Modeling and Forecasting Ghana's Inflation Rate Under Threshold Models

by

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Abstract

Over the years researchers have been modeling inflation rate in Ghana using linear models such as Autoregressive Integrated Moving Average (ARIMA), Autoregressive Moving Average (ARMA) and Moving Average (MA). Empirical research however, has shown that financial data, such as inflation rate, does not follow linear patterns. This study seeks to model and forecast inflation in Ghana using nonlinear models and to establish the existence of nonlinear patterns in the monthly rates of inflation between the period January 1981 to August 2016 as obtained from Ghana Statistical Service. Nonlinearity tests were conducted using Keenan and Tsay tests, and based on the results, we rejected the null hypothesis of linearity of monthly rates of inflation. The Augmented Dickey-Fuller (ADF) was performed to test for the presence of stationarity. The test rejected the null Hypothesis of unit root at 5% significant level, and hence we can conclude that the rate of inflation was stationary over the period under consideration. The data were transformed by taking the logarithms to follow nornal distribution, which is a desirable characteristic feature in most time series. Monthly rates of inflation were modeled using threshold models and their fitness and forecasting performance were compared with Autoregressive (AR) models.

Two Threshold models: Self-Exciting Threshold Autoregressive (SETAR) and Logistic Smooth Threshold Autoregressive (LSTAR) models, and two linear models: AR(1) and AR(2), were employed and fitted to the data. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) were used to assess each of the fitted models such that the model with the minimum value of AIC and BIC, was judged the best model. Additionally, the fitted models were compared according to their forecasting performance using a criterion called mean absolute percentage error (MAPE). The model with the minimum MAPE emerged as the best forecast model and then the model was used to forecast monthly inflation rates for the year 2017.

The rationale for choosing this type of model is contingent on the behaviour of the time-series data. Also with the history of inflation modeling and forecasting, nonlinear models have proven to perform better than linear models.

The study found that the SETAR and LSTAR models fit the data best. The simple AR models however, out-performed the nonlinear models in terms of forecasting. Lastly, looking at the upward trend of the out-sample forecasts, it can be predicted that Ghana would experience double digit inflation in 2017. This would have several impacts on many aspects of the economy and could erode the economic gains made in the year 2016. Our study has important policy implications for the Central Bank of Ghana which can use the data to put in place coherent monetary and fiscal policies that would put the anticipated increase in inflation under control.

Keywords: Inflation, Nonlinear Models, Self-Exciting Threshold Autoregression Model, Logistic Smooth Threshold Autoregression model.

Declaration

I, Emmanuel Antwi, declare that this research is my original work and has not been submitted for any degree at any other university or institution. The research does not contain other persons' writing unless specifically acknowledged and referenced accordingly.

Signed (Student): .... Date: ....

List of Figures

List of Tables

Contents

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Dedication

This work is dedicated to the Lord God Almighty for the divine wisdom and strength given me to go through this research successfully. It is also dedicated to my lovely wife, Olivia Frimpong, my children, Rachel, Samuel and Emmanuel junior, my sweet mother, Akua Badu and also to my siblings, Emmanuel, Mary, Ernest,Patrick and Samuel for their love, care and support.

Abbreviation

List of abbreviations

The following table describes the significance of various abbreviations and acronyms used throughout the dissertation. The page on which each one is defined or first used is also given. Nonstandard acronyms that are used in some places to abbreviate the names of certain white matter structure are not included in this list.

Abbreviation	Meaning	Page
ADF	Augmented Dickey-Fuller	49
ACF	Autocorrelation Function	21
AIC	Akaike Information Creterion	36
ARCH	Auto Regressive Conditional Heteroscedastic	14
ARFIMA	Auto Regressive Fractional Integrated Moving Average	14
ARIMA	Autoregressive Integrated Moving Average	76
AR	Autoregressive	5
BET	Bucharest Stock Exchange	17
BoG	Bank of Ghana	14
BIC	Bayesian Information Creterion	36
BVAR	Bayesian Vector Auto Regressive	14
COICOP	classification of individual consumption by purpose	10
CPI	Consumer Price Index	10
DIS	Databank Stock Index	16
DJ-AIGCI	Dow Jones AIG Commodity Index	15
DJIA	Dow Jones Industrial Average	15
EGARCH	Exponential Generalized Auto Regressive Conditional Heteroscedastic	14
ERP	Economic Recovery Programme	13
EWMA	Risk Metric Exponential Weighted Moving Average	15

GARCH	Generalized Auto Regressive Conditional Heteroscedastic	14
GDP	Gross Domestic Product	10
GJR-GARCH	G Glosten-Jagannathan Runkle Generalized Auto Regressive Conditional Heteroscedastic	14
GSE	Ghana Stock Exchange	16
GSS	Ghana Statistical Service	10
GSSNL	Ghana Statistical Service News Letter	10
IGARCH	Integrated Generalized Auto Regressive Conditional Heteroscedastic	14
i.i.d.	independent and identically distributed	24
IT	Inflation Targeting	2
LSTAR	Logistics Smooth Threshold Autoregressive	5
MA	Moving Average	19
MAPE	Mean Absolute Error	38
MOFEP	Ministry of Finance and Economic Planning	10
MSE	Mean Square Error	38
MT	Monetary Targeting	14
PACF	Partial Autocorrelation Function	24
PARCH	Power Auto Regressive Conditional Heteroscedastic	14
RW	random walk	14
SAP	structural Adjustment Programme	13
SAR	Simple Auto Regressive	14
SARIMA	Seasonal Auto Regressive Integrated Moving Average	14
SETAR	Selt-Exciting Threshold Autoregressive	5
S and P	Standard and Poors	15
SVAR	structural Vector Auto Regressive	14
VAR	Vector Auto Regressive	14
VAT	Value Added Tax	13
QTM	Quantum Theory of Money	17

1. Introduction

Hardly does a day go by without government officials, politicians and economists talking about inflation. In some cases, we are told that inflation is high for a particular month and low for another month. There are several important variables that help to describe the state of an economy. These include inflation, unemployment, the budget balance, the interest rates and the balance of payments.

Price stability (stable inflation) is one of the main objectives of every government as it is an important economic indicator that the government, politicians, economists and other stakeholders use as their basis of argument when debating on the state of the economy (Suleman and Sarpong, 2012). In recent years, rising inflation has become one of the major economic challenges facing most countries in the world, especially, developing countries like Ghana. David (2001) describes inflation as a major focus of economic policy worldwide. This is rightly so, as inflation is the frequently used economic indicator of the performance of a country's economic performance as it has a direct impact on the state of the economy. In Ghana, the debate of achieving a single digit inflation value has been on going for both the government and the opposition parties. Despite these different opinions on the inflation figures, it is important to point out that, both the government and the opposition parties are concerned about the inflation in the country as it affects all sectors of the economy. In view of this, a reliable model needs to be developed to model and forecast the monthly rates of inflation in Ghana.

In the midst of this heated argument, Ghana parliament in May 2007,adopted inflation targeting policy as a remedy to bring price stability in Ghana and became the second African country to adopt this policy after South Africa.

Since New Zealand adopted inflation targeting in 1990, a steadily growing number of industrial and emerging economies have explicitly adopted an inflation targeting as their nominal anchor. Eight industrial countries and twenty emerging economies had full-fledged inflation targeting in place in early 2016. Many other emerging emerging economies are planning to adopt inflation targeting in the near future. This trend has triggered an intensifying debate over whether inflation targeting makes difference. Opinions diverge widely over whether central banks are better off after adoption of inflation targeting as an explicit and exclusive anchor for conducting monetary policy.

Analysts are demanding hard evidence that inflation targeting improves macroeconomic performance relative to countries without explicit inflation targeting (Mishkin F.S. and Klaus Schmidt-Hebbel, 2007).

This study uses the nonlinear approach of modeling time series to model and forecast inflation rate with respect to a particular case in Ghana. The finding of this research will be important for policy debates on the appropriateness and future of inflation targeting regime for monetary policy formulation in Ghana.

In order to solve these differences among politicians, they must have complete understanding of better forecsating what it actually is and its importantance.

Having good forecasting information at your disposal also gives you the ability to explore possibilities to increase both revenue and net income.

The benefits or importance of forecasting accurate inflation underscores potential significance. First of foremost, forecasting accurate inflation help better financial planning. If you accurately forecast your sales, not only by volume, but also by product or service type and time of the year, you can make sure you have enough cash or credit to pay your bills on time. This not only help you to avoid penalties and potential loss of access to important vendors, but it also helps you to rely less on credit, reducing year interest payments. Meet with your salespeople and clients each quarter to accurately determine your potential sales and production needs to help you brtter manage your finances.

In addition to this,accurate forecasting helps to improve staffing. If your business is seasonal or you know you will have slow or busy periods at certain times during the year, plan on increasing or reducing staff when necessary. this can help you bring additional workers early enough to train them or use contractors during peak periods instead of hiring employees you would need for a short period. Knowing your demand can also help you to plan the sales levels at which adding another shift or expanding your production capacity is profitable. If you can accurately forecast your office needs, you can create a better organization chart, allowing you to hire and proactively.

Moreso, forecasting inflation accurately enable organizations to meet their targeted market. Just because sales are slow does not mean you can afford to cut back on advertising and promotions. If you can accurately plot your revenue the year, you will be able to schedule more low-cost guerrilla marketing during slow periods and plan more expensive marketing when you have the cash. If you create an annual, or quarterly, media schedule, you will be able to ensure that contractors you rely on are available when you need them and you will not scrambling to find replacement or stop-gap designer or copywriter for an important project.

Accurate forecasting of inflation enhanced production management. Just-in-time method of inventory management schedules near expected sales or delivery needs to reduce inventory carrying costs. Accurate sales and demand forecasting lets you to spread out to ensure your customers have product when they need it.

Again, good forecasting help to provide internal control of the economy. Having a gasp on the projected inflation rates, it is possible for you to have better control of internal operations. By anticipating future inflation increase you can make decisions about having permanent or temporary marketing and expansion.

Finally, accurate forecasting provide continuous improvement. Continuous improvement is a goal of many if not most businesses. By forecasting sales and continuous revising the process to improve the accuracy, you can improve all aspects of your business.

Inflation is the percentage change in the value of the Wholesale Price Index (WPI) on a year-on -year basis. It effectively measures the change in the prices of a basket of goods and services in a year.

Inflation is calculated by taking the WPI as base, thus, the formula for calculating inflation is given as:

$$\frac{\text{(WPI in month of current year- WPI in month of previous year)}}{\text{WPI in month of previous year}} \times 100$$

Inflation occurs due to an imbalance between demand and supply of money, changes in production and distribution cost or increase in taxes on products. When an economy experiences inflation, that is, when the price level of goods and services rises, the value of the currency reduces. This means that now, each unit of currency buys fewer goods and services. It has its worst impact on consumers since high prices of day-to-day goods make it difficult for consumers to afford even the basic commodities in life. This leaves them with no choice but to ask for higher incomes, hence, the government tries to keep inflation under control.

Contrary to its negative effects, a moderate level of inflation characterizes a good economy and beneficial for an economy as it encourages people to buy more and borrow more, because during times of lower inflation, the level of interest rates also remains low, hence the government, as well as the Central Bank always strive to achieve a limited level of inflation.

Inflation may also be defined as a rise in the average level of a group of prices in a country, and the term "inflation" is sometimes restricted to prolonged or sustained rises. Inflation creates a problem because the purchasing power of money falls as the price level rises. It imposes an opportunity cost on holders of money, thus inflation reduces the real value of money wage, and savings accounts making holders of these instruments to lose. Inflation also demotivates a wasteful increase in the volume and frequency of transactions people undertake and because it is difficult to foresee, it adds to the uncertainties of economic life. In real terms, inflation means your money cannot buy as much as what it could have bought yesterday and retards economic growth because any economy needs a certain level of savings to finance investments to boost economic growth. Inflation causes global concerns because it can distort economic patterns and can result in the redistribution of wealth when not anticipated. The process can also discourage investors within and without the country by reducing their confidence level in investments; this is because investors expect a high possibility of returns so that they can make good financial decisions.

Inflation can be grouped into two: 'creeping' or 'moderate' inflation and 'hyper-inflation'. The creeping inflation, also known as "mild inflation", is when the rates of price change is not very severe. Examples of creeping inflation that Ghana experienced were in 1992, 1999, 2002; 2004-2007 and 2010; a rate of inflation of about 10% annually can be described as creeping inflation. The hyper-inflation is the type in which the price rises are so severe and a typical example of this is what happened in Zimbabwe from 2007 to 2008; this country had rates of inflation of about 8000% this means that if you buy an item in the morning, the price of the item will change in the evening. Hyper-inflation is the worst economic problem any country can experience. The effect of inflation is considered a crucial issue for a country since its related problems make living in a country very difficult for the average person. People who are living on fixed income suffer most when prices of commodities rise, since they cannot buy as much as they could, previously.

Inflation can have both positive and negative effects, although, usually, the negative effects outweigh the positive ones. With this in mind, countries have over the years adopted many forms of measures to control inflation, and Ghana is no exception. One of these methods of controlling inflation adopted by most countries is Inflation Targeting. Inflation Targeting is a monetary policy in which the central bank has an explicit target inflation rate for the medium term and announces this target to the public (Bernanke et al, 1999) . The assumption is that the best monetary policy to support long-term growth of the economy is the maintaining of price stability.

In this study, the aim is to model and forecast monthly rates inflation using nonlinear time series Self-exciting Threshold Autoregressive (SETAR) and Logistics Smooth Threshold Autoregressive models (LSTAR) models and compare them with linear models such as Autoregressive (AR1) and (AR2) to find out the one that can predict inflation rates in Ghana. In the process of forecasting, there is an extensive number of methods and approaches available and their relative success or failure to outperform each other is, in general, conditional upon the problem at hand. The rationale for choosing this type of model is contingent on the behaviour of the time series data, with the history of inflation modeling and forecasting, these models have proved to perform better than other linear models.

1.1 Statement of the Problem

Modeling and forecasting inflation rate of economic growth is one of the most central point of macroeconomic issues that need to be resolved. Inflation is a measure of the persistent and continuous rise in the general price levels of goods and services in an economy or a country and is one factor that affects all other levels of the economy (Webster,2000).

Every country or government, therefore, aims to control inflation levels to its bearest minimum, due to the fact that these levels affect all other sectors of the economy, especially business transactions. It is therefore, important to model and forecast or estimate the value of inflation rates in the future so that such values are incorporated in decision-making from all sectors. Various researches have been carried out in the area of inflation modeling and forecasting in Ghana; examples include, Aidoo (2010), Alnaa and Ahiakpor (2011) and Suleman and Sarpong (2012). All these researchers attempted to model inflation in Ghana, however, they were using models that did not account for conditional heteroscedasticity in the data.

Gujarati (2004) asserts that the underlying characteristic of most financial time series is that, in their level form they are random walks, that is, they are nonstationary.

It is argued by Campbell, Lo and MacKinlay (1997) that it is both statistically inefficient and logically inconsistent to use models that are based on the assumption of constant variance over some period when the resulting series progress over time. In the case of financial data, for example, large and small errors occur in clusters, which imply that large returns are followed by larger returns and small returns are also followed by further smaller returns. When applied to inflation time series data, it is equivalent to saying that periods of high inflation are usually followed by further periods of high inflation while low inflation is likely to be followed by further periods of low inflation (Amos, 2010). Time series models that capture variability of time series data such as Threshold models had to be developed to model and forecast the rates of inflation using time series analysis. These models have been used and empirical evidence of their relative performance has been given for the success of developed economies like the US and Europe. However, limited studies have been done in the context of developing countries. This indicates a gap in literature or in the relative performance of these models in developing countries and poses a challenge as to which of these models is the optimal choice for modeling and forecasting economic and financial data (in particular inflation rates) for developing countries. A review of the inflation modeling in Ghana using nonlinear modeling is limited. In view of this, the study intends to model inflation in Ghana using the SETAR and LSTAR-type models and to recommend the most appropriate model for inflation modeling and forecasting of future inflation rate in Ghana.

1.2 Significance of the Study

The empirical findings from this study will be significant to industry practitioners, policy makers, the government, businesses and the general public as well as academics and researchers. This is due to the following reasons: firstly, identifying the optimal model based on their relative performance, better and robust forecasts of inflation values will be obtained which will be very useful in the planning activities of the government, businesses and the public in general; secondly, the results from this study will benefit academia and provoke further research. Finally, the results will benefit the industry practitioners by contributing to existing literature and close or eliminate the gap in literature or information on the relative performance of these models in the context of developing countries.

1.3 Scope and limitation of the study

The study was limited to the developing country, Ghana. Secondary data consisting of year-on-year inflation data for each month from January 1981 to August 2016 was used in the study. The main reason why the scope was limited to the years 1981 to 2016 was due to the lack of data. Many of the variables used in the inflation models do not have adequate quarterly data. The total number of data points was therefore 428. The year-on-year inflation is the percentage change in the consumer price index (CPI) over a twelve-month period, which is used to measure changes over time in the general price level of goods and services that households acquire for the purpose of consumption. The monthly year-on-year inflation was collected by the Ghana Statistical Service.

1.4 Research questions

The study will address the following questions:

1. Are there nonlinear patterns in Ghana's monthly rates of inflation?
2. Which model would best fit Ghana's inflation rate?
3. Are Ghana's inflation rates best forecasted with linear or nonlinear?

1.5 Aims and Objectives

The specific objectives of the study are as follows:

1. To establish the existence of nonlinear pattern in the monthly rates of inflation in Ghana.
2. To model inflation rate in Ghana using nonlinear approach and compare its fitness to linear models
3. To identify the optimal model that would fit inflation rates in Ghana.
4. To predict a one year out-sample forecast based on an optimal model for Ghana.

1.6 Organization of the study

The study was divided into five chapters. Chapter one introduces the research study, providing the background to the study, a statement of the problem, objectives, the significance, scope and methodology used in the study. Chapter two focuses on the conceptual framework and review of related literature that pertains to the study whilst Chapter three presents the detailed methodology used for the study. Chapter four covers the data analysis, presentation and discussion of the results. Finally, Chapter five encompasses the summary, conclusion, recommendations, and direction for future research.

2. Literature Review

This chapter reviews the relevant theories, concepts and related works associated with inflation that have been carried out by other researchers. The chapter is divided into two main headings: the Concept of inflation and review of related works.

2.1 The Concept of Inflation

There are different theories that have been proposed by economists to explain the occurrence of an inflationary situation. These numerous theories can be grouped into two main broad theories, the excess-demand theory and the cost-push theory. The excess-demand theory argues that the excess demand for goods and services above supply in the economy is the main source of inflation (Hall,1982). While, the cost-push theory of inflation is based on the belief that inflation can be triggered by the increase in the cost of production by firms. The increase in the cost of production will affect the profit margins of these firms and hence they will have to pass on the extra cost to consumers by increasing the prices of their products.

According to Webster (2000), inflation is the persistent and continuous rise in the levels of the consumer prices in an economy. Inflation can also be seen as the persistent decline in the purchasing power of money. Inflation, therefore, means that your money cannot buy today as much as what it could have bought yesterday.

The effects of inflation include, among other things, people losing confidence in the currency as its real value is severely reduced. Inflation can also lead to the wage-price spiral. This is the situation in which there are higher wage demands as people try to maintain their real living standards. This leads to businesses increasing prices to maintain profits and higher prices then put further pressure on wages. Furthermore, inflation can lead to a build-up of inflation expectations that can worsen the trade-off between unemployment and inflation. Lastly, the uncertainty created by the rising inflation can also disrupt business planning since budgeting becomes difficult. Bailey (1956) observes that inflation has additional negative effects on the economy through its cost on welfare. He continues that the costs associated with unanticipated inflation are the distributive effects from creditors to debtors and increasing uncertainty affecting consumption, savings, borrowing and investment decisions.

2.2 Consumer Price Index (CPI) as a Measure of Inflation

Various indexes have been devised to measure inflation. These indexes include consumer price index (CPI), producer price index, cost of living index, commodity price index and the Gross Domestic Product (GDP) deflator. The consumer price index however, is the most common way of measuring inflation. It is a measure for capturing changes overtime (monthly, quarterly, yearly) in the general price level of goods and services. This is determined at the beginning period called the' base period' and according to a fixed pattern of consumption called 'weight' which is assigned to a representative sampled basket of goods and services. The consumer price index is then used to calculate the inflation rate. Let P_t be the current average price level of an economic basket of goods and services and P_{t-1} be the average price level of the same basket a year ago, then the inflation rate at that time is calculated as:

$$I_t = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100\%$$

2.3 Construction of Consumer Price Index in Ghana (CPI)

In Ghana, the consumer price index (CPI) is calculated by the Ghana Statistical Service (GSS) which is a department of the Ministry of Finance and Economic Planning (MOFEP). The Ghana Living Standards Survey generates the basket of goods and services classified into 12 main classes using the classification of individual consumption by purpose (COICOP) system. These baskets of goods and services are then used in the construction of the CPI based on the weight assigned to each item in the basket. There are 242 items and the weight assigned to each item depends on the expenditure on that item, such that high volume expenditure items carry the most weight and therefore would have the most material impact on the calculated index.

The CPI cover prices collected from a national sample of 40 markets. The markets are made up of 9 urban and 31 rural markets across the country. Prices are collected every first and third week of the month from 6 traders in the urban markets and 3 traders in the rural markets for all goods, those with fixed prices, such as, stamps. The prices are then used to construct the CPI, which is in turn used to calculate the inflation rates. Currently, the construction is based on the 2002 base year which was changed in 1963, in 1977 and then in 1997. Table 2.1 below presents the construction of Consumer Price Index. Table 2.1 represents the construction of CPI in Ghana.

CLASS	WEIGHT	NUMBER OF ITEMS
Food and Non-alcoholic beverages	44.91	76
Alcoholic-beverages, tobacco and narcotics	2.23	11
Clothing and Footwear	11.29	59
Housing, Water, Electricity, Gas and Other Fuels	6.98	10
Furnishings, Household Equipment and Routine Maintenance of the house	7.83	43
Health	4.33	9
Transport	6.21	9
Communication	0.31	3
Recreation and Culture	3.04	6
Education	1.60	2
Restaurants and Hotels	8.28	7
Miscellaneous Goods and Services	2.99	7
Total	**100**	**242**

Table 2.1: construction of CPI -Ghana

2.4 The Ghanaian Experience of Inflation

This section attempts to take a look at Ghana's inflation experience since the attainment of independence. This discussion is relevant as the need to model and forecast Ghana's inflation requires an understanding of where Ghana has come from, where Ghana is currently, in terms of inflation rates. Ghana experienced high rates of inflation for several decades. However, since July 2009, inflation has fallen consistently even to a single digit in June 2010.

Ocran (2007) asserts that the inflation in Ghana from independence to 2003 can be characterised as episodic, identifying four distinct episodes: the immediate post-independence period which was up to 1966; immediate post Nkrumah period (1966-1972); the deterioration period of 1972-1982 and the most recent period (1982-2003), which he termed the 'stabilization inflationary experience'.

This study has adopted Ocran's (2007) episodic characterisation of inflation in Ghana and modified it slightly by adding a new episode. The study hence would review Ghana's inflation experience under five distinct episodes as follows: the immediate post-independence period up to 1966; immediate post-Nkrumah period (1966-1972); the deterioration period (1972-1985); recent period (1986-2001) and the most recent period of 2002-2015 which would be termed the 'single-digit inflationary experience'.

The first five years in the post-independence period (1957-1962) saw inflation centring on a single-digit. This stability in prices could be attributed to the trickling effects of Ghana being a member of the West African Currency Board (WACB) which consisted of the former four British Colonies of West Africa - Ghana, Nigeria, Sierra Leone and The Gambia. The WACB had no control of the discretionary monetary policy and as a result, market forces determined the money supply in the member countries. Ocran (2007) points out that during the currency board years, inflation was in the single digits and indeed, inflation rates were typically estimated at less than 1%. Between 1960 and 1962, inflation averaged 8% per annum and then increased to 23% per annum between 1964 and 1966, by which time the trickling benefits of Ghana having been a member of the WACB had been eroded.

From 1966 to 1972, inflation rates were in the range of 31.2% (January 1966) to -12.1% (July 1967), with annual averages in the range of -8.3% to 10.2%. During this period, there was a devaluation of the cedi by 30% against the US dollar and massive retrenchment exercise in the public sector (Hutchful, 2002). This led to a deflation of about 8% in 1962. It is worth noting that from December 1966 through to December 1967, the inflation was less than zero (negative). This led to a deflation of about 8% in 1962.

The period, 1972 to 1983 arguably has been the period that inflation rates increased the most in the economic history of Ghana. According to Apaloo (2001), inflation was running at about 100% at the beginning of 1979. In the mid-1979, however, the rate dropped dramatically by about 25% following the *coup d' etat* in June 1979. With the exception of the first five months in 1980, the inflation rates ranged between 40% and 88% in 1980. Throughout the period, 1981 to 1983, inflation rates were over 100% for all the months with the rates reaching a peak of 174% at the end of June 1983 before declining to 142% in December. This was largely due to expansionary fiscal and loose monetary policies and the attempt to using controls, such as, fixed exchange rate, import licensing and administered prices for goods and services to hold down inflation (Ocran, 2007).

In particular, the high rate of inflation recorded in 1983 could be attributed to the devaluation of the cedi, the drought and famine. The episode of inflation started on the backdrop of the introduction of the Economic Recovery Programme (ERP) and the successive Structural Adjustment Programme (SAP) in 1983. The ERP had two stages of implementation. The first stage, ERP I (1983-1986) had a stabilization package aimed at reducing inflation and fostering external balance. The second stage, ERP II had a structural adjustment which was undertaken with the aim of removing the distortions in the incentive structure and thereby facilitating production as well as restoring the broken-down social and economic infrastructure (Ocran, 2007). The ERP was able to bring down inflation to an average of about 40% and subsequently lowered it to single digit at the end of 1985. The success of ERP and SAP at bringing down inflation was short-lived, though, as between 1986 and 1990, year-on-year inflation was in the range of 19% to 46%. The average inflation was between 25% and 40% per annum, far exceeding the official targets set within ERP (Apaloo, 2001). The rates of inflation fell continually from the beginning of 1991 till the beginning of 1992 (the first two quarters) where single digit inflation rates were recorded. The rate of inflation was relatively kept under control till the end of 1992, largely on account of the good harvests of the previous year and conscious efforts at monetary control (Apaloo, 2001).

In January 1993, there was over 60% increase in inflation rate as it stood at 21.50% compared to 13.3% in December 1992. The average rate of inflation in the 1993 was 24.9% compared to the average of 10% in 1992. This increase in inflation rate was attributed to an increase in petroleum prices. The next two years (1994-1995), Ghanaian were even worse off as there was a sharp increase in inflation rates from 22.80% in January 1994 to 70.80% in November 1995. According to Aidoo (2010), this sharp increase could be attributed to several factors. These factors included a triple year-to-year increase in petroleum prices in 1993, 1994 and 1995, the depreciation of the local currency (cedi) at the exchange rate level relative to the US dollar the same year, a poor performance of agriculture in 1995 and the introduction of a new tax system known as the Value Added Tax (VAT). The value of the VAT was higher than the previous sales tax and that led to an increase in general prices of commodity.

Inflation fell consistently from 69.1% from the beginning of January 1996 till May 1999 except for a brief increase in March and April 1998. In May 1999, the rate was 9.4%. This drastic drop in inflation rate was due to the improvement in agriculture productivity giving credence to the fact that the food component plays a significant role in the level of inflation rates. This was short-lived as the level of prices started rising again in June 1999 from 10.3% to 40.5% by the end of December 2000. This was attributed to the increase in world oil prices and a decrease in world market cocoa prices, as well as a reduction in agriculture performance in the year 2000 (Aidoo, 2010).

The inflation episode started with a new government in office in January 2001. In the first quarter of 2001, inflation was still high ranging between 40.1% and 41.9%. However, the rates started dropping from 39.5% in the second quarter of 2001 to 12.9% in the third quarter of 2002. In the last quarter of 2002, inflation rose again ending the year at 15.2%. Between 2003 and 2006, inflation ranged from 33.6% (August, 2003) to 10.7% (November, 2004) with an average of 29.8%, 18.2%, 15.5% and 11.7% for 2003, 2004, 2005 and 2006 respectively. Ghana adopted a monetary policy called the 'Inflation Targeting (IT)' in 2007. This was after the Monetary Targeting (MT) framework used in the management of inflation was not effective due to the intractability of the underlying causes (Kwakye, 2004). The aim of the Bank of Ghana (BoG) was to target inflation rate and then attempt to direct actual inflation rate towards the target. The target set by the BoG was to bring inflation rate below 10%. This target of an inflation below 10% was not successful until June 2010 as the inflation figures hovered between 10.1% (October, 2007) and 20.7% (June, 2009) with an annual average of 10.7% (2007), 16.5% (2008), 19.3% (2009).

In conclusion, it could be seen that Ghana has had its own share of unstable and high inflation rates, however, in the last few years, the country can be seen to be winning the fight against inflation as inflation has been kept at single digits. This notwithstanding, the authorities in charge of price stability in the country should, however, take note of the potential threat posed by the oil production, boost in the government expenditure through the implementation of the single-spine salary structure as these factors could exert both demand and cost pressures on inflation. Currently, Ghana's inflation is 18.2%.

2.5 Review of related works in Ghana and other countries in the world

In this section, a review of the numerous related works that have been carried out by other researchers using time series techniques and other forecasting techniques will be taken into consideration. These include Vector Auto Regressive (VAR), Bayesian Vector Auto Regressive (BVAR), Structural Vector Auto Regressive (SVAR), Seasonal Auto Regressive Integrated Moving Average (SARIMA), Simple Auto Regressive (SAR), random walk (RW) and Auto Regressive Fractionally Integrated Moving Average (ARFIMA). The rest are the ARCH-typed models including the traditional Auto Regressive Conditional Heteroscedastic (ARCH) model with extensions such as Generalized ARCH (GARCH), Exponential GARCH (EGARCH), Integrated Generalized ARCH (IGARCH), Power ARCH (PARCH), Glosten - JagannathanRunkle GARCH (GJR - GARCH) and Self-excited Threshold Autoregressive (SETAR) Model.

The related works reviewed will be categorised into two: works done in Ghana and other works done in the rest of the world.

Minkah (2007) examined the forecasting ability of three widely used time series volatility models namely, the Historical Variance, the Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model and the Risk Metrics Exponential Weighted Moving Average (EWMA). The characteristics of these volatility models were explored using data on the Standard and Poors (S and P) 500 Index, Dow Jones Industrial Average (DJIA), OMX Swedish Stock Exchange (OMXS30) index, Dow Jones-AIG Commodity Index (DJ-AIGCI), the 3 Months US Treasury Bill Yield, the Ghanaian Cedi and the US Dollar (CEDI/USD) exchange rates. It was reported that the complex models, that is, GARCH(1, 1) and Risk Metrics EWMA outperformed the simple Historical Variance in the In-Sample volatility forecasts. The Out-of-Sample forecasting accuracy comparisons also revealed that for shorter forecasting horizons, the GARCH (1, 1) performed better whereas at longer horizons the simple Historical Variance outperformed all in most markets. This was due to the fact that complex models have more parameters and thus add to the estimation errors and its forecasts are consistently poor in Out-of-Sample.

Owusu (2010) used the ARIMA models to model inflation and forecast the monthly inflation on short-term basis. The study used different ARIMA models to model the inflation rates from 1990-2009. The period under consideration was split into two sub-periods: 1990-2000 and 2001-2009. The results showed that the best inflation model for the period of 1990-2000 was ARIMA (1, 2, 2) whilst that of the period 2001 2009 was ARIMA (2,2, 1). Furthermore, the study concluded that the inflation rate for the period of January 2001 to December 2009 was less than that of January 1990 to December 2000.

Ocran (2007) modeled Ghana's inflation experience and sought to ascertain the key determinants of inflation in Ghana for the past 40 years. Stylized facts about Ghana's inflation experience indicated that Ghana's exit from the West African Currency Board soon after independence made inflation management ineffective, despite two decades of vigorous reforms. Ocran (2007) used the Johansen co-integration test and an error-correction model and the results identified inflation inertia, changes in money supply and changes in government Treasury Bill rates, as well as changes in the exchange rate, as determinants of inflation in the short run. Of these determinants, inflation inertia was the most dominant and therefore the study suggested that to make Treasury bill rates more effective as a nominal anchor, inflationary expectations, ought to be reduced considerably. In an attempt to analyse and forecast the macroeconomic impact of oil price fluctuations in Ghana, using annual data from 2000 - 2011, Abledu and Agbodah (2012) focused on the feasibility forecast using nested conditional mean (ARIMA) and conditional variance (GARCH, EGARCH and GJR) family of models as the market conditions were too volatile. The best model was the ARIMA (1, 1, 0) and it was used to predict the oil prices in Ghana National Petroleum Authority (GNPA) till the end of 2016. Using the Seasonal Autoregressive Integrated Moving Average (SARIMA) model, Aidoo (2010) examined the inflation rates in Ghana. Monthly inflation data from July 1991 to December 2009 were used. The results revealed that the ARIMA (1, 1, 1) (0, 0, 1) can best represent the behaviour of inflation rates in Ghana.

Frimpong and Oteng-Abayie (2006) studied volatility of returns on the Ghana Stock Exchange (GSE) using the random walk (RW), GARCH, EGARCH and TGARCH models. The unique three days a week Databank Stock Index (DSI) was used to study the dynamics of the GSE volatility over a 10-year period. Their results revealed that the DSI exhibited stylized facts, such as, volatility clustering, leptokurtosis and asymmetry effects associated with stock market returns on more advanced stock markets. The random walk hypothesis was also rejected and overall, the GARCH (1, 1) model outperformed the other models under the assumption that innovation follows a normal distribution.

Suleman and Sarpong (2012) applied the Box-Jenkins approach to model monthly inflation data in Ghana. The study applied the SARIMA model to inflation rates from January 1990 to January 2012. The study concluded that the best model was the ARIMA (3, 1, 3) (2, 1, 1). Alnaa and Ahiakpor (2011) also used the ARIMA approach to predict inflation in Ghana. The monthly data from June 2000 to December 2010 was used and it was found that ARIMA (6, 1, 6) was the best fitted model for forecasting inflation in Ghana. Inflation was predicted as being the highest for the months of March, April and May at 8.95%, 10.07% and 10.24% respectively. The researchers recommended that the appropriate measures must be put in place to prevent inflation spiral from setting in motion, since, their model suggests that, inflation has a long memory and that once the inflation spiral is set in motion, it will take at least 12 periods (months) to bring it to a stable state.

Ahmad (2016) investigated current account deficit in Mauritius looking at risks and prospects. He investigated the constant decline, over the last decade, in the exports to gross domestic product (GDP) ratio , which was a cause for concern. Using a three-regime SETAR model and comparing it to the STAR model revealed that the SETAR model performed better than STAR model and predicted that the Mauritius current account was more likely to switch from surplus to deficit equilibrium than from deficit to surplus.

Hui and Jia (2003) studied the forecasting performance of non-linear time series using SETAR model and comparing to linear model, ARIMA using Canadian GDP from 1965 to 2000. The results demonstrated that the SETAR model forecast the GDP of Canada better than the linear model, ARIMA after fitting the preferred model.

Marius and Petre (2011) examined the existence of nonlinear patterns in the dynamics of the main stock index returns in Romania using the daily closing data of Bucharest Stock Exchange (BET) index series, from 2004 to 2010.They used the Keenan and Tsay tests and based on the results for nonlinearity test rejected the null hypothesis. They used several threshold models and compared their fitness and forecasting performance with basic AR models, they found that, the SETAR and LSTAR models fit best the data, however, they cannot outperform the simple AR model in forecasting.

Apergis, Katrakilidis and Tabakis (2010) investigated the sustainability of current account deficit in Greece over the period 1960 to 1994. They employed various unit root and cointegration tests which allow for structural breaks.The results showed evidence in favour of the sustainability of current account deficit against a required devaluation of the drachma. They fitted the SETAR and LSTAR models to the data. Based on the AIC and BIC values, the results revealed that the best fit models tend to be the SETAR and LSTAR. However, after diagnostic and forecast accuracy tests were performed, the SETAR model was adjudged to be the best model for forecasting.

Mugume and Kasekende (2009) examined inflation and forecasting in Uganda from 1993 to 2009. They employed various inflation-forecasting models like Philips curve, P-star model based on Quantum Theory of Money (QTM), the price equation and ARIMA model. They also employed M3 and the results of both short-run dynamics and long-run equilibrium showed that inflation had not been a result of money growth. The long-run inflation equation seems to show that exchange rate depreciation could have a strong impact in driving inflation upwards, than money supply, although it had no short-run impact.

Rothman (1998) studied the asymmetric behaviour of quarterly unemployment rate in postwar in U.S. using nonlinear time-series model, LSTAR and compared to linear models. He reported that improved forecast performance can indeed be achieved by using nonlinear time-series model for the U.S. unemployment rate. Thus, there is support for the claim that nonlinear forecast dominate the linear forecast. However, the relative performance of the model is sensitive to whether or not unemployment rate series is transformed to stationarity. Without such transformation, several of the nonlinear time-series produce biased forecasts. For untransformed unemployment rate series, a simple linear AR(2) model performed better.

3. Material and Methods

This chapter presents an overview of the statistical methods used in analyzing inflation values. We employed a quantitative research method. In this section, we discuss the theory of time series analysis Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA) and nonlinear SETAR and LSTAR models.

3.1 The Theory of Times Series

Time series is a time-dependent sequence. $Y_1, Y_2, Y_3, ..., Y_n \ or \ Y_t, Y \in N$

where $Y_1, ..., Y_n$ denote time steps.

3.1.1 Components of Time Series

Traditional methods of time series analysis are mainly concerned with decomposing the variation in series into the various components of trend, seasonal and cyclic.

Periodic Component

If

$$Y_t = Y_{t-1} + e_t \tag{3.1}$$

for $t \in N$, then the time series has a periodic component of period T.

Trend Component

If

$$Y_t = y_t + \beta_t + e_t \tag{3.2}$$

then there exists a linear trend with the slope being β.

Stochastic Component

If

$$Y_t = y_t + e_t \tag{3.3}$$

then e_t is the stochastic component for time series.

There are several objectives in analyzing a time series. The objectives may be classified as descriptive, explanative, predictive and to control.

3.1.2 Description

When presented with a time series data, the first step in the analysis is usually to plot the data to obtain simple descriptive measures of the main properties of the series such as seasonal effect and trend. Apart from trend and seasonal variation, the outliers to look for, in the graph of the time series are the possible presence of turning points, where, for example, an upward trend has suddenly changed to a downward trend.

3.1.3 Explanation

When observations are taken on two or more variables, it may be possible to use the variation in one-time series variable. This may lead to a deeper understanding of the mechanism which generated a given time series. For example, it is interesting to see how sales are affected by price and economic conditions.

3.1.4 Prediction

Given an observed time series, one may want to predict the future values of the series. This is an important task in sales forecast and the analysis of economic and industrial time series. Prediction is closely related to control problems in many situations. For example, if one can predict that a manufacturing process is going to move off target, then appropriate corrective action can be taken.

3.1.5 Control

When a time series is generated which, for example, measures the quality of a manufacturing process, the aim of the analysis may be to control the process. Control procedures are of several different kinds. In statistical quality control for instance, the observations are plotted on control charts and the controller takes action which is based on fitting a stochastic model of the series, from which future values of the series are predicted. The values of process variables predicted by the model are taken as the variables conform to the target values.

3.1.6 Stationary Time Series

A time series is said to be strictly stationary if the joint distribution of $X_{t1}, ..., X_{tn}$ is the same as joint distribution of $X_{t1+T}, ..., X_{tn+T}$ for all $t_{n+T}, ..., t_n + T$. In other words,, shifting the time origin by an amount T has no effect on the joint distribution, which must therefore depend only on the intervals between $t_1, ..., t_n$

3.1.7 Achieving Stationarity

If there is trend in the mean, then differencing the time data will remove the trend and stationarity will be achieved. For non-seasonal data, first order differencing is usually sufficient to attain stationarity. Occasionally second order differencing is required using the operator, ∇^2 where

$$\nabla^2 X_{t+2} = \nabla X_{t+2} - \nabla X_{t+1} = X_{t+2} - 2X_{t+1} + X_t. \tag{3.4}$$

Stationarity can also be achieved by taking logarithms of the data. If there is trend in variance, the series is made stationary by transforming the data as follows

$$Y_t = U_t \tag{3.5}$$

where $U_t = log X_T$

3.2 Autocorrelation function (ACF)

The autocorrelation function measures the degree of correlation between neighbouring observations in a time series. Given measurements, $y_1, y_2, ..., y_n$ at time $x_1, x_2, ..., x_n$, the lag k autocorrelation function is defined as:

$$\rho_k = \frac{\Sigma_{i=1}^{n-k}[(y_i - \mu_y)(y_{i+k} - \mu_y)]}{\Sigma_{i=1}^{n}(y_i - \mu_y)^2}] \tag{3.6}$$

$$\rho_k = \frac{Cov(Y_i, Y_i + k)}{\delta_{i+k}} \tag{3.7}$$

According to Hamilton (1994), the autocorrelation coefficient is estimated from sample observations using the formula:

$$\rho_k = \frac{\sum_{i=2}^{n}(y_i - \mu_y)(y_i + k - \mu_y)}{\sum_{i=1}^{n}(y_i - \mu_i)^2} \tag{3.8}$$

The auto coefficient of a random data are approximately normal with

$$\mu\rho^k = 0 \tag{3.9}$$

and

$$\frac{\sigma^2}{\rho^k} = \frac{1}{n}, \tag{3.10}$$

where n is the size of the sample. Thus for random sample of size 40 we expect

$-2\sigma_\rho < \gamma_k < 2\sigma_\rho$,

for significant limits of two standard errors which is :

$\frac{-2}{\sqrt{40}} \leq \gamma_k \leq \frac{2}{\sqrt{40}}$ or $-0.316 \leq \gamma_k \leq 0.316$.

Hence any value of γ_k lying outsides this interval is said to be significantly different from zero according to Hamilton (1994).

3.3 Sampling Distribution of Autocorrelation Coefficient

The autocorrelation coefficients of random data are approximately normal with

$${}^{\mu}\phi_{pk} = 0 \tag{3.11}$$

and

$$\sigma^2_{pk} = \frac{1}{n} \tag{3.12}$$

where n is the size of the sample.Thus for a random sample of size 40 we expect $-2^{\sigma}\phi_{kk}$ for significant limits of two standard errors which is

$\frac{-2}{\sqrt{40}} \leq \phi_{kk} \leq \frac{2}{\sqrt{40}} = -0.316 \leq \phi kk - 0.316.$

Hence any value of ϕkk lying outside this interval is said to be significantly different from zero, according to Hamilton (1994).

3.4 Partial Autocorrelation Coefficient

Autocorrelation function measures the degree of association between

Y_t

and

Y_{t+k}

when the effects of other time lags on Y are held constant. The partial autocorrelation function PACF is denoted by

$\phi_{kk} : k = 1, 2, ...$

The set of partial autocorrelation at various lags k are defined by:

$$\phi_{kk} = \frac{|\rho_k^*|}{|\rho_k|}, \tag{3.13}$$

where ρ_k is the $K \times K$ autocorrelation matrix and ρ_k^* is ρ_k with the last column replaced by

$[\rho_1, \rho_2, ..., \rho_k]^T$

and an example is

$\phi_{11} = \phi_1 = \rho_1$

and

$$\phi_{22} = \frac{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & \rho_2 \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{vmatrix}} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$$

3.5 An Autoregressive Model of Order p $AR(p)$

An autoregressive model of order p denoted by $AR(p)$ is a special kind of regression in which the explanatory variables are past values of process. An autoregressive model of order p is given by:

$$Y_t = \sum_{k=1}^{p} = \alpha_k Y_{t-k} + \mu + e_t, \tag{3.14}$$

where μ is the mean of the time series data and e_t is the white noise. The order of $AR(p)$ process is determined by the partial autocorrelation function (PACF). An $AR(p)$ process has its PACF cutting after lag and the ACF decays. For example, the PACF of an $AR(1)$ process cuts off after lag one (1) according to Hamilton (1994).

3.6 Autoregressive Process of order one $AR(1)$

From equation (3.14), the $AR(1)$ process is

$Y_t = \alpha_1 Y_{t-1} + \mu + e_t.$

Putting

$\mu = 0$

we have

$Y_t = \alpha_1 Y_{t-1} + e_t.$

Multiplying through by

Y_{t-k}

we have

$Y_{t-k}Y_t = \alpha_1 Y_{t-1} + e_t Y_{t-k}$. $Cov(Y_{t-k}, Y_t) = \alpha_1 cov(Y_{t-k}, Y_{t-1}) + cov(Y_{t-1}, e_t)$.

But

$cov(Y_{t-k}, e_t) = 0$

since

Y_{t-k}

depends only on

$e_{t-k}, e_{t-k-1}, \ldots$

which are not correlated with e_t for $k > 0$.

Hence

$\gamma_k = \alpha_1 \gamma_{k-1}$.

Dividing through by

γ_0

we have $\frac{\gamma_k}{\gamma_0} = \alpha_1 \frac{Y_{k-1}}{\gamma_0}$. $\rho_k = \alpha_1 \rho_{k-1}$,

where

$\rho_0 = 1$.

We have

$\rho_1 = \alpha_1 \rho_0 = \alpha_1$ since $(\rho_0 = 1)$.

This implies

$\rho_1 = \alpha_1$.

For

$k = 2$, $\rho_2 = \alpha_1 \rho_1 = \alpha_1(\alpha_1) = \alpha_1^2$.

For

$k = 3$, $\rho_3 = \alpha_1 \rho_2 = \alpha_1 \alpha_1^2 = \alpha_1^3$.

In general,

$$\rho_k = \alpha_1^k, \tag{3.15}$$

according to Box and Jenkins (1971).

3.7 Estimating $AR(p)$ Parameters Using the method of Ordinary Least Squares

The method of ordinary least squares can be employed to estimate the parameters of the $AR(p)$ process. In multiple regression we have,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k + e_t \tag{3.16}$$

and

$$\beta = (\beta^T X)^{-1} X^T Y, \tag{3.17}$$

where

$\beta = [\beta_0, \beta_1, ..., \beta_k]^T$

and

$X = [X_1, X_2, ..., X_n]^T$

and

$Y = [Y_1, Y_2, ..., Y_n]^T$.

Similarly, with AR process the X vector transpose is formed using the past values of Y. For example, consider the $AR(1)$ process:

$Y_t = \alpha_1 Y_{t-1} + \mu$.

Hence

$Y_2 = \alpha_1 Y_1 + \mu$, $Y_3 = \alpha_1 Y_2 + \mu$. $Y_t = \alpha_1 Y_{t-1} + \mu$.

This equation is over-determined and it is solved using the ordinary least squares method. The X vector transpose is given by:

$(Y_1, Y_2, ..., Y_{t-1})^T$

and the Y vector transpose is given by:

$$(Y_2 - \mu, Y_3 - \mu, ..., Y_t - \mu)^T . Then \alpha_1 = (X^T)^{-1} X^T Y, \tag{3.18}$$

as proposed by Hamilton (1994).

3.8 Moving Average of Order q $MA(q)$

According to Hamilton (1994), MA models provide predictions of Y_t based on a linear combination of past forecast errors. In particular, the MA model of order q is given by:

$$Y_t = \sum_{k=1}^{q} \theta_k e_{t-k} + \mu + e_t \tag{3.19}$$

3.9 Autocorrelation Function (ACF) of $MA(q)$

$\gamma_k = cov(Y_t, Y_{t+k}) = cov(\theta_1 e_{t-1} + \theta_2 e_{t-2}) + ... + \theta_q e_{t-q}, \theta_1 e_{t+k-1} + \theta_2 e_{t+k-2} + ... + \theta_q e_{t+k-q}$

$$= \begin{cases} 0 ....k > q \\ \sigma_e^2 \sum_{t-1}^{q-k} \theta_i \theta_{i+k} ....k = 0, 1, ..., q \\ \gamma(-k) ....k > o \end{cases}$$

Since

$$cov(e_s, e_t) = \begin{cases} \sigma_e^2 ....s = t \\ 0 ....s \neq t \end{cases}$$

Hence the autocorrelation function (ACF) of $MA(q)$ process is given by:

$$\rho(k) = \begin{cases} 1 ....k = 0 \\ \frac{\sum_{i=1}^{q-k} \theta_i \theta_{i+\mu}}{\sum_{t=1}^{q} \theta_i^2} ....k = 1, 2, ..., q \\ \rho(-k) ....k < 0 \end{cases} \tag{3.20}$$

The order of the $MA(q)$ is given by the autocorrelation function. The ACF cuts after lag q and the partial autocorrelation function decays to zero. Thus an $MA(1)$ process cuts off after lag one. In the other words, the ACF after lag one will not be significantly different from zero.

3.10 Moving Average Process of Order One $MA(1)$

The $MA(1)$ process is given by:

$$Y_t = \theta_1 e_(t-1) + \mu + e_t. \tag{3.21}$$

and its autocorrelation is given by:

$$\rho(k) = \begin{cases} 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots k = 0 \\ \frac{\theta_1}{1+\theta^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots k = 1 \\ 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots otherwise \end{cases} \tag{3.22}$$

Thus

$\rho_1 = \dfrac{\theta_1}{1+\theta_1^2}\ \rho_1 + \rho_1\theta_1^2 - \theta_1 = 0. \Rightarrow \rho_1\theta_1^2 - \theta_1 = 0.$

The parameters are thus roots of a quadratic. This means that we can find two $MA(1)$ processes that corresponds to the same ACF. To establish a one-to-one correspondence between the ACF and the model and obtain a converging autoregressive representation, we restrict the moving average parameter such that $\theta < 1$.

This restriction is known as the invertibility which implies that the process can be written in terms of an autoregressive model, according Hamilton (1994).

3.11 Estimation of Model Parameters of $MA(q)$ Process

For an $MA(1)$ process, an iterative method is used since the ordinary least squares cannot be used as the residual sum of a square is not a quadratic function. The approach suggested by Box and Jenkins is used.

Given the $MA(1)$ model:

$Y_t = \theta_1(e - t) + \mu + e_t$

where μ and θ_1

are constants and

$\gamma = \frac{\theta_1}{1+\theta_1^2}$.

Then the residual sum of sequences is calculated using

$Y_t = \mu + e_t + \theta_1 e_{t-1}$

recursively in the form:

$e_t = Y_t - \mu - \theta_1 e_{t-1}$.

With

$e_0 = 0$

we have

$e_1 = Y_1 - \mu, e_2 = Y_2 - \mu - \theta_1 e_1,\ e_3 = Y_3 - \mu - \theta_1 e_2,\ e_n = Y_n - \mu - \theta_1 e_{n-1}$.

Then

$\sum_{t-1}^{n} e_t^2$ is calculated.

This procedure is then repeated for other values of and $_1$ the sum of squares

$\sum_{t-1}^{n} e_t^2$

computed for grid of points in the μ, θ_1plane.

We then determine by inspection the least squares estimates of μ and θ_1

which minimizes $\sum_{t-1}^{n} e_t^2$

according to Box and Jenkins (1971).

3.12 The Duality of AR and MA Processes

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The Random Walk process is given by :

$$Y_t = Y_{t-1} + e_t. \tag{3.23}$$

It can be rewritten as an infinite moving average. Indeed, consider the following moving average:

$$Y_t = e_t + e_{t-1} + e_{t-2} + ... = \sum_{i=0}^{\infty} e_{t-i}$$

$$= (1 + B + B^2 + B^3 + ...)e_t = (\sum_{i=0}^{\infty} B^i)e_t \tag{3.24}$$

We recall that

$\sum_{t=0}^{\infty} y^t = \frac{1}{1-y}$, is valid when $|y| < 1$.

Hence $Y_t = (\frac{1}{1-B})e_t$.

So that $(1 - B)Y_t = e_t$,

$Y_t - B(Y_t) = e_t$,

$Y_t - Y_{t-1} = e_t$,

$$Y_t = t_{t-1} + e_t. \tag{3.25}$$

This is a random walk process. This means a finite autoregressive process. For example, we show that $MA(1)$ process is an infinite autoregressive process.

For such a process.

$Y_t = e_t - \theta_1 e_{t-1}$

Using B operator notation we have

$Y_t = (1 - \theta_1 B)e_t$

$\frac{Y_t}{1-\theta_1 B} = e_t$

$(1+\theta_1 B+\theta_2^2 B^2+...)Y_t = e_t$

$Y_t+\theta_1 Y_{t-1}+\theta_2^2 Y_{t-2}+... = e_t$

$$Y_t+\theta_1 Y_{t-1}+\theta_2^2 Y_{t-2}+... = e_t. \tag{3.26}$$

This is an infinite autoregressive process.

3.13 ARMA or Mixed Process

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Consider the process given by:

$$Y_t = \alpha_1 Y_{t-1}+\theta_1 e_{t-1}+e_t \tag{3.27}$$

This can be rewritten as:

$Y_t-\alpha_1 Y_{t-1} = e_t+\theta_1 e_{t-1}$ or

$(1-\alpha B)Y_t = (1+\theta B)e_t$

$$AR(B)Y_t = MA(B)e_t \tag{3.28}$$

Equation (3.28) is called a mixed or autoregressive moving average (ARMA) process of order (1,1), since equation (3.27) is ARMA (1,1) if $\theta < 1$.

It can be rewritten as:

$(1-\alpha B)(\frac{1}{1+\theta B})Y_t = e_t$

$(1-\alpha B)(1-\theta B+\theta^2 B^2-\theta^3 B^3+...)Y_t = e_t$

$$[(1-\alpha+\theta)B+(\alpha\theta+\theta^2)B^2+...]Y_t = e_t \tag{3.29}$$

This is an infinite order AR process. This is true if

$a < 1$ and $\theta < 1$

i.e. if the AR is stationary and MA is invertible. If we have two polynomial in B, MA(B) and AR(B), and an ARMA model,

$$AR(B)Y_t = MA(B)e_t \tag{3.30}$$

It is possible to write the model as an infinite AR process:

$$\left(\frac{AR(B)}{MA(B)}\right)e_t \tag{3.31}$$

or an infinite MA process:

$$\left(\frac{MA(B)}{AR(B)}\right)e_t, \tag{3.32}$$

and they approximate either by finite processes ARMA processes are parsimonious however identifying those using ACF and PACF may be difficult. The condition necessary for dividing by AR(B) is that the AR process be stationary and by MA(B) is that the MA process be invertible.

3.14 Autoregressive Moving Average Model (ARMA)

A more general model is a mixture of the $AR(p)$ and $MA(q)$ models and is called an autoregressive moving average model (ARMA) of order (p,q).

The $ARMA(p, q)$ is given by:

$$Y_t = \sum_{i=1}^{p} \alpha Y_{t-i} + \sum_{i=1}^{q} \theta e_{t-i} + \mu + e_t \tag{3.33}$$

From equation (3.31) $ARMA(1, 1)$ can be written as :

$$Y_t = \alpha_1 Y_{t-1} + \theta_1 e_{t-1} + \mu + e_t \tag{3.34}$$

An important characteristic of the ARMA models is that both the ACF and PACF do not cut off as in AR and MA models, according to Box and Jenkins (1971).

3.15 $ARMA(1, 1)$ model

An example of an $ARMA(p, q)$ model is the $ARMA(1, 1)$ model given by:

$$Y_t = \alpha_1 Y_{t-1} + \theta_1 e_{t-1} + \mu + e_t \tag{3.35}$$

The $ARMA(1,1)$ model is stationary if $-1 < \alpha_1 < 1$.

Its theoretical autocorrelation function (ACF) and partial autocorrelation (PACF) trail off to zero in a damped exponential fashion.

In an $ARMA(1,1)$ model both, the ACF and the PACF trail off to zero, according to Hamilton (1994).

3.16 Estimating Parameters of an ARMA Model

The procedure for estimating the parameters of the ARMA model is like the one for the MA model; it is an iterative method. Like the MA the residual sum of squares is calculated at every point on a suitable grid of the parameters' values, and the values that give the minimum sum of squares are used as the estimates.

$ARMA(1,1)$ is given by:

$$Y_t - \mu = \alpha_1 (Y_{t-1} - \mu) e_t + \theta_1 e_{t-1} \tag{3.36}$$

Given n observation $Y_1, Y_2, ..., Y_n$, we guess values for , α_1, θ_1, set $e_0 = 0$ and then calculate the residuals recursively by:

$$e_1 = Y_1 - \mu \tag{3.37}$$

$$e_2 = Y_2 - \mu - \alpha_1 (Y_1 - \mu) e_t + \theta_1 e_t \tag{3.38}$$

$$e_t = Y_n - \mu - \alpha_1 (Y_1 - \mu) - \theta_1 e_{n-1} \tag{3.39}$$

The residual sum of squares,

$\sum_{t=1}^{n} e_t^2$

is calculated. Then other values of

$\mu, \alpha_1, \theta_1,$

are tried until the minimum residual of squares is found.

Note: It has been found that most of the stationary time series occurring in practice can be fitted by $AR(1), AR(2), MA(1), MA(2), ARMA(1,1)$ or white noise models that are customarily needed in practice as proposed by Hamilton (1994).

To identify the resulting ARMA process,the principle tools for putting e_1 and e_2 into effect is the sample autocorrelation function and the sample partial autocorrelation function. Apart from helping to guess the model form, they are used to obtain approximate estimates of the parameters of the model. These approximations are useful at the estimates stage to provide starting values for iterative procedures employed at that stage.

3.17 Use of ACF and PACF in Identification ARMA Process

A stationary mixed autoregressive moving average process of order (p,0,q):

$$\alpha(B)Y_t = \theta(B)e_t, \tag{3.40}$$

the autocorrelation function satisfies the difference equation

$$\alpha(B)\rho_k = 0, k > q \tag{3.41}$$

Also, if

$$\alpha(B) = \Pi_{i=1}^{p}(1 - G_i B) \tag{3.42}$$

The solution of difference equation for the kth autocorrelation is, assuming distinct roots, of the form:

$$\rho_k = A_1 G_1^k + A_2 G_2^k + ... + A_p G_p^k, k > q - p \tag{3.43}$$

The stationarity requirement that the zeros of $\alpha(B)$ lie outside the unit circle, implies that the roots G1, G2, G3,...,Gk lie inside the unit circle. Inspection of the equation (3.43)

$\rho_k = A_1G_1^k + A_2G_2^k + ... + A_pG_p^k, k > q - p$

shows that in the case of a stationary model in which none of the roots lies close to the boundary of the unit circle, the autocorrelation function will quickly die out or decay for moderate and large k. Suppose that a single real root, say G_1 approaches unity, so that

$G_1 = (1 - \delta)$ where

δ

is a small positive quantity.

Then, since for k large,

$\rho_k = A_1(1 - k\delta)$

the autocorrelation function will not die out quickly and will fall off slowly and very nearly linearly. Similarly, if more than one root approaches unity the autocorrelation function will decay slowly, therefore if the autocorrelation functions dies out slowly, it implies there is at least a root which is approaching unity. As a result failure of the estimated autocorrelation function to die out rapidly might logically suggest that the underlying stochastic process is non-stationary in Y_t but possible stationary in ∇Y_t or in some higher difference. It is therefore assumed that the degree of differencing d_1 necessary to achieve stationarity, has been reached when the autocorrelation function of $W_t = \nabla^d Y_t$

die out fairly quickly.

3.18 Identifying the Stationary ARMA Process

The autocorrelation function of an autoregressive process of order tails off when its partial autocorrelation function has a cut off lag . The autocorrelation function of a moving average process of order$q$ cuts off after lag q and its partial autocorrelation tails off. Furthermore the autocorrelation function for a mixed process, containing a p^{th} order autoregressive component and q^{th} order moving average components, is a mixture of exponential and damped sine waves after the first $q - p$ lags conversely, the partial autocorrelation function for mixed process is dominated by a mixture of exponentials and damped sine waves after the first $p - q$ lags.

3.19 Model selection

3.19.1 Akaike's Information Criteria (AIC)

The AIC which was proposed by Akaike uses the maximum likelihood method. In the implemention of the approach, a range of potential ARMA models are estimated by the maximum likelihood method, and for each, the AIC is calculated as :

$$AIC(d,p) = \frac{-2\ln(maximumlikelihood) + 2r}{n} \quad (3.44)$$

$$AIC(p,q) = \ln(\sigma_e^2) + r\frac{2}{n} + constant, \quad (3.45)$$

where n is the sample size or the number of observations in the historical time series data

σ_e^2 is the maximum likelihood estimate of

σ_e^2 , and it is the residual or shock variance,

$r = p + q + 1,$

denotes the number of parameters estimated in the model. Given two or more competing models the one with the smaller AIC value will be selected.

3.19.2 Schwarzs Bayesian Information Criterion (BIC)

Schwarzs BIC like the AIC uses the maximum likelihood method. It is given by:

$$BIC(p,q) = \ln(\hat{\sigma_e^2}) + r\frac{\ln(n)}{n}, \quad (3.46)$$

where $\hat{\sigma}_e^2$ is the maximum likelihood estimate of σ^2

$r = p + q + 1$,

denotes the number of parameters estimated in the model, including a constant term and n is the sample size or the number of observations in the time series data. The BIC imposes a greater penalty for the number of estimated model parameters than does AIC.

The use of minimum BIC for model selection results in a chosen model whose number of parameters is less than that chosen under AIC. One disadvantage of the information criteria approach is the enormous work involved in computing the maximum likelihood estimates of several models which is time consuming and expensive. However, the problem has been solved by the introduction of computers which lends itself to several software which compute these information criteria values. Information criteria are useful tools in model selection; they should not, however, be substituted for the careful examination of the autocorrelation and partial autocorrelation functions.

3.20 Estimation of Parameters of the Model Identified

Once a model is identified the next stage is to estimate the parameters. In this study the estimation of the parameters was done using a statistical package called R to estimate the varions parameters.

3.21 Model Diagnostic Checks and Adequacy

The model diagnostic checks are performed to determine the adequacy or goodness of fit of a chosen model. The models' diagnostic checks are performed on residuals and more specifically on the standardized residuals (Talke, 2003). The residuals are assumed to be independently and identically distributed following a normal distribution (Tsay, 2002). Plots of the residuals such as the histogram, the normal probability plot and the time plot of residuals can be used. If the model fits the data well the histogram of residuals should be approximately symmetric. The normal probability plot should be a straight line while the time plot should exhibit random variation (Bowerman and OConnell, 1997). The ACF and the PACF of the standardized residuals are used for checking the adequacy of the conditional variance model.

The Lagrange multiplier and the Ljung Box Q-test are used to check the validity of the SETAR effect as well as test for autocorrelation in the data. To test the presence of SETAR effects, the null hypothesis of no SETAR effects is rejected if the significance probability value (p-value) is less than specified level of significance. In case of testing for the presence of autocorrelation, the null hypothesis of no autocorrelation is rejected if the Ljung Box (Q) statistics of some of the lags are significant. Thus if the probability value of Ljung Box (Q) statistics of some of the lags are less than the specified level of significance, then the null hypothesis of no autocorrelation is rejected. Once the estimated model satisfies all these model assumptions, it can be seen as an appropriate representation of the data. Having established that the model fits the data well, the model can then be used to compute forecasts of the series under consideration.

3.22 Model Validation

The data set was divided into two parts; an initialization or training set and a verification or test set. The training set was used to estimate the model parameters whilst the test set was used to validate the model. This validation process is necessary to evaluate the model for how accurate it is in forecasting. If a chosen model is able to describe the testing set well, then the model is considered valid and adequate and hence it can be used in forecasting the series under consideration.

3.23 Assessment of Predictiveness or Forecast Accuracy of the Model

Forecast accuracy test can be used as criteria for selecting the best model. Several measures for assessing the forecast accuracy of threshold models have been proposed. Some of these measures are the mean absolute percentage error (MAPE), mean square error (MSE) and mean absolute error (MAE). This study considered the MAPE. The MAPE is defined by:

$$M = \frac{100}{n} \Sigma_{t=1}^{n} |\frac{A_t - F_t}{A_t}|, \tag{3.47}$$

where A_t is the actual value and F_t is the forecast value. The difference between A_t and F_t is divided by the actual A_t again. The absolute value in this calculation is summed for every forecasted point in the time and divided by the number of fitted points, n. Multiplying by 100 makes it a percentage error.

3.24 Forecasting

The fourth stage is to forecast with the model selected. Suppose the model chosen to fit a hypothetical data is:

$$Y_t = Y_{t-1} + \alpha_1(Y_{t-1} - Y_{t-2}) + e_t \tag{3.48}$$

And suppose further that the data is of length 60, $\alpha = 0.2178$

$Y_{60} = 131.2$

$Y_{59} = 134,8$

Then from equation (3.46) Y_{61} can be written as:

$Y_{61} = Y_{60} + 0.2178(Y_{60} - Y_{59})$

$Y_{61} = 131.2 + 0.2178(131.2 - 134.8)$

$Y_{61} = 130.097,$

hence, a forecast value for period 61 is 130.097.

3.25 Testing for the Existence of Nonlinearities of Monthly Rates of Inflation

Before nonlinear model is used, it is important to test for the existence of nonlinearities in the data. If there is no evidence of nonlinear dynamics, the nonlinear approach is hardly justifiable and rather classical linear methods should be used. There are several methods in the literature, but we applied two of the tests, namely Keenan test and Tsay test.

3.25.1 Keenan Test for Nonlinearity

To perform tests for nonlinearity we have to specify m, the autoregressive order. Under the null hypothesis that the process is linear, the order can be specified by using the information criterion such as AIC. The presentation of Keenan test follows the Cryer and Chan (2008). The Keenan test is based on a second-order Volterra type of expansion. The Volterra and Taylor expansion are similar in concept. Volterra expansion is used for nonlinear modeling and its unique feature is its ability to capture memory effects. The Keenan test can be written as :

$$y_t = \mu + \sum_{i=-\infty}^{\infty} \theta_i \varepsilon_{t-i} + \sum_{i.j=-\infty}^{\infty} \theta_{i,j} \varepsilon_{t-i} \varepsilon_{t-j} + \sum_{i,j,k=-\infty}^{\infty} \theta_{ijk} \varepsilon_{t-i} \varepsilon_{t-j} \varepsilon_{t-k} + ..., \tag{3.49}$$

where ε_t and $-\infty < t < \infty$

are a sequence of i.i.d. random variables with mean zero while

$y_1, ..., y_n$

are the observations.

The process Y_t is linear if the double sum on the right-hand side of the equation disappears.

Basically, testing for the nonlinearity of a series y_t

is to find out whether the double sum is zero or not. Alternatively, as Cryer and Chan (2008) pointed out, Keenan test can also be systematically deduced as follows:

$$Y_t = \theta + \psi_1 Y_{t-1} + ... + \psi_m Y_{t-m} + exp[\eta(\sum_{j=1}^{m} \psi_j Y_{t-j})^2] + \varepsilon_t, \tag{3.50}$$

where

ε_t

are i.i.d. with zero mean and finite variance. If the regression coefficient,

$\eta = 0$, then the exponential term becomes 1 and it can be absorbed in the intercept so that the previous model becomes an autoregressive model AR_m.

If the regression coefficient η is different from zero, then the previous model is nonlinear. Using the expansion

$exp(x) \approx 1 + x$,

which holds for x of small magnitude, we can see that for small η, Y_t follows approximately a quadratic AR model:

$$Y_t = \theta + 1 + \psi_1 Y_{t-1} + ... + \psi_m Y_{t-m} + \eta\{(\sum_{j=1}^{m} \psi_j Y_{t-j})^2\} + \varepsilon_t \tag{3.51}$$

Keenan (1985) highlights the potential limits of testing for nonlinearity: yet, it is powerful in detecting nonlinearity in the form of the square of the approximating linear conditional mean function, the strength of the test may be sometimes low.

The test statistic

$$F = \eta^2 (\frac{n - 2m - 2}{RSS - \eta^2}), \tag{3.52}$$

where F is approximately distributed as an F-distribution with degrees of freedom 5 and $(n - 2m - 2)$.

3.25.2 Tsay Test for Nonlinearity

The Keenan test was improved by Tsay (1986). Again, we follow the description from Cryer and Chan (2008).

The Tsay test enhanced the Keenans approach by replacing the term

$$\eta\{(\sum_{j-1}^{m} \phi_j Y_{t-j})^2\}$$

by

$$exp(\varsigma_{1,1}Y_{t-1}^2 + \varsigma_{1,2}Y_{t-1}Y_{t-2} + ... + \varsigma_{1,m}Y_{t-m} + \varsigma_{2,2}Y_{t-2}^2$$

$$+Y_{2,3}Y_{t-2}Y_{t-3} + ... + \varsigma_{2,m}Y_{t-2}Y_{t-m} + ... + \varsigma_{m-1,m-1}Y_{t-m+1}$$

$$+\varsigma_{m-1,m}Y_{t-m+1}Y_{t-m+1}Y_{t-m} + \varsigma_{m,m}Y_{t-m}^2 + \varepsilon_t$$

Using the approximation, we can deduce that the nonlinear model is approximately a quadratic AR model but the coefficients of the quadratic terms are unconstrained.

The Tsay test considers the following quadratic regression model:

$$Y_t = \theta_0 + \phi_1 Y_{t-1} + ...\phi_m Y_{t-m} \qquad (3.53)$$

$$+\varsigma_{1,2}Y_{t-1}^2 + \varsigma_{1,2}Y_{t-1}Y_{t-2} + ...\varsigma_{1,m}Y_{t-1}Y_{t-m}$$

$$+\varsigma_{2,2}Y_{t-2}^2 + \varsigma_{2,3}Y_{t-2}Y_{t-3} + ...\varsigma_{2.m}Y_{t-2}Y_{t-m} + ...$$

$$+\varsigma_{m-1,m-1}Y_{t-m+1}^2 + \varsigma_{m-1,m}Y_{t-m+1}Y_{t-m+1} + \varsigma_{m,m}Y_{t-m}^2 + \varepsilon_t$$

and test whether all

$$m(\frac{m+1}{2}) coefficients \zeta_{ij} = 0.$$

3.26 Modeling the inflation rate using SETAR

There are several time series models that have been proposed for describing the different regimes generated by a stochastic process. Tong (1978) and Tong and Lim (1980) proposed a threshold autoregressive (TAR) model in which the regime was determined by the value of an observable variable relative to a threshold value. First order SETAR Model would be applied. The application is based upon first-order SETAR model. The model is given by:

$$Y_t = \begin{cases} \mu_{1,0} + \rho_{1,1} Y_{t-1} + \sigma_1 \varepsilon_t, if Y_{t-1} < \theta \\ \mu_{2,0} + \rho_{2,1} Y_{t-1} + \sigma_2 \varepsilon_t, if \theta < Y_{t-1} \end{cases} \quad (3.54)$$

where ρ are the autoregressive parameters, σ are noise standard deviations, θ is the threshold parameter and e_t is a sequence of i.i.d. random variables with zero mean and unit variance. Therefore, if the lag 1 value of y_t is not greater than the threshold, then the conditional distribution of y_t is similar to the first AR(1) process and we say that we are in the lower regime, but when the lag 1 value of y_t is greater than the threshold, then the second AR(1) model is operational and we are in the upper regime. Thus, the process switches between two linear models depending on the position of the lag 1 value. Since the error variance may be different in the regimes, the SETAR model can account for conditional heteroscedasticity in the data. This study considered two regimes SETAR models to model monthly rates of inflation in Ghana.

3.27 Modeling the inflation rate LSTAR

If $G(Y_{t-1,\gamma,\theta})$ is a continuous and logistic function, then the resulting model is called a Logistic Smooth Transition AR LSTAR model.

The presentation follows Franses and Van Dijk (2000). The transition logistic function is given by:

$$G(Y_{t-1,\gamma,\theta}) = \frac{1}{1 + exp(-\gamma[Y_{t-1} - \theta])}, \quad (3.55)$$

where θ is the location parameter and tells us where the transition occurs and interpreted as threshold between the two regimes corresponding to

$G(Y_{t-1}, \gamma, \theta) = 0$

and

$G(Y_{t-1}, \gamma, \theta) = 1$

as the logistic function changes from 0 to 1.

γ is the slope indicator and it determines the smoothness of the change in the value of the logistic function, namely the speed of the transition from one regime to another. As γ becomes larger, then the transition from 0 to 1 occurs almost immediately at

$Y_{t-1} = \theta$

and,consequently, the logistic function

$G(Y_{t-1}, \gamma, \theta)$

approaches the indicator function

I_a

$Y_{t-1} > \theta$

and the SETAR model may be approximated by a LSTAR model.

According to Terasvirta (2006), if the slope,$\gamma \to \infty$ then a two-regime SETAR model becomes a special case of the LSTAR.

The chapter has provided an overview of time series and its basic concepts as well as a detailed description order determination, estimation and forecasting. Furthermore, various selection criteria that help to select the best fit model among a class of competing models were discussed. Finally, there was an exposition on model validation as well as assessing the predictiveness or forecast accuracy of a model.

4. DATA ANALYSIS AND DISCUSSION OF RESULTS

Introduction

This chapter presents the analysis and the discussion of the results obtained from the study. The chapter is further organised into five subsections excluding the introductory section. A description of the data with respect to basic statistics was done under section 4.1. Section 4.2 dealt with the preliminary analysis of the data. Model estimation and fitting as well as model evaluation and diagnostics are presented under section 4.3 and 4.4, respectively. The last section, 4.5, focused on forecasting of monthly inflation rate based on model selected as the most appropriate model under section 4.4. The analysis was carried out using the R-package to obtain the various graphs and descriptive statistics, estimation and others.

4.1 Data description

The sample data consists of 428 observations of monthly rates of inflation in Ghana. The data span January 1981 to August 2016 and covered a thirty-five years and eight months period. The rates from January 1981 to December 2015 were used as training sets and January to August 2016 as the validation sets. The data were obtained from the Ghana Statistical Service (GSS) as published in their official website (www.gss.gov.org). The GSS is the official government institution mandated to provide the rate of inflation in Ghana on periodic bases, such as, monthly rates of inflation.

Table 4.1 presents the descriptive statistics of monthly rate of inflation.

4.2 Descriptive Statistics of monthly Rates of Inflation (1981M01-2016M08)

Statistics	Value
Mean	27.80
S.E. mean	1.34
Std. deviation	27.67
Median	18.40
Maximum	174.10
Minimum	1.14
Range	172.96
Skewness	2.80
Kurtosis	11.02
Jarque-Bera	1705.60
Probability	0.00
Sample	428

Table 4.1: Descriptive statistics of Monthly rate of Inflation (1981M01-2016M08)

From Table 4.1, the results showed that the mean of the monthly inflation rate was 27.80 with standard deviation of 27.67. The Maximum rate of inflation rate was 174.10 whilst the minimum rate of inflation was 1.14 The range of the rate of inflation over the 35 year period under consideration was 172.96. Also the median of the monthly rate of inflation was 18.40. Again,the data had positive skewness of 2.80 implying that the distribution of the data has a long right tail; kurtosis of 11.02 indicates that the distribution of monthly rate of inflation was leptokurtic. The Jarque-Bera statistic of 1705.6 is statistically significant at 1 percent level of significance. It can be concluded that the monthly rate of inflation did not follow a normal distribution. These confirmed the non-normality and positive skewness of the monthly rate of inflation as revealed by Figure 4.1. The histogram of the monthly rate of inflation has a long right tail implying positive skewness whilst the normal probability (QQ) plot of the monthly rate of inflation shown that the data was curvilinear indicating a deviation from normality as shown in Figure 4.2.

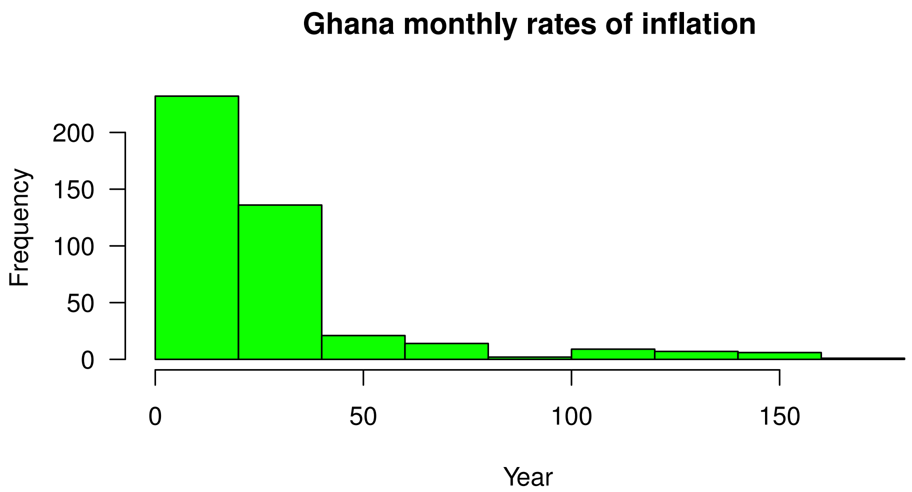

Figure 4.1: Histogram of Monthly Rates of Inflation (1981-2016)

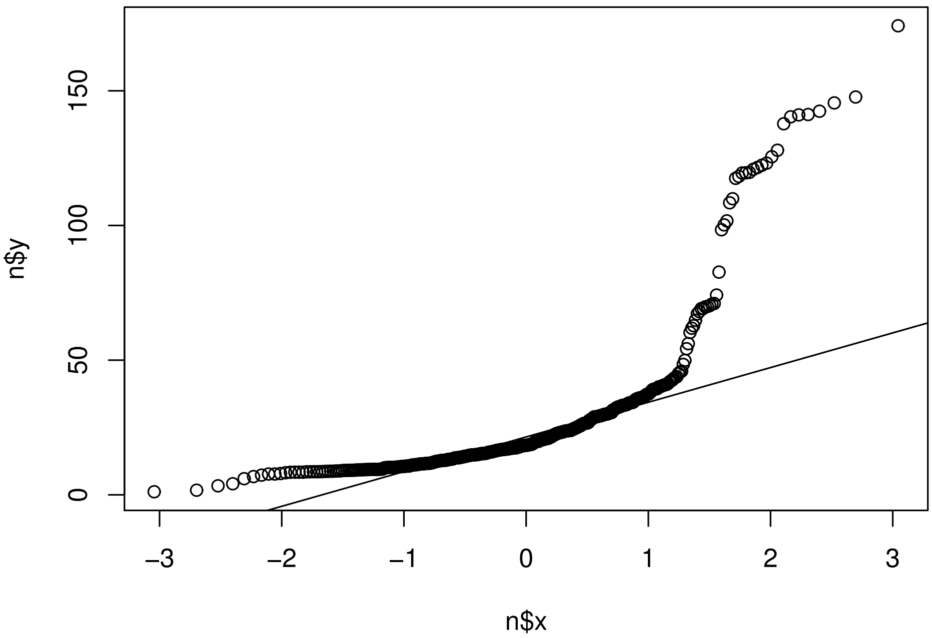

Figure 4.2: Normal probability QQ Plot of Ghana Monthly Rates of Inflation (1981-2016)

4.3 Preliminary Analysis of Data and Time Series Plot of the Monthly Rates of Inflation in Ghana (1981-2016)

From the Figure 4.3, it was evident that both the mean and variance were changing over time. This means that, the monthly rates of inflation were characterised by non-constant mean and an unstable variance. The changing mean and variance over time were an indication of the non-stationarity of the monthly rates of inflation.

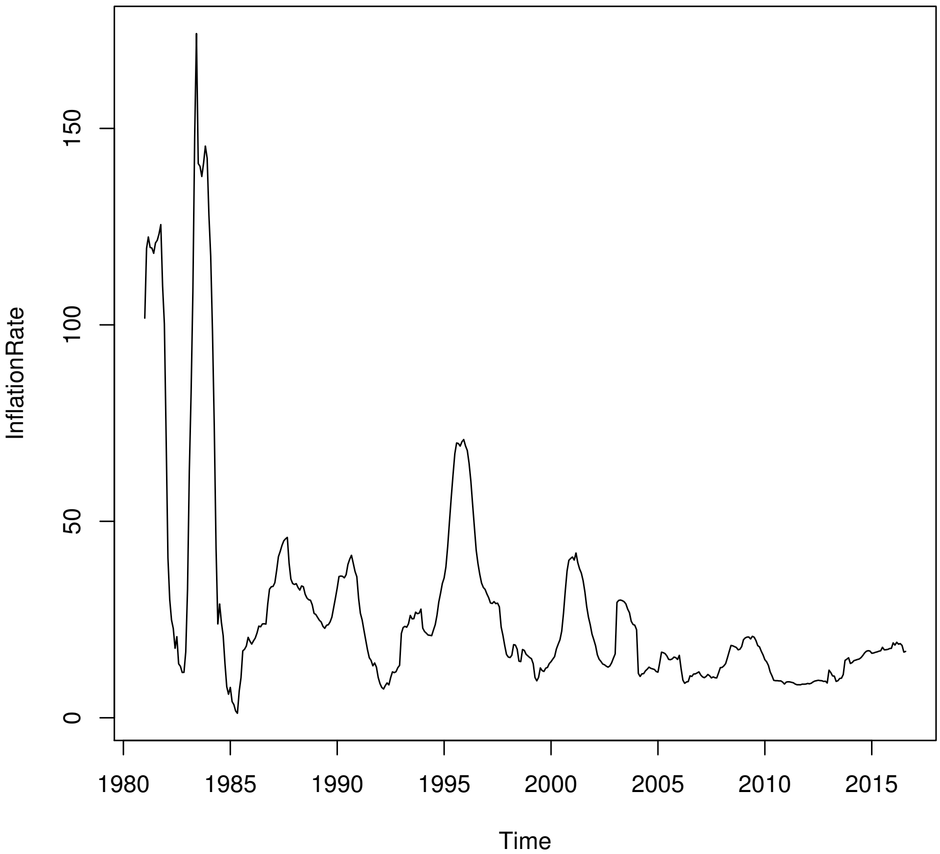

Figure 4.3: Time Series Plot of Ghana Monthly Rates of Inflation (1981-2016)

4.4 Augmented Dickey-Fuller(ADF) Unit Root Test for the monthly Rates of Inflation in Ghana (1981-2016)

As a way of confirming the non-stationarity in the monthly rates of inflation, the Augmented Dickey Fuller (ADF) test was performed. The test revealed a critical value of -7.178 which was more negative than tabulated -2.87. The test rejected the null hypothesis of unit root at 5% significant level. This can be concluded that the rate of inflation was stationary over the period under consideration.

The model that excluded trend was estimated using the sample of 428 observations. The null hypothesis of unit root was rejected.

The data were transformed by taking the logarithms to follow normal distribution, which is a desirable characteristic feature in most time series. Stationarity can be achieved in time series using methods such as ordinary differencing, seasonal differencing, and taking of logarithms. Logarithm was chosen because the data did not reveal any form of seasonality and hence seasonal differencing was not necessary. The result of the ADF test is shown in Table 4.2.

Model Type	t-statistic	p-value
Constant	-7.178	0.01

Table 4.2: ADF of Monthly Rates of Inflation (1981-2016)

4.5 Time Series Plot of logarithm of Monthly Rates of Inflation in Ghana (1981-2016)

Again, ADF test was performed after taking the logarithm of monthly rates of inflation. The test revealed a critical value of -5.5702 which is more negative than tabulated value of -2.78. The test again rejected the null hypothesis of of unit root at 5% significant level. The plot in Figure 4.4 also revealed that the logarithm of monthly rates of inflation series appears to be stable in both mean and variance over time indicating that there is stationarity in logarithm of monthly rates of inflation. This was also confirmed by the Q-Q plot of logarithm of monthly rates of inflation in Figure 4.5. From Figure 4.3, the monthly rate of inflation series exhibits heteroscedasticity, thus, changing variance over time. The SETAR model can account for conditional heteroscedasticity in the data.

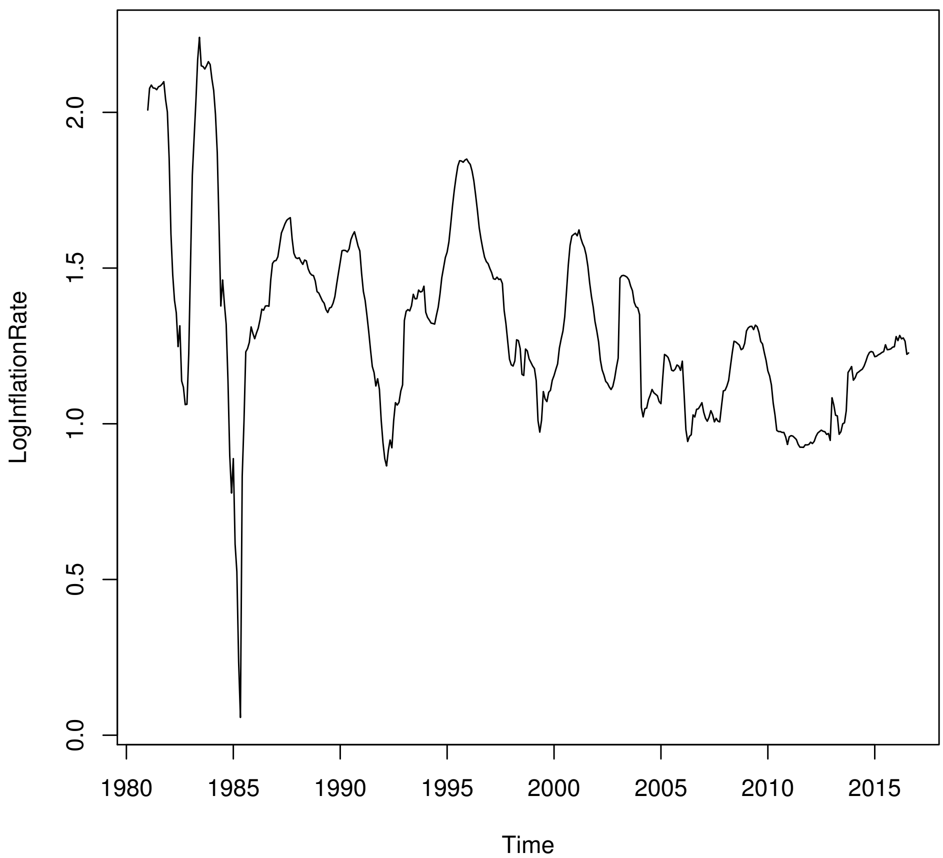

Figure 4.4: Time series Plot of logarithm of Monthly Rates of Inflation (1981-2016)

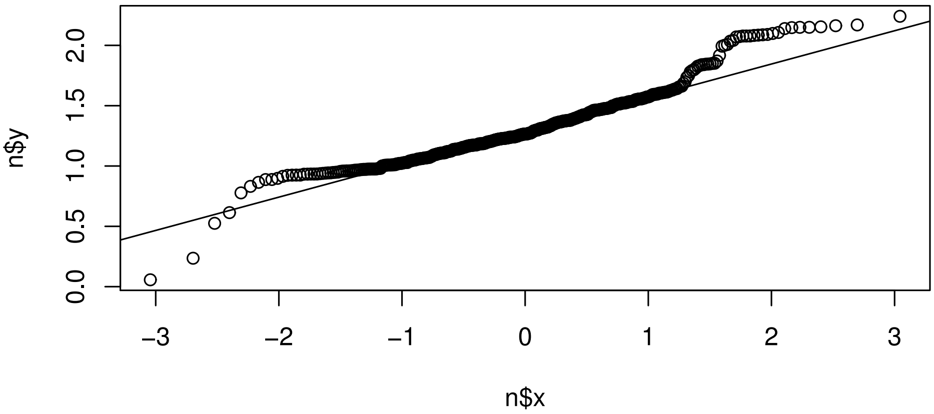

Figure 4.5: Q-Q plot of logarithm of Ghana Monthly Rates of Inflation (1981-2016)

Furthermore, the test for autocorrelation in the monthly rate of inflation series was also performed. This was done by obtaining the autocorrelation Function (ACF) and Partial autocorrelation Function (PACF) plots of the monthly rate of inflation series as shown in the Figures 4.6 and 4.7 respectively. Significant spikes at lag 1 to 10 of PACF shown in Figure 4.8 show an indication of seasonal variation, however, the data did not reveal any form of seasonality and therefore the spikes could be attributed to random effects. Also, from the plots of the ACF and PACF shown in Figures 4.7 and 4.8 respectively, there were indications of correlation in the monthly rates of inflation.

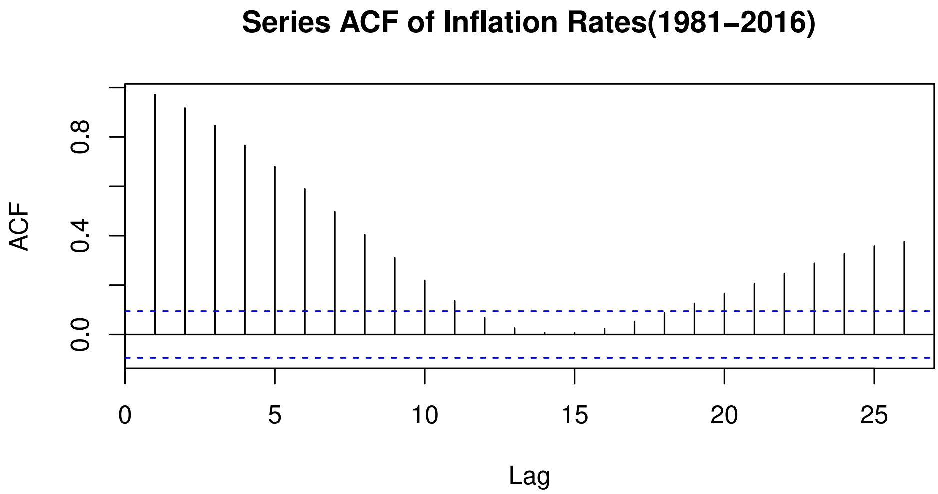

Figure 4.6: ACF Series Plot of Ghana Monthly Rates of Inflation (1981-2016)

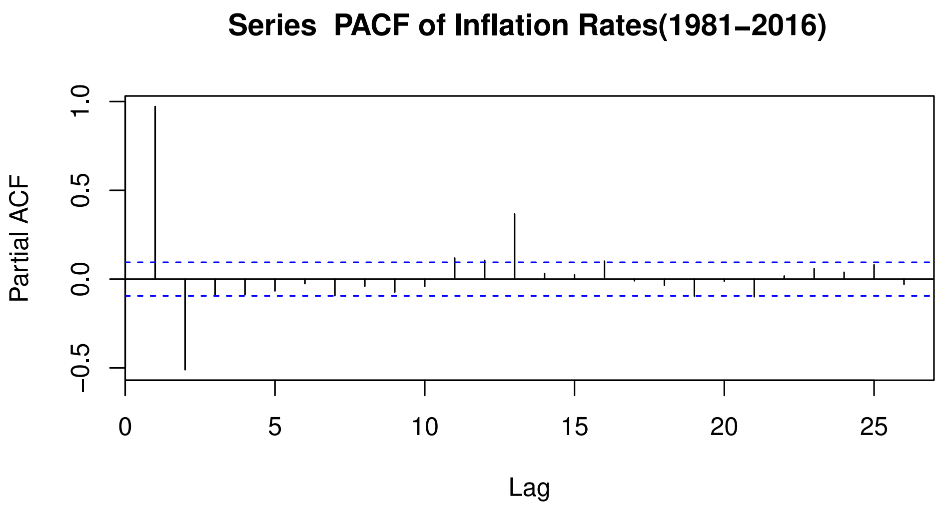

Figure 4.7: PACF Series of Ghana Monthly Rates of Inflation (1981-2016)

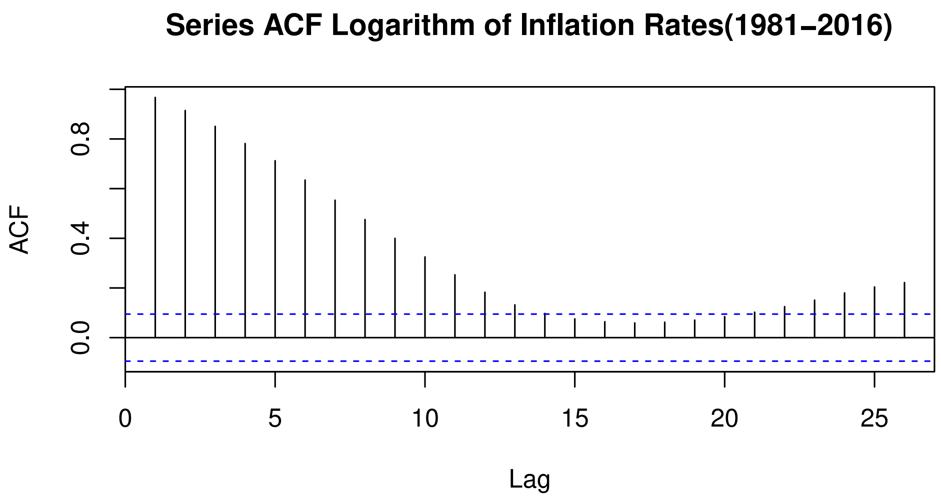

Figure 4.8: ACF Series of logarithm of Ghana Monthly Rates of Inflation (1981-2016)

The graph of ACF in Figure 4.6 dies in a sine wave form, whilst the PACF in Figure 4.7 shows a significant nunber of spikes also dying down in a sine fashion. This implied that there was no significant correlation between monthly rates of inflation. We see from the correlogram in Figure 4.8 that autocorrelation for lags 1-12 exceeded the significance bounds and that autocorrelation tails off to zero after lag 12. The autocorrelation for lags 1-12 were positive and decreasing in magnitude with increasing lags. The autocorrelation for lags 21-26 also exceeded the significance bounds with increasing lags, but it is likely that this was due to chance, since they just exceeded the significance bounds. The autocorrelation for lags 14-20 did not exceed the significance bounds and we could expect 1 to 26 lags to exceed 95% significance bounds by chance alone.

From the partial autocorrelogram in Figure 4.7, we see that the partial autocorrelation at lag 1 was positive and exceeded the significance bounds while the partial autocorrelation at lag 2 was negative and also exceeded the significance bounds. The partial autocorrelation tails off to zero after lag 13. Since the correlogram tails off to zero after lag 12 and the partial correlogram was zero after lag 13, the following models were possible for the time series:

- an ARMA (13,0) model, since the partial autocorrelogram was zero after lag 13 and correlogram tails off to zero after lag 12.
- an ARMA (0,13) model, since the autocorrelogram was zero after lag 12 and the partial correlogram tails off to zero after lag 13.

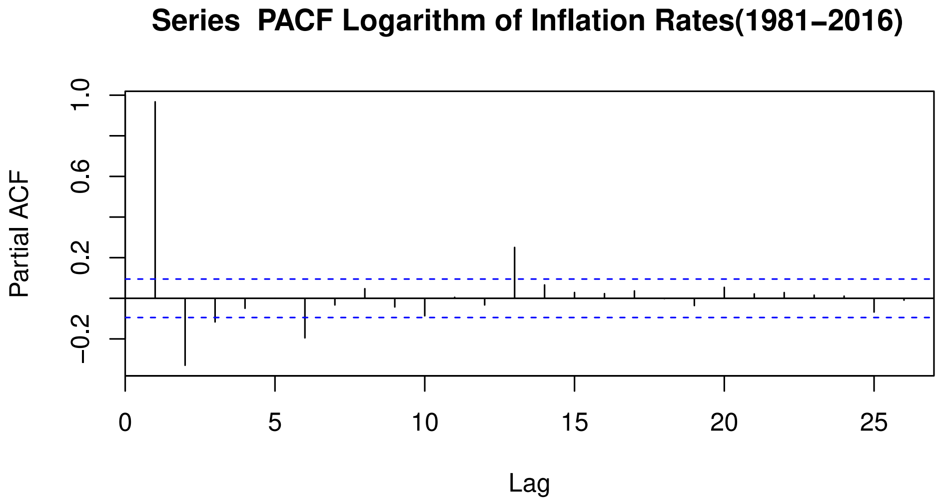

Figure 4.9: PACF Series of logarithm of Ghana Monthly Rates of Inflation (1981-2016)

From the fore-going analysis, it can be concluded that the logarithm of monthly rates of inflation satisfied all the data assumptions or characteristics of threshold models such as SETAR and LSTAR. The next step in the time series model building procedure after all assumptions or properties of the series have been satisfied is the determination of the order of the model. The ACF and PACF plots of the series were used to determine the order. From the ACF and PACF plots, the ACF tails at lag 13 while the PACF spikes at lag 13. This suggests that the order m=13 and s=13 and hence therefore an AR(13) and MA(13) models were suspected of combining to give ARMA(13,13) model around which to build the SETAR and LSTAR models. In short, it has revealed that the monthly rates of inflation was non-stationary and logarithm transformation brought stationarity. It was also observed that the logarithm of monthly rates of inflation have both significant heteroscedasticity and autocorrelation present in the series. Lastly, from the ACF and PACF plots of the logarithm of the monthly rates of inflation revealed, the AR(13) and MA(13) give an indication of an ARMA (13,13) model around which to build the SETAR and LSTAR models as shown in Figure 4.9.

4.6 Model Fitting and estimation

After the determination of the order of the model and consequently the model identification has been done, the parameters of the model can be estimated. The method used to estimate the parameters was the maximum likelihood method. The maximum likelihood function that was used in the estimation was based on the distributional assumption of normality on error term and residuals. In order to test for the performance of the nonlinear models described above, the simple AR models were used as benchmarks. In order to perform the test for nonlinearity, we have to specify m, the autoregressive order, under the null hypothesis that the process is linear, the order can be specified by using the information criterion, such as AIC.

4.7 Linear Models

The autoregressive model of order one, AR(1) was fitted to the data and the model output is presented below.

4.7.1 Residuals output of AR(1)

Model Output 4.1a

```
Residuals:
          Min            1Q        Median            3Q           Max
-0.3134064040 -0.0175215292 -0.0008779744  0.0198714658  0.7352374625
```

From the above residuals out, the fitted data revealed a minimum of -0.3134135 and a maximum value of 0.7352228. These implied that, the monthly rates of inflation vary from -0.3134135 to 0.7352228. There was a strong relationship between the monthly rates of inflation as revealed by the residuals of the data.

4.7.2 Fitted Residuals of AR(1)

Model output 4.1b

```
 Fit:
residuals variance = 0.004971,  AIC = -2266, MAPE = 4.134%
Coefficient(s):

       Estimate  Std. Error  t value  Pr(>|t|)

const  0.040935    0.014993   2.7302  0.006592 **

phi.1  0.967537    0.011082  87.3033 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Again, from the fitted residuals, it takes an average of 0.967537 for the monthly rates of inflation to increase. Also the monthly rates of inflation went up by 0.04095 every month for the period under consideration. The standard error of the monthly rates of inflation was 0.01106. This implied that, it takes an average month of 0.011082 for the inflation to be increased. This shows that inflation rate within a particular month can change as the standard error was less than 1. The t-statistic value for the first lag was 87.3033. This value was far from zero which is an indication of a strong relationship in the monthly rates of inflation. The larger the absolute value t-statistic, the less likely that the actual value of the parameter could be zero.

From the model output 4.1b, the p-value for the first lag was less than 5% of the tabulated value. This implied that, the probability of obtaining the estimated value of the parameter if the actual parameter value is zero. The smaller the value of a probability value, the more significant the parameter and the less likely that the actual value is zero. There exists a strong relationship in the monthly rates of inflation.

From the model output 4.1b, the best model that described the data is given by:

$$X_t = 0.040935 + 0.967537X_t + \varepsilon_t \tag{4.1}$$

4.7.3 Model Output of AR(2)

Furthermore, autoregressive model of order two, AR(2) was also fitted and the model output is presented in model 4.2a .

Model Output4.2a

```
Non linear autoregressive model
AR model
Coefficients:
      const       phi.1       phi.2
 0.05384635  1.32847204 -0.37039926
AR model
Coefficients:
      const       phi.1       phi.2
 0.05384635  1.32847204 -0.37039926
Residuals:
          Min            1Q        Median            3Q           Max
-0.3319678365 -0.0176558598 -0.0005198949  0.0186207748  0.7890264039
```

4.7.4 Fitted Residuals of AR(2)

Model Output 4.2b

```
Fit:
residuals variance = 0.00426,  AIC = -2330, MAPE = 3.417%
Coefficient(s):
       Estimate  Std. Error  t value  Pr(>|t|)
const  0.053846    0.014022   3.8403 0.0001417 ***
phi.1  1.328472    0.044905  29.5838 < 2.2e-16 ***
phi.2 -0.370399    0.044646  -8.2964 1.451e-15 ***
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

From the model output 4.2b, the model that fitted the data is given by:

$$X_t = 0.053846 + 1.328472X_t - 0,370399X_{t-2} + \varepsilon_t \quad (4.2)$$

Autoregressive of order 1 and 2 models were chosen for this study due to the desirable results obtained when the data was fitted as compared to other orders in terms of AIC and BIC criteria.

4.8 Nonlinearity Test for Monthly Rates of Inflation (1981 to 2016)

Before the nonlinear model is used, it is important to test for the existence of nonlinearities in the data. If there is no evidence of nonlinear dynamics, the nonlinear approach is hardly justifiable and rather classical linear methods should be used. There are several methods in the literature, but we applied two of the tests, namely Keenan test and Tsay test. Tables 4.3 and 4.4 present the Keenan and Tsay tests of monthly rates of inflation respectively.

4.8.1 Keenan Test for Monthly Rates of Inflation (1981-2016)

Number of lags	F-test	P-value
1	3.122	0.077964
2	27.809	2.144512e-07
3	30.023	7.371371e-08
4	33.011	1.767361e-08
5	34.345	9.403056e-09
6	34.723	7.885516e-09
7	38.161	1.565095e-09
8	39.919	6.906682e-10
9	43.266	1.467128e-10
10	45.166	6.146297e-11
11	37.815	1.87058e-09
12	37.520	2.15649e-09
13	30.735	5.380194e-08
14	33.403	1.511745e-08
15	30.300	6.664911e-08

Table 4.3: Keenan Test for nonlinearity

4.8.2 Tsay Test for Monthly Rates of Inflation (1981-2016)

Number of lags	F-test	P-value
1	0.81	0.3672
2	17.18	1.533e-10
3	14.66	3.304e-15
4	14.91	1.006e-22
5	13.04	1.041e-26
6	29.28	2.418e-67
7	23.16	8.796e-66
8	25.47	1.559e-79
9	26.88	4.161e-90
10	29.63	5.311e-102
11	28.18	1.781e-103
12	24.06	3.091e-97
13	17.62	7.792e-81
14	17.04	6.15e-81
15	15.50	4.757e-76

Table 4.4: Tsay Test for nonlinearity

The empirical studies on financial time series revealed that the inflation rate indexes may be nonlinear. We applied Keenan and Tsay tests for detecting nonlinearity. Tables 4.3 and 4.4 present the nonlinearity test of the monthly rates of inflation.The results from the test above supports our conviction of nonlinearity of the inflation rate since the p-value was less than 0.05. This may be attributed to diverse economic factors. The results from Table 4.3 and 4.4 revealed that the monthly rates of inflation were nonlinear. The p-values for both Keenan and Tsay tests were less than 0.05 which implies that the data was nonlinear The above results were consistent and strong in favour of nonlinearities in the time series over the chosen sample. Both Keenan and Tsay tests for lags 1 to 15 strongly rejected the null hypothesis of linearity. This was also true for larger number of lags. These results permit us to carry on our investigation using nonlinear features of the time series from Ghana's inflation rate by using nonlinear models.

4.9 Nonlinear Models

4.9.1 2-Regime SETAR Model of order 2

After carefully investigating the data, the number of regimes for the SETAR model was set to 2 regimes. The maximum autoregressive order for low regime was set at 2, while the maximum autoregressive order for the higher was also set at 2. We searched for 279 possible threshold values within regimes with sufficient 15% number of observations and then searched on 1116 combinations of thresholds 279, with a threshold delay 1, for a maximum autoregressive order for the low regime 2, and a maximum autoregressive order for the high regime also set at 2. The number of regimes for the SETAR model may be explained by the fact that there were no derivatives on the inflation index. The number of regimes depends on the data. If the data allows, it is possible to observe more regimes. Table 4.5 presents the grid search of logarithm of montly rates of inflation of order 2 .

4.9.2 Grid Search for SETAR Model of oder 2 of logarithms of Monthly rates of Inflation (1981-2016)

Nr.crt	threshold delay	Low regime	High regime	Threshold	Pooled-AIC
1	0	2	2	1.017451	-1250.427
2	0	2	2	1.017868	-1248.751
3	0	2	2	1.021189	-1247.041
4	0	1	2	1.017451	-1246.893
5	0	2	2	1.021603	-1245.497
6	0	1	2	1.017868	-1245.142
7	0	2	2	1.022016	-1244.282
8	0	2	2	1.025306	-1244.177

Table 4.5: Grid Search for SETAR Model for Monthly Rates of Inflation (1981-2016)

```
Proportion of points in low regime: 17.84%       High regime: 82.16%
```

No delay was found in the threshold function after the grid search. The threshold function is given by:
Variable:

$$Z(t) = +(1)X(t) + (0)X(t-1) = 1.042. \tag{4.3}$$

The threshold was found to be positive and was estimated at 1.042, at the proportion points in low regime of 17.84% and high regime of 82.16%, respectively. This signified that the investors behaviour changes any time the inflation index is below or above 1.042%. No delay was found after the grid search and this can be explained as investors' ability to quickly react to the changing inflation rates and its associate market conditions.

The results from the estimated lower regime also depict an increasing phase while the higher regime corresponds to the decreasing phase. When the inflation rate is low, investors take an advantange of entering into the market in order to secure high profit. Again, it was observed that both coefficients in the lower regime were positve and this can be attributed to rebust growth in profit. There was also a significant number of observations in the upper regime 82.16%, while the remaining number in the lower regime 17.84% was less than the number in the higher regime. This implies that more investors were leaving the market in the high regime as a result of less opportunities available on the market. At this point investors were more vulnerable to high risks of loss. The coefficient in the higher regime indicates decreasing profit for investors. The switching regime indicates that the investors were attracted by high profit and unwilling to bear higher risks. Figure 4.10 represents the best SETAR of the monthly rates of inflation and its associated threshold values.

The best fitted residual variance of the data was 0.00382 with AIC of -2368, BIC of 2339.442 and MAPE of 4.207%.

4.9.3 Residuals of SETAR (2)

Model Output 4.3a

```
 Residuals:
      Min         1Q     Median        3Q        Max
-0.3965858 -0.0140903  0.0011165  0.0177213  0.5590700
```

From the model output 3a , the residuals revealed a minimum of -0.3947023 and a maximum value of 0.5629535. These signified that the monthly rates of inflation vary from -0.3947023 to 0.5629535.

Model Output 4.3b

```
Fit:
residuals variance = 0.003764,  AIC = -2375, MAPE = 4.152%
Coefficient(s):
         Estimate  Std. Error  t value  Pr(>|t|)
const.L  0.278477    0.046210   6.0263 3.659e-09 ***
phiL.1   0.982384    0.065139  15.0815 < 2.2e-16 ***
phiL.2  -0.264169    0.068520  -3.8554 0.0001336 ***
const.H  0.040267    0.017537   2.2962 0.0221542 *
phiH.1   1.547591    0.057432  26.9464 < 2.2e-16 ***
phiH.2  -0.578601    0.057648 -10.0369 < 2.2e-16 ***
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Threshold
Variable: Z(t) = + (0) X(t) + (1) X(t-1)
Value: 1.042
```

Note: Where Phi.1 and Phi.2 are the lower order coefficients for lower regime, constL is the constant for the lower regime. PhiH.1 and PhiH.2 are the autoregressive coefficients for the higher regime with constL the constant.

Again, from the model output 4.3a of the fitted residuals, it takes an average of 0.278477 for the monthly rates of inflation to switch from a lower regime to a higher regime and 0.028863 to switch from a higher regime to a lower regime. The standard error of the monthly rates of inflation were 0.046210 and 0.017537 for low and high regimes respectively. This implied that the required month for inflation rate to increase vary by 0.046210 for lower regime and 0.017537 for higher regime. This shows that inflation rate can change within a particular month as the standard errors were less than 1.

The t-statistic value for the lower regime was 6.0263 and that of high regime was 2.2962. These values were far from zero which is an indication of strong relationship in the monthly rates of inflation. The larger the absolute value t-statistic, the less likely that the actual value of the parameter could be zero.

From the model output 4.3b, the p-values for both regimes were less than 5% of the tabulated value.This implied that, the probability of obtaining the estimated value of the parameter if the actual parameter value is zero. The smaller the value of a probability value, the more significant the parameter and the less likely that the actual value is zero. There exists strong relationship in the monthly rates of inflation of the period under consideration.

From the model output 4.3b, the SETAR model that described the sampled data is given by:

$$X_{t+1} = \begin{cases} 0.278477 + 0.982384X_t - 0.264169X_{t-1}, X_{t-1} \leq 1.042 \\ 0.28 + 1.541X_t - 0.564X_{t-1}, X_{t-1} > 1.042 \end{cases} \quad (4.4)$$

Figure 4.10 is the graphical representation of SETAR (2,2,2) of logarithm of the monthly rates of inflation with various thresholds.

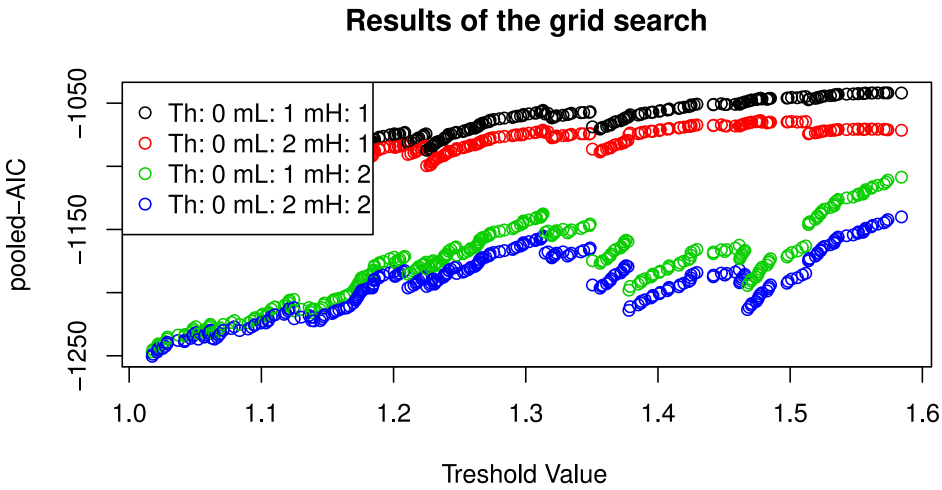

Figure 4.10: 2-Regime SETAR Model Representation of order 2 of logarithms of Monthly Rates of Inflation(1981-2016)

4.9.4 LSTAR Models

The grid search was performed for the following fixed starting values: gamma=40, threshold=0.0214,SSE=0.5554, and the optimization algorithm converged, then the optimized values fixed for regime 2 were gamma=40 and threshold 0.2498. The results of the grid search of LSTAR is presented in the Table 4.6 below.

4.9.5 Grid search for LSTAR Model of Monthly Rates of Inflation

Nr.crt	Threshold delay	Low regime	High regime	Pooled-AIC
1	0	2	2	-14443.50
2	0	1	2	-14412.13
3	0	1	1	-14400.20
4	0	2	1	-14398.33
5	1	2	1	-14396.00
6	1	2	2	-14394.14
7	1	1	1	-14390.70
8	1	1	2	-14389.62
9	2	1	1	-14377.11

Table 4.6: Grid search for LSTAR

From Table 4.6, the best LSTAR model according to AIC criterion is the model in position 1.

4.9.6 Residuals of LSTAR Model

Model Output 4.4a

```
 Residuals:
 Min        1Q    Median       3Q       Max
-0.306743 -0.014561  0.002972  0.018168  0.657501
```

From the model output 4.4a, the residuals revealed a minimum of -0.306743 and a maximum value of 0.657501. These implied that the monthly rates of inflation vary from -0.306743 to 0.657501.

4.9.7 The Fitted Residuals of LSTAR Model for Logarithm of Monthly Rates of Inflation (1981-2016)

Model Output 4.4b

```
Fit:
residuals variance = 0.003847,  AIC = -2364, MAPE = 4.096%
Coefficient(s):
         Estimate  Std. Error  t value Pr(>|z|)
Const.L  0.00048787  0.00048132   1.0136 0.310769
phiL.1  -2.12858354  1.39545776  -1.5254 0.127168
phiL.2   3.15494208  1.39956833   2.2542 0.024182 *
Const.H -0.00060171  0.00059034  -1.0192 0.308085
phiH.1   4.46643223  1.38692051   3.2204 0.001280 **
phiH.2  -4.50253872  1.38363791  -3.2541 0.001137 **
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Non-linearity test of full-order LSTAR model against full-order AR model
 F = 26.567 ; p-value = 1.3625e-11
Threshold
Variable: Z(t) = + (1) X(t) + (0) X(t-1)
Value: -0.2106
   Const.L        phiL.1        phiL.2
```

```
 0.0004878665 -2.1285835403  3.1549420830
High regime:
      Const.H        phiH.1        phiH.2
-0.0006017066  4.4664322274 -4.5025387213
Smoothing parameter: gamma = 0.9837
```

The estimation of the smooth parameter was found at gamma = 0.9837. Such a value for gamma implies a quick speed of transition between the two regimes, as in the SETAR case, no delay was found in the threshold function.

The threshold function is given by:

$$Z(t) = +(1)X(t) + (0)X(t-1) = 0.2106 \tag{4.5}$$

The threshold value was also estimated within positive range of values at a slighter high value at a 2.5%. The estimated results from the SETAR and LSTAR models revealed that investors respond quickly to seize business opportunities available in the market as no delay was found in SETAR and LSTAR models and threshold values were very close. The best fitted residual variance was found to be 0.003847 with AIC of -2364 and MAPE of 4.096%.

From the model output 4.4b, the monthly rates of inflation could be modeled as:

$$X_{t+1} = \begin{cases} -28.474 - 11.025X_t + 1.256X_{t-1}, X_{t-1} \leq 0.2498 \\ 61.625 + 7.548X_t - 2.791X_{t-1}, X_{t-1} > 0.2498 \end{cases} \tag{4.6}$$

Figure 4.11 shows the lags monthly rates of inflation and associated threshold values. Figure 4.12 also shows the regime switching of LSTAR of monthly rates of inflation.

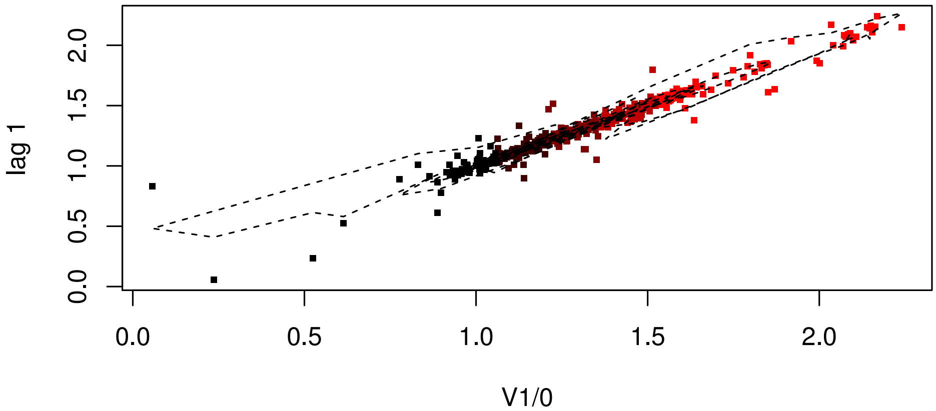

Figure 4.11: LSTAR Representation of Monthly Rates of Inflation(1981-2016)

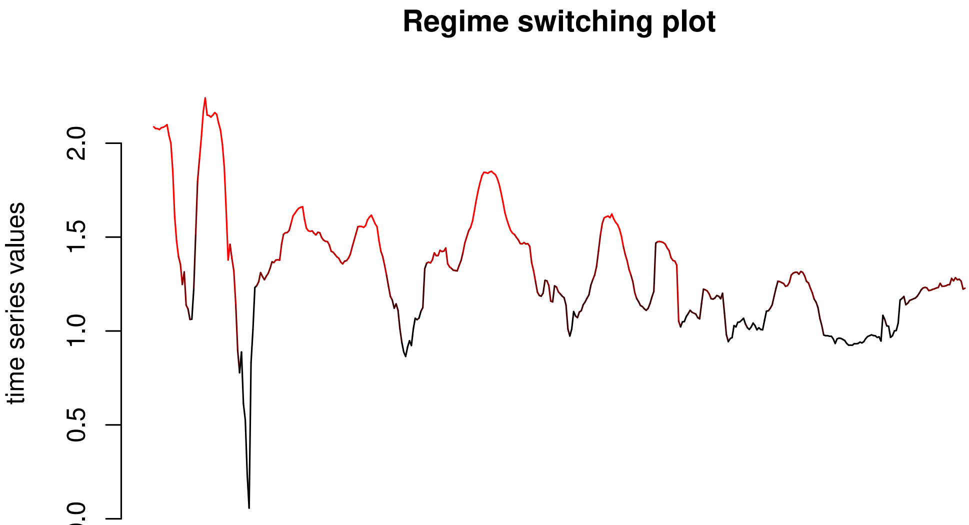

Figure 4.12: Regime Switching of 2-Regime LSTAR Model Representation of Monthly Rates of Inflation (1981-2016)

4.10 Comparing nonlinear and linear models

The best SETAR and LSTAR models within their class of models were compared with the estimated models with AR models as the benchmark. Two AR models were used as benchmark, namely, AR(1) and AR(2). Table 4.7 compared the results based on two criteria, the AIC criterion and BIC criterion for forecasting. Both AIC and MAPE values were obtained from the model behaviour on the sample.

4.11 Model Comparison for the Inflation rates series

Model	AIC	BIC	MAPE
AR1	-2266.0	-2258.1	4.1
AR2	-2330.2	-2318.1	3.4
SETAR	-2375.2	-2346.8	4.2
LSTAR	-2363.1	-2331.4	4.1

Table 4.7: Comparison of Fitted Models

From the Table 4.7 the behaviour of the time series were quite similar. The best models were SETAR and LSTAR in terms of AIC and BIC criteria with SETAR having a slight advantage over LSTAR. Again, from forecasting capability, the ranking changes with the basic AR models ranked as the best, based on MAPE. Moreover,the AR(2) class has a better forecasting performance relative to the LSTAR, SETAR and AR(1) models type. The LSTAR model has a better forecasting performance relative to SETAR model. The nonlinear SETAR model was adjudged as parsimonious model fitted for the data and AR(2) was chosen as the best prediction model for future monthly rates of inflation.

4.12 Diagnostic Checks and Adequacy for Estimated Models

The model diagnostic checks were performed to determine the adequacy of the chosen model. This was done through the analysis for the residuals from the fitted model. If the model fits the data well, the residuals are expected to be random i.i.d. following normal distribution. Plot of residuals such as the histogram, the normal probability plot and the time plot of the residuals were used. The histogram of the residuals as well as the normal probability plot were used to check for normality approximately symmetric. The ACF and PACEF of the standardized residuals were used for checking the adequacy of the conditional variance model.

4.13 Diagnostic Checks and Adequacy for SETAR(2) Model

The time series plot of the residuals was used to check whether residuals were random and the result is given by Figure 4.13. From the plot, the residuals exhibit random variation about their mean and hence it can be concluded that the residuals appear to be random. Also from the normal probability plot of the residuals in Figure 4.14, the plot of the residuals was almost linear. The linearity of the plot implied that the distribution of the residuals is normal. This was confirmed by the histogram of the residuals shown by Figure 4.15. From the Figure 4.15, the histogram is symmetric implying that the residuals have a normal distribution.

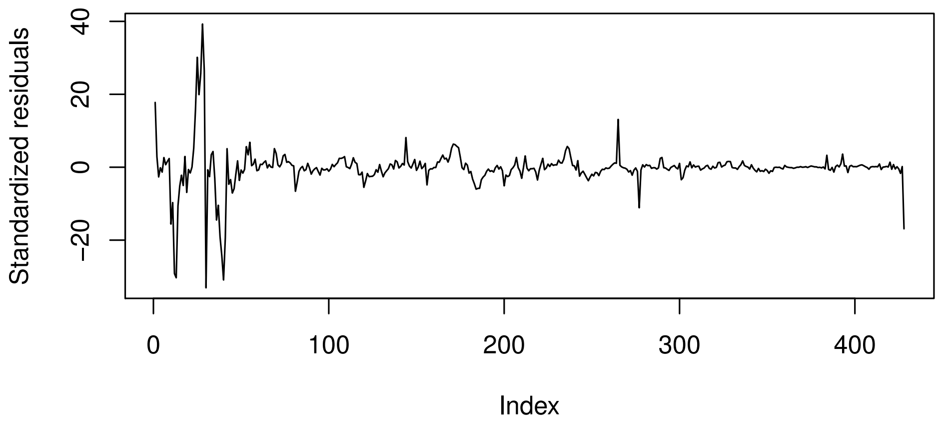

Figure 4.13: Time Series Plot of Residuals

Based on the diagnostic checks above, 2-regime SETAR of order 2 was the most appropriate model to represent the data.

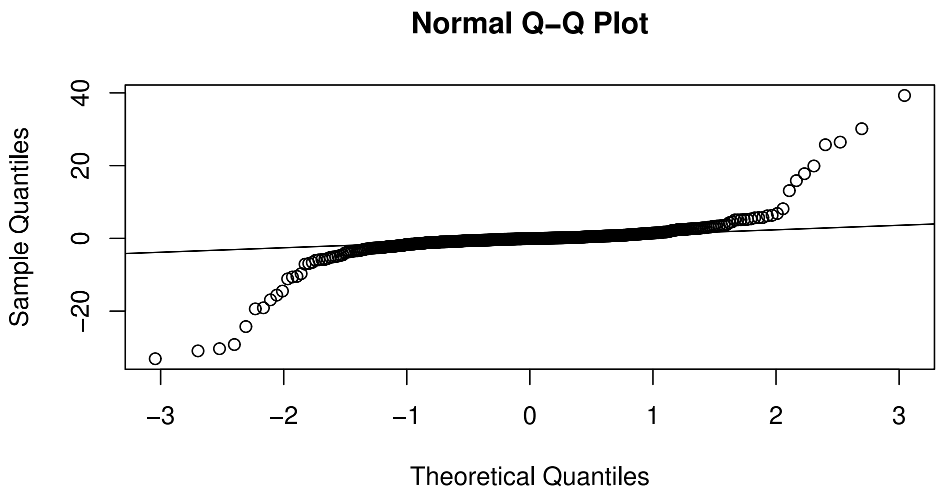

Figure 4.14: Normal probability Plot of Residuals

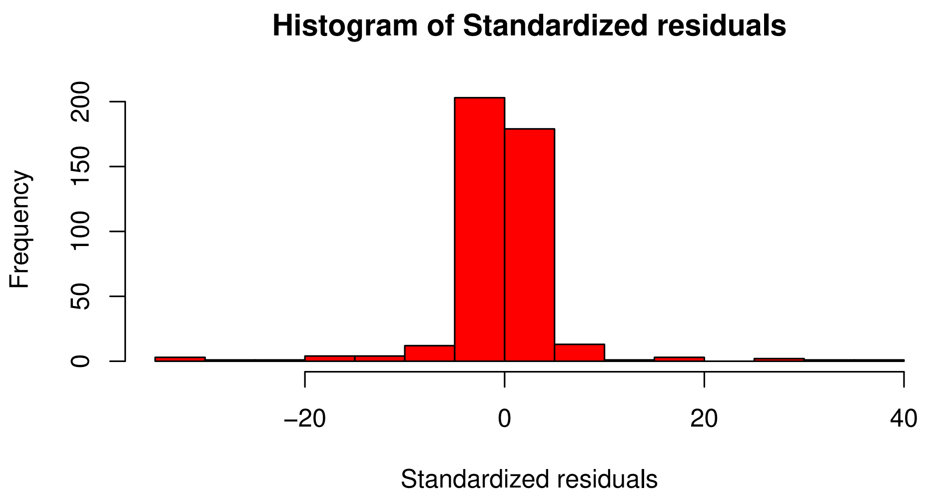

Figure 4.15: Histogram standardized of Residuals

4.14 Forecasting Evaluation and Accuracy Criteria

The models were also evaluated in terms of their forecasting ability of the future monthly rates of inflation. This was necessitated as most previous research had found that the selected model was not necessarily the model that provides best forecasting. Common measure of forecast evaluation of MAPE was used to compare the models. The model that exhibits the lowest value of the error measurement is considered to be the best. Table 4.7 shows the results of the forecast performance of the various models considered. The results from Table 4.7 revealed that AR(2) had the least value of the performance measurement. It had MAPE of 3.42%. Thus, the AR(2) model out-performed all the other models and was adjudged the best performing forecasting model.

4.15 One Year Out-Sample Forecast of Monthly Rates of Inflation

The AR model of order 2 emerged as the optimal and most adequate forecasting model representation of the data. AR(2) model was then used to predict the monthly rates of inflation based on monthly rates of inflation of June and July, 2016 using prediction method called 'point forecast'.

4.16 Forecast Output of One Year Monthly Rates of Inflation (September 2016-August 2017)

Forecast Output

Point	Forecast
429	17.37836
430	18.04823
431	18.81247
432	19.61222
433	20.40861
434	21.17689
435	21.90211
436	22.57612

437	23.19536
438	23.75933

The results from the forecast output revealed that the monthly rates of inflation of the period were high. However, there is high amount of variation in the monthly rates of inflation and this might pose great challenges to other economic variables, such as, exchange rates, stock returns and insurance premium. Also the linear models were superior in forecasting the monthly rates of inflation. The superiority in performance of linear models was attributed to their ability to capture the stochastic nature of the monthly rates as is evident in the pattern of the forecast errors.

Again, looking at the upward trend of the out-sample forecasts, it can be predicted that Ghana would expereince double digit inflation in 2017. This would have several impact on aspects of the economy and could erode the economic gains made in the year 2016. The policy makers (Bank of Ghana) need to put in place coherent monetary and fiscal policies that would put the anticipated increase in inflation under control. Figure 4.16 represents the graphical representation of monthly rates from September, 2016 to August, 2017 financial year.

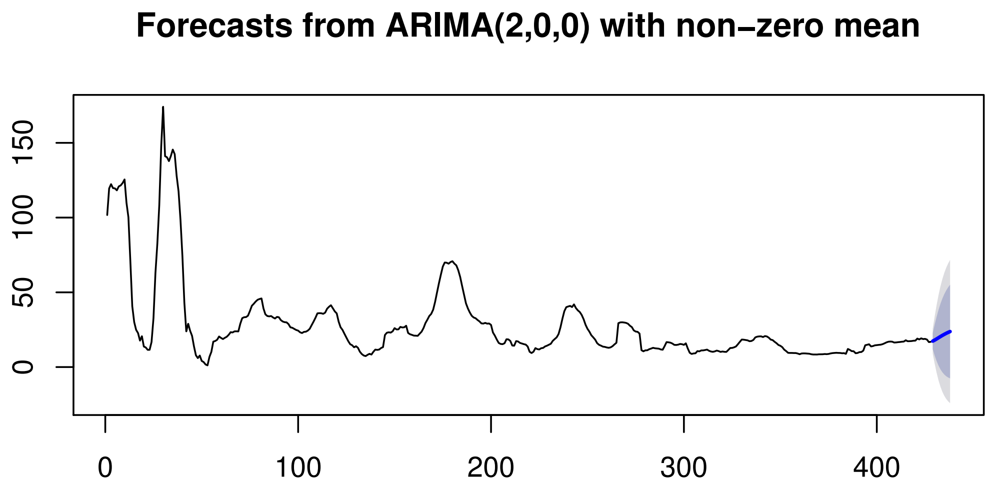

Figure 4.16: Forecast Representation of Monthly Rates of Inflation

Chapter 5 deals with the summary, conclusions and recommendations from the main results and the findings on the performance of the selected nonlinear Models are also presented.

5. Summary, Conclusions and Recommendations

The chapter presents a summary of the findings from the study as well as the conclusions, recommendations and the areas for further research, thus, the main results and the findings on the performance of the selected nonlinear models are presented.

5.1 Summary

Most empirical studies carried out to investigate the inflation rates in Ghana based their argument on the standard economic approach and linear methods such as Autoregressive Integrated Moving Average (ARIMA) models of Box and Jenkins (1976), and Owusu (2010) until the introduction of threshold models. In Ghana, several of such empirical research have been carried out in the area of inflation modeling and forecasting, however, these Box and Jenkin's models are based on the assumption of constant variance, which is uncharacteristic of most financial series.

The study was designed to investigate the existence of nonlinear patterns in the monthly rates of inflation and model inflation rate using nonlinear models, SETAR and LSTAR models, in modeling the monthly rates of inflation in Ghana from January 1981 to August 2016 and in comparison to linear models. Also the study sought to identify the optimal model that would fit the Ghana inflation rate and finally to forecast a one year out-sample forecast based the optimal model.

Series of tentative models were developed for each of the selected nonlinear models based on regime as allowed by the data. After considering the AIC, BIC and the MAPE values of the tentative models, the model with minimum AIC and BIC was adjudged the optimal among the nonlinear family type model; the model with the minimum MAPE was considered as the best forecasting model.

Based on the data analyses, the key findings were summarised as follows: the average monthly rate of inflation for the period under study was 27.80 with standard deviation of 27.67. The high value of the standard deviation implied that, Ghana's monthly rates of inflation always remained high over the period under study.The distribution of the monthly rates of inflation was leptkurtic with long-tail to the right.

The distrbution of monthly rate of inflation was characterised by a nonconstant mean and an unstable variance implying a non-stationary series. Also the series exhibited the presence of heteroscedasticity and autocorrelation.

Two regimes SETAR, LSTAR, AR(1) and AR(2) models were developed. The AIC values for SETAR, LSTAR, AR(1) and AR(2) were -2375.2 and -2363.0, -2266 and -2330.2 respectively whilst the BIC values were also -2346.8, -2331.4, -2258.1 and -2318.1 respectively. Again, the MAPE values were 4.2% and 4.1% respectively, hence, SETAR had the minimum AIC and BIC and was adjudged the best-fitted model for the data. AR(2) had the minimum MAPE and emerged as the optimal forecasting model for the monthly rates of inflation in Ghana.

5.2 Conclusions

The study was designed to ascertain whether there is the existence of nonlinear patterns in the monthly rates of inflation in Ghana. Monthly rates of inflation from January 1981 to August 2016 were used. Based on nonlinearity tests, the Keenan and Tsay tests, the null hypothesis of the linearity was rejected. The threshold models were compared according to their fitness and forecasting performance with AR models as benchmark. It was revealed that the SETAR and LSTAR models fit best the data and SETAR model was selected as the best fit model for the data, however, the nonlinear models could not outperform the simple AR models in terms of forecasting.

The estimated SETAR and LSTAR models can be interpreted economically. The lower regime depicts an increasing phase whilst the upper regime depicts decreasing phase. When the regime is low, investors take advantage and enter into the market to make more profit, as no delay was found in both models.

Lastly, looking at the upward trend of the out-sample forecasts, it can be predicted that Ghana would experience double digit inflation in 2017. This would have several impacts on the aspects of the economy and could erode the economic gains made in the year 2016. The policy makers (Bank of Ghana) need to put in place coherent monetary and fiscal policies that would put the anticipated increase in inflation under control.

5.3 Recommendations

Based on the findings and conclusions from the study, the following recommendations are made both in the areas of policy formulation and further research.

Firstly, policy makers, industry players and all those interested in modeling and forecasting future rates of inflation in Ghana should consider using threshold models instead of the traditional Box and Jenkins models since threshold models are able to capture the nonlinearities in the monthly rates of inflation. Also by using threshold models, policy makers and industry players would be able to estimate and forecast the monthly rates of inflation accurately and prepare ahead of time.

Secondly, in the area of further studies, similar research could be carried out using other threshold models such as smooth threshold autoregressive model (STAR) model and others. This would help identify the best model among the family of threshold models that fit and predict the monthly rates of inflation better for Ghana.

Thirdly, it is recommended that multivariate time series models, where other economic variables that could influence the monthly fluctuation of inflation rate such as exchange rates, interest rates, amount of money supply and others should be modeled along with rates of inflation. The inclusion of these other variables could identify which of them contribute more to the persistent increase in the monthly rates of inflation in Ghana.

Finally, juxtaposing the above mentioned recommendations vis-a-vis the literature review, there is no doubt, if the recommendations are adhered to, would help contribute more to the persistent increase in monthly rates of inflation in Ghana as others have already said in the literature review.

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