aquifers in Africa, Geological Society Special Publication no. 66, pp 1-27.

Wright, K.A. and Xu, Y. (2000) A water balance approach to the sustainable management of groundwater in South Africa, *Water SA*, vol. 26 (2), pp. 167-170.

Wu, H. and Zhang, J-T. (2006) *Nonparametric regression methods for longitudinal data analysis: mixed-effects modeling approaches*, John Wiley and Sons, New York.

Wurbs, R.A. (2005) Modeling river/reservoir system management, water allocation, and supply reliability, *Journal of Hydrology*, vol. 300, pp. 100–113.

Xu, Y., Lin, L. and Jia H. (2009) Groundwater flow conceptualization and storage determination of the Table Mountain Group (TMG) Aquifers, WRC Report No. 1419/1/09, Water Research Commission, Pretoria, 268 pp.

Yan, C-A., Zhang, W., Zhang, Z. (2014) Hydrological modeling of the Jiaoyi Watershed (China) using HSPF model, *The Scientific World Journal*, http://dx.doi.org/10.1155/2014/672360, pp. 9.

Yang, G., Guo, S., Li, L., Hong, X. and Wang, L. (2016) Multi-objective operating rules for Danjiangkou Reservoir under climate change, *Water Resources Management*, vol. 30, pp. 1183–1202.

Yazdanian, A. and Peralta, R.C. (1986). Sustained-yield groundwater-pumping strategy by goal programming, *Groundwater*, vol. 24(2), pp. 157-165.

Zhou, C., Sun, N., Chen, L., Ding, Y., Zhou, J., Zha, G., Luo, G., Dai, L. and Yang, X., (2018) Optimal operation of cascade reservoirs for flood control of multiple areas downstream: A case study in the upper Yangtze River basin, *Water*, vol. 10(9), doi:10.3390/w10091250.

Zhou, Y., Guo, S., Liu, P., Xu, C. and Zhao, X. (2016) Derivation of water and power operating rules for multi-reservoirs, *Hydrological Sciences Journal*, vol. 61(2), pp. 359-370.